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Financial Mathematics

Pricing autocallable structured products

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by

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Abstract

The purpose of this end of master project is to show the methodologies of pricing the autocallable structured products.
Therefore, in this work, we will present the two most common ways of pricing this special type of financial products. First of all, we will highlight the famous Monte Carlo simulations, which reproduce as probable as possible, the future product through simulations. Then we will study the pricing of an autocallable product with the closed form solution which is based on the probability calculations.
In the second part of this work we will compare numerical results of those two methods. In the third part of this work, we will study the risk of a concrete autocallable contract.

Keywords: Autocallable products, Structured products, Monte Carlo methods, Pricing methods, Options, Black Scholes, Risk analysis.

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Contents

Introduction

Arfima Spain is a privately held global proprietary company. All of the trading is exectued for their own account. They are present in a wide range of assets classes, both domestically and internationally. I had the opportunity to spend my off-cycle internship in this company and more precisely in their financial solutions departement. The "Arfima financial solutions" departement provides a wide range of structured products to their investors by pricing and assessing risks related to these products.

In my internship, I focused on the pricing of some structured products mainly reverse convertibles and autocalls. In this work, I will discuss the two pricing methodologies I have been using :

- The Monte Carlo Path Generation
- The Closed Form Solution

Framework

We first start with a product description, presenting derivatives products and some financial notions. Beyond all the structured products, we introduce particularly the so called Autocallable structured products.

In the second part, we will discuss the theoretical aspect in order to understand the assumptions that have been made and the methods which have been used in this work. We will implement the scripts related to the models. Then we will apply this theory on a concrete example taken from real trades done in the OTC market.

In the numerical part, we will study the effectiveness of the different methods and we will show the convergence of the Monte Carlo prices to the closed form solutions prices.

Then, to finish with we will do a little study on a specific autocallable contract risk.

Finally the conclusions and possible studies will be presented for the reader interested in continuing in this field.

Part I

Financial and derviative products

1 The market

1.1 The regulated Market

The regulated market is a market on which are exchanged standardized contracts, regulated by markets authorities. Counterparties do not trade directly with each other but place their buy (bid) and sell (ask) orders on display for all market participants. The market operator defines the rules, authorises membership, organises and supervises trading and ensures the proper functioning of the market and its technical facilities.

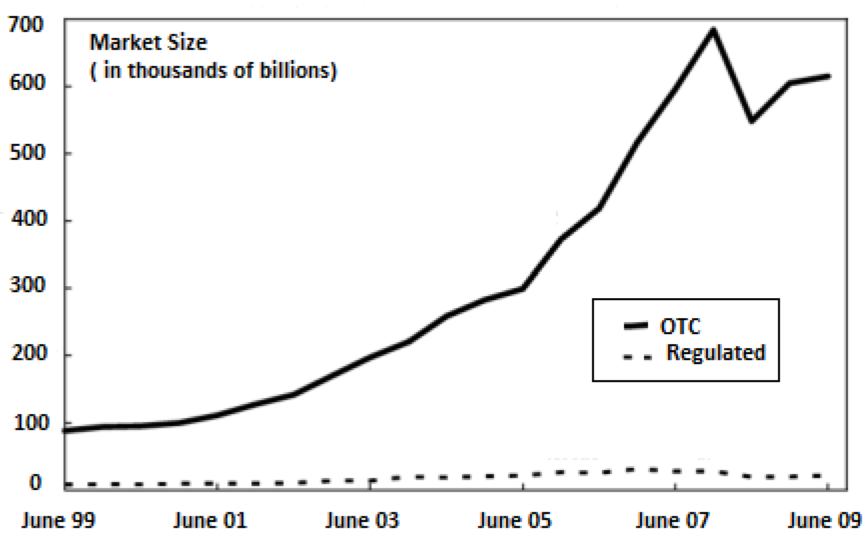
The main element of this regulated market is the clearing room which encompasses all the required steps between the trade and settlement: order verification and matching, compensation and management of the counterparty risk concerning the default of one party in the transaction.

The caracteristics of the regulated market are:

- The existence of a regulation which normalize the contract(underlyings,maturity, strike)
- The existence of a compensation room which realize the contracts liquidation and manage the counterparty default risk by providing to the market the necessary liquidity.
- The availability of a transparent information about the supply and the demand.

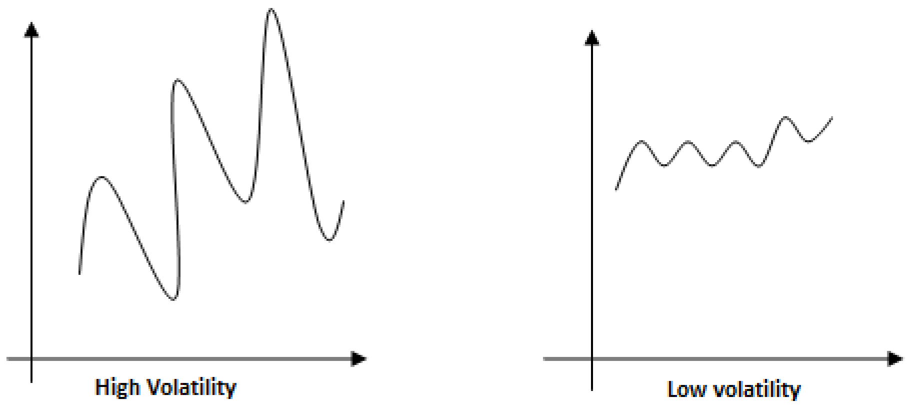
1.2 The OTC Market

The Over The Counter (OTC) market is a market on which exchange is done directly between two parties, whithout any supervision. In an OTC trade, the price is not necessarily made public information. Moreover, contracts are bilateral(only between two parties). OTC derivatives can lead to significant risks. Especially counterparty risk has gained particular emphasis due to the credit crisis in 2007. Counterparty risk is the risk that a counterparty in a derivatives transaction will default prior to expiration of the trade and will not make the current and future payments required by the contract. The advantages of OTC derivatives over exchange-traded ones are mainly the lower fees and taxes, and greater freedom of negotiation and customization of a transaction, as it involves only a seller and a buyer and no standardization authority. All the exotics products are traded on the OTC market.



2 The concept of volatility

The volatility is a statistical measure of the dispersion of returns for a given security or market index. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. Commonly, the higher the volatility, the riskier the security. In other words, the volatility is the measure of range of motions of an underlying.



Moreover, the volatility is one of the parameters in the option pricing formulas, showing the extent to which the return of the underlying asset will fluctuate between now and the option's expiration. Volatility, as expressed as a percentage coefficient within option-pricing formulas, arises from daily trading activities.

2.1 Historical volatility

The realized volatility of a financial instrument over a given time period corresponds to the volatility realized in the past. Generally, this measure is calculated by determining the average deviation from the average price of a financial instrument in a given time period in the past. From a mathematical point of view, standard deviation of percentage changes in price is used to calculate historical volatility within the considered timeframe.

For the pricing, the realized volatility is generally calculated when there is no option prices available. The calculation is made from the last 180 days applying the methodology widely used by variance swap market participants,

$$\sigma^2_{realized} = \frac{252 * \sum_{i=1}^{N} Return_i^2}{N}$$

and,

$$\sigma_{realized} = \sqrt{\sigma^2_{realized}}$$

where:

$$Return_i = Ln(\frac{S_t}{S_{t-1}})$$

N is the number of trading days over the period.

2.2 Implied volatility

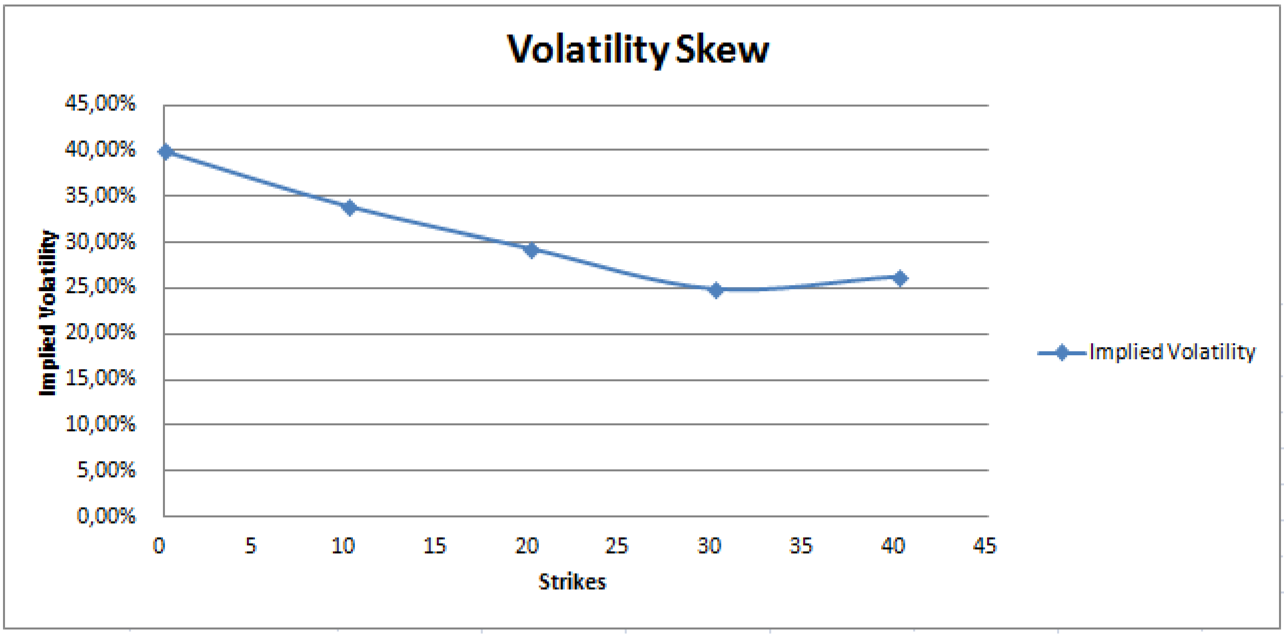
The one parameter on Black-and-Scholes pricing formulas that cannot be directly observed is the volatility of the stock price. Market participants usually work with what is known as *implied volatilities*, which are the volatilities implied by option prices observed on the market.

In financial mathematics, the implied volatility of an option contract is that value of the volatility of the underlying instrument which, when input in an option pricing model will return a theoretical value equal to the current market price of the option. The implied volatility shows how volatile the market might be in the future. In general, implied volatility increases when the market is bearish and decreases when the market is bullish. This is due to the common belief that bearish markets are more risky than bullish markets.

2.3 The volatility skew

The Black scholes model assumes the volatility constant which is not very realistic. This is at odds with what happens in the market where traders know that the formula misprices deep in-the-money and deep out the-money options. The mispricing is rectified when options (on the same underlying with the same expiry date) with different strike prices trade at different volatilities. Traders say volatilities are skewed when options of a given asset trade at increasing or decreasing levels of implied volatility as you move through the strikes. The empirical relation between implied volatilities and exercise prices is known as the volatility skew. A plot of the implied volatility of an option as a function of its strike price is known as a volatility skew. The volatility skew used by traders to price options has this form:

Strikes	0	10	20	30	40
Maturity(years)	1	1	1	1	1
Implied Volatility	40,09%	34,02%	29,39%	24,93%	26,28%



The volatility decreases as the strike price increases.

3 Basics about Options

An option is a derivative financial instrument that specifies a contract between two parties. It gives the right not the obligation to buy or sell a specific financial product officially known as the option's underlying instrument. The contract itself is very precise. It establishes a specific price, called the strike price, at which the contract may be exercised, or acted on. And it has an expiration date. When an option expires, it no longer has value and no longer exists.

There are two basic types of options:

- **Call Option:** The holder has the right to buy the underlying asset at a certain time(the expiration date) for a certain price(the strike price).
- **Put Option:** The holder has the right to sell the underlying asset at a certain time(the expiration date) for a certain price (the strike price).

The holder pays a fee (called a premium) for this right.

Options can have different types of features:

- **European type:** The option can be exercised only at the expiry date.
- **American type:** The option can be exercised at any time until expiry date
- **Autocallable type:** The option may terminante prior to maturity due to a barrier condition one or several underlyings.

In this thesis, we will look in more details the autocallable type.

Options terminology

- S_0: is the spot price, the current price of the underlying
- K: the strike price is the pre-agreed price at which the underlyings are exchanged if the option is exercised.
- T: is the time before maturity. It represents the time remaining to exercise the option and it is expressed in days.
- σ : The volatility measures the variations of the spot price. It is expressed in percentage.
- r : The risk free interest rate.

Options Payoffs

The payoff is the pay the option's holder receives when exercised. This payoff depends on the position we have on the option and the type of option we have. We have to distinguish between two types of positons: The Long and the short position.

- Call Payoffs :

If we are **long a call**, it means that we are buying a call (see Figure 1.).

Mathematically the payoff is written as :

$$\text{Payoff} = \max(0, S - K)$$

If we are **short a call**, it means that we are selling a call (see Figure 2.).

Mathematically the payoff is written as :

$$\text{Payoff} = \min(0, K - S)$$

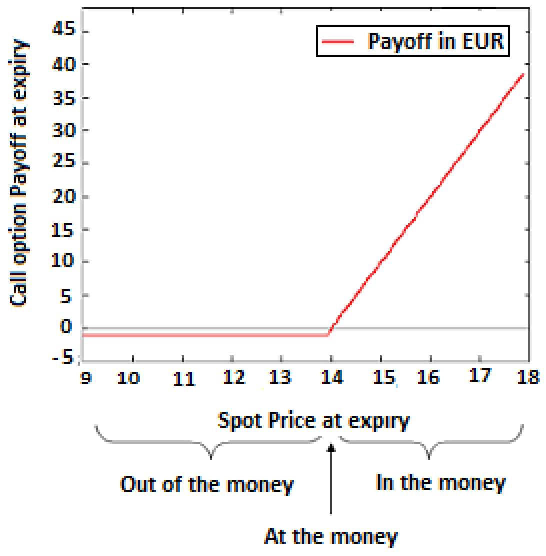


Figure 1. Long Call, Strike K=14

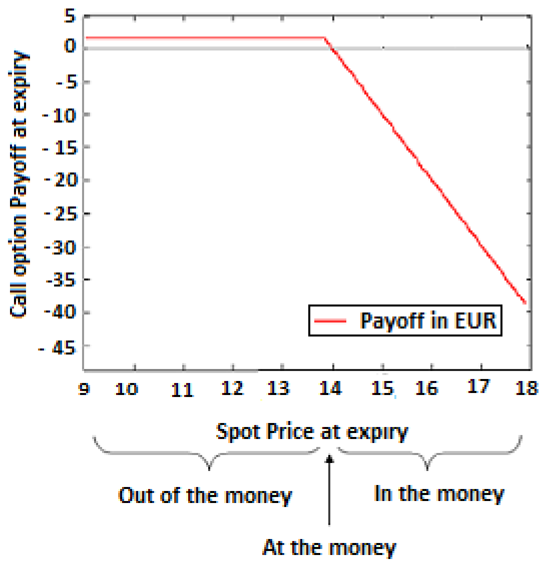


Figure 2. Short Call, Strike K=14

- Put Payoffs :

If we are **long a put**, it means that we are buying a put (see Figure 3.).

Mathematically the payoff is written as :

$$\text{Payoff} = \max(0, K - S)$$

If we are **short a put**, it means that we are selling a put (see Figure 4.).

Mathematically the payoff is written as :

$$\text{Payoff} = \min(0, S - K)$$

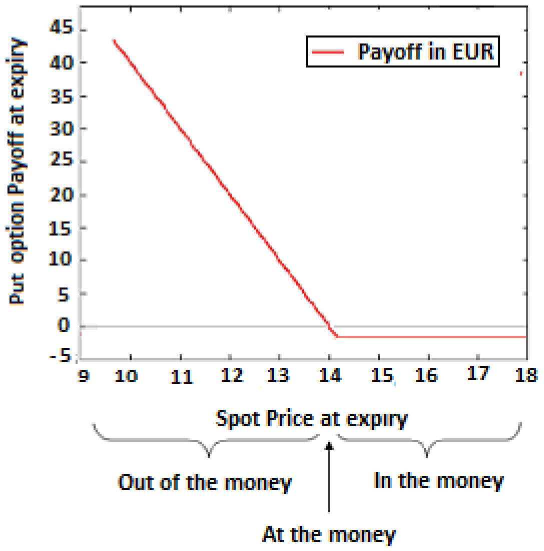


Figure 3. Long Put, Strike K=14

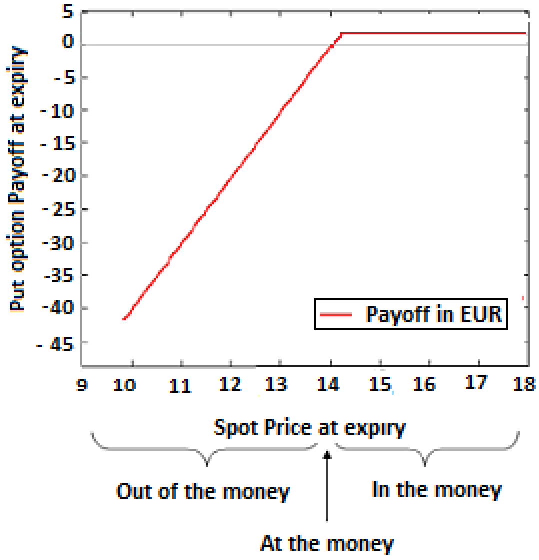


Figure4. Short Put, Strike K=14

4 Standard structured products

In structured finance, a structured product, also known as a market linked investment, is generally a pre-packaged investment strategy based on derivatives, such as a single security, a basket of securities, options, indices, commodities, debt issuance and/or foreign currencies, and to a lesser extent, swaps. The variety of products just described is demonstrative of the fact that there is no single, uniform definition of a structured product.

The standard structured product is a combination of mainly two products. The first element is a zero coupon bond, Z_T, also called a discount bond. The bond is bought at a price lower than its face value, and guarantees the repayement of this face value at the time of maturity. The second item in those structured products is an option or a set of options of some underlying assets.

Usually, the difference between the bond price and the face value of the bond is the amount invested in the risky assets, that way, the complete product will be capital guaranteed seen in the long perspective as the investor will, in the worst-case-scenario, receive the notational amount i.e. the face value of the bond.

This difference between the bond price and the face value is in reallity the participation rate.

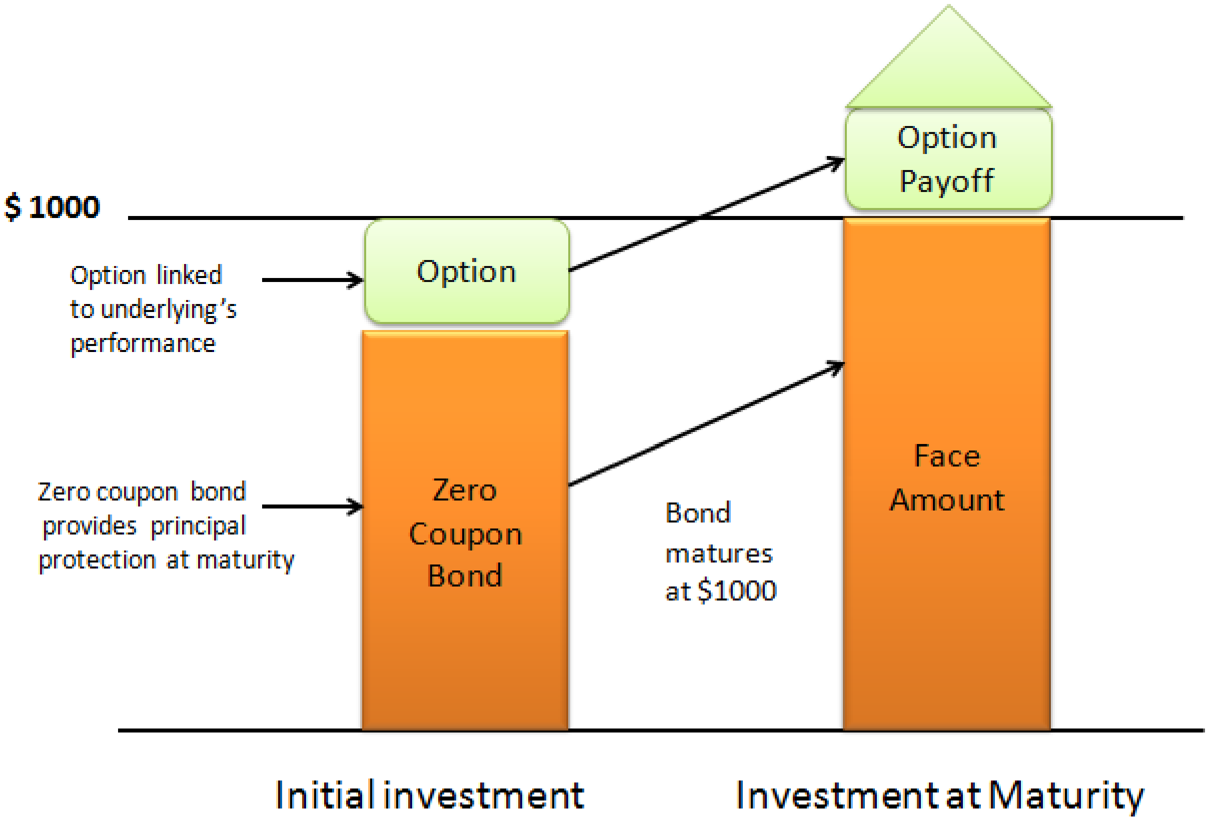


Figure 5. Standard structured product

From this figure we can see how a zero coupon bond can make the product capital protected. The option part represents the possible profit. This part generally adds value to the products that will be payed off at maturity.

5 Autocallable structured products

Since first introduced in 2003, the number of autocallable structured products has increased exponentially and became an often seen investment vehicle in any investors portfolio.

The autocall feature causes the product to be called and redeemed automatically as soon as a certain barrier condition is fulfilled on one of some predefined observation dates. More precisely, the idea of this type of financial instrument is as follows (see Figure 6.)

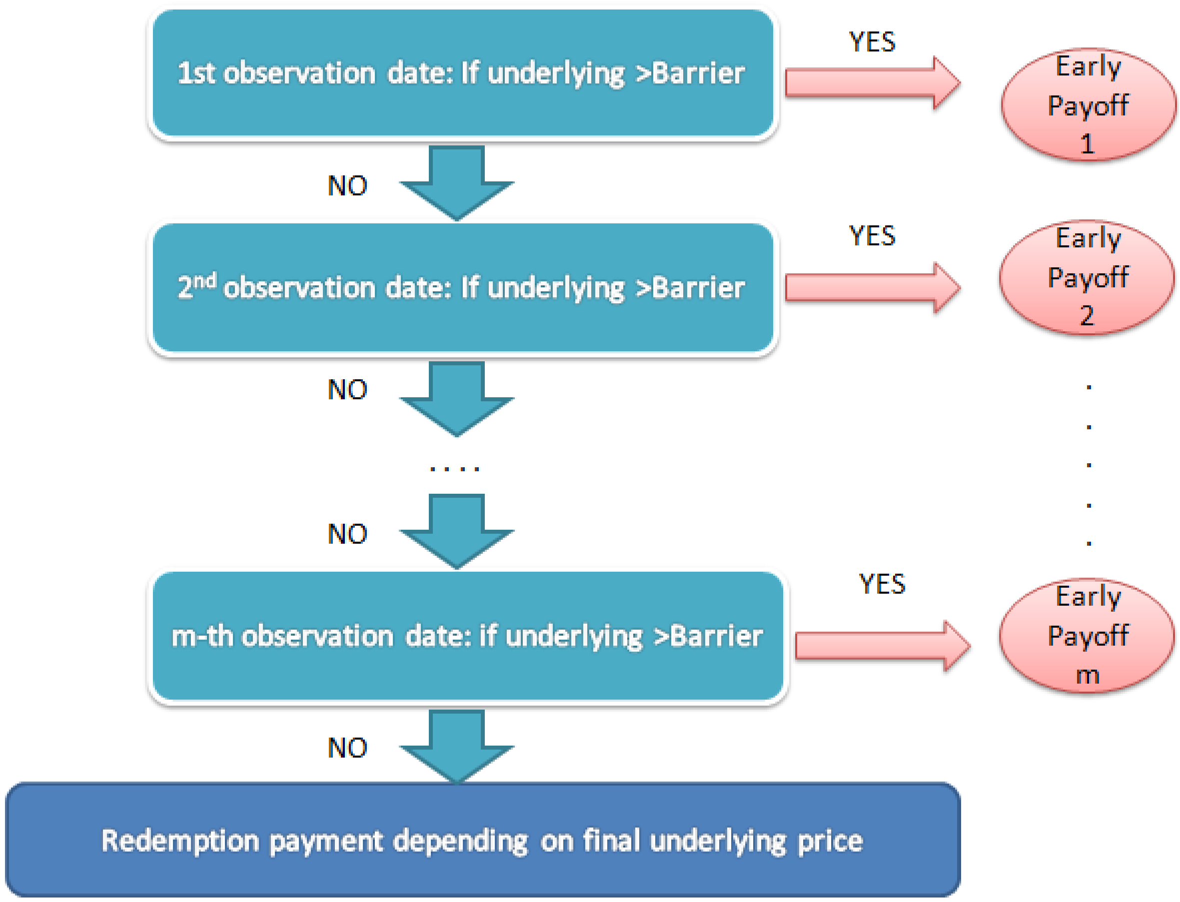


Figure 6. Autocallable payoff profile

The figure below shows the payoff profile of an autocallable structure. At the observation dates, it is checked whether the underlying(s) reaches a certain barrier. If this is the case, the buyer of the autocallable gets the pre-agreed early payoff and the contract terminates. Otherwise the instrument continues to exist until the next observation date. In the case, where the autocallable survives until maturity, the buyer gets a payoff depending on the underlying's performance. Autocallables can be considered as an exotic type of barrier options.

In the following part, we will introduce the concept of pricing autocallables.

Part II

Concepts and assumptions for pricing autocallables

5.1 The Wiener process and Ito Lema

The Wiener process $W(t)$ is a continuous-time stochastic process. This model is often called the brownian motion and is used to describe the behavior of particles and so the progression of the state over the time.

The Wiener process plays an important role both in pure and applied mathematics. In pure mathematics, the Wiener process gave rise to the study of continuous time martingales. It is a key process in terms of which more complicated stochastic processes can be described. As such, it plays a vital role in stochastic calculus and diffusion processes. In applied mathematics, the Wiener process is used to represent the integral of a Gaussian white noise process, and so is useful for pricing and modeling financial products.

Characterizations of the Wiener process

The Wiener process is characterized by three properties:

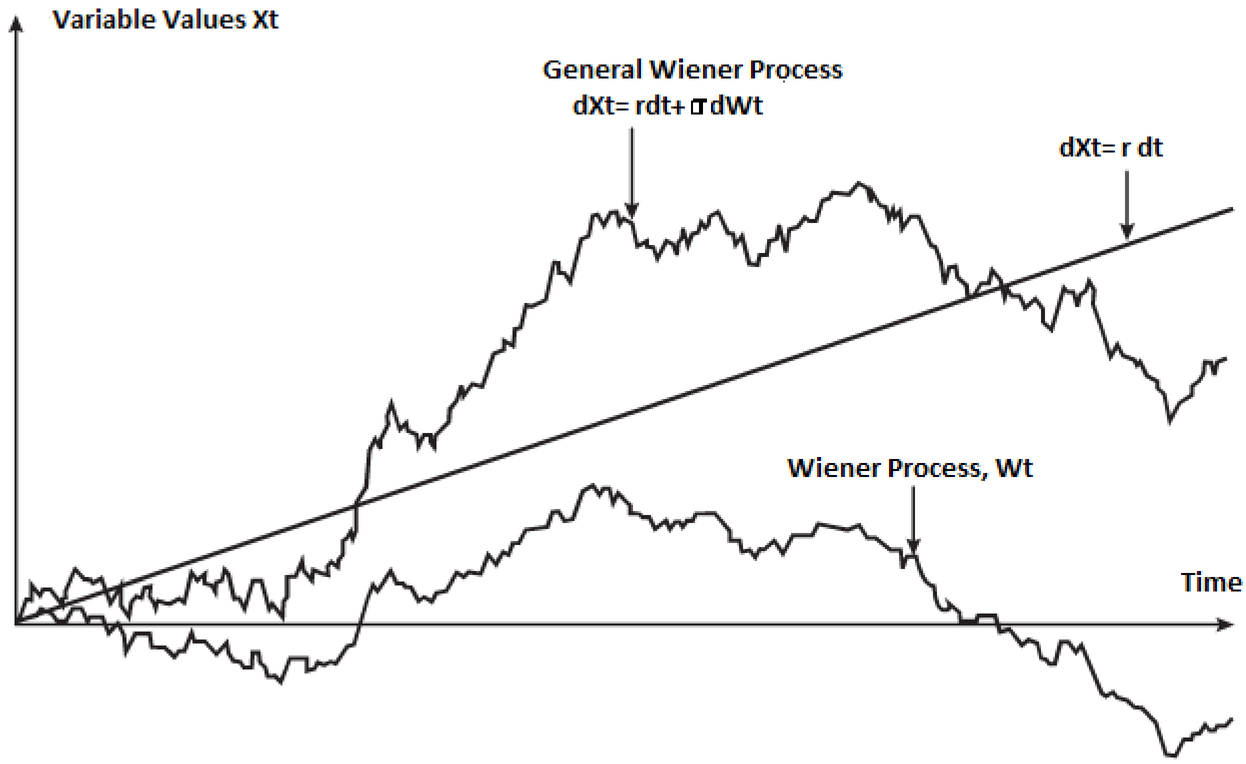
- $W_0 = 0$
- The function t$\rightarrow$ W_t is almost surely everywhere continuous.
- W_t has independant increments with $W_t - W_S \sim N(0, [t-s])$ where $N(r, \sigma^2)$ denotes the normal distribution with r representing the expected value and σ^2 representing the variance.

In general, the Wiener process is defined with this expression:

$$dx = rdt + \sigma dWt$$

where σ is known as the volatility of the process.

To understand the equation above, here is a representation of the Wiener process.



Parameters

The process for a stock price involves two parameters, r and σ. The parameter r is the expected return (annualized) earned by an investor in a short period of time. In most of the cases, higher is the expected return, higher is the risk taken by investors. It follows that the value of r should depend on the risk of the return from the stock. It also depends on the interest rate of the economy. The parameter σ is the volatility and is important to the determination of the value of many derivatives.

Ito Lemma

The value of a stock option is a function of the stock price and the time. More generally, we can say that the price of any derivative is a function of the stochastic variables underlying the derivative and time [2].

We suppose that the value of a variable x follows the Ito process:

$$dx = r(x,t)dt + \sigma(x,t)dWt$$

Ito lemma shows that a function G of x and t follows this process:

$$dG = \left(\frac{\partial G}{\partial x}r + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2\right)dt + \frac{\partial G}{\partial x}\sigma dWt$$

Thus by identification , we recognize that G has a drift rate of :

$$\frac{\partial G}{\partial x}r + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2$$

and a variance rate of:

$$(\frac{\partial G}{\partial x})^2\sigma^2$$

(See Appendix for the proof of the Ito lemma)

If we note that S the stock price at time t , we have that the variation of a stock price follows the following model :

$$dS = rSdt + \sigma SdWt$$

with r and σ constant.

From the previously introduced Ito lemma, we get the process followed by a function G of S and t is :

$$dG = \left(\frac{\partial G}{\partial S}rS + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial G}{\partial S}\sigma SdWt$$

This stochastic differential equation is very important in the derivation of the Black scholes results.

The lognormal property

We define :

$$G = lnS$$

Since,

$$\frac{\partial G}{\partial S} = \frac{1}{S}, \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2}, \frac{\partial G}{\partial t} = 0$$

Then remplacing in this equation,

$$dG = \left(\frac{\partial G}{\partial S}rS + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial G}{\partial S}\sigma SdWt$$

We get ,

$$\mathrm{d}(\ln S) = (r - \frac{\sigma^2}{2})dt + \sigma dWt$$

After integrating between time 0 and some future time T, we have:

$$ln(S_T) = ln(S_0) + (r - \frac{\sigma^2}{2})dt + \sigma dWt$$

Thus, we have the following formula for a stock price

$$S_T = S_0 \times e^{(r-\frac{\sigma^2}{2})T+\sigma W_T}$$

5.2 The Black Scholes Model

In the early 70's, F.Black and M.Scholes achieved a major breakthrough in the pricing of stock options. This involved the development of the famous Black Scholes model. The model of stock price behavior used by Black Scholes and Merton is the model we introduced previously.

Hypotesis

The hypotesis used in the Black Scholes model are:

- The stock price follows the process developed previously with r and σ constant.
- There are no transactions costs or taxes. All securities are perfectly divisible.
- There are no dividends during the life of the derivative.
- There are no riskless arbitrage opportunities
- The risk free rate of interest, r is constant and the same for all maturities.

  Some of these assumptions can be relaxed. For example σ and r can be known as functions of t, also the interest rate can be stochastic provided that the stock price distribution at maturity of the option is still lognormal.

Black Scholes Partial differential equation (PDE)

We introduce V(S, t), the price of a derivative as a function of time and stock price. We define a portfolio Π where the holder is short one derivative and long an amount $\frac{\partial V}{\partial S}$ of shares :

$$\Pi = -V + \frac{\partial V}{\partial S} S$$

The change $\Delta\Pi$ in the value of the portfolio in the time interval is:

$$\Delta\Pi = -\Delta V + \frac{\partial V}{\partial S} \Delta S$$

We have that :

$$\Delta S = rS\Delta t + \sigma S \Delta W t$$

and from the Ito lemma,

$$\Delta V = \left(\frac{\partial V}{\partial S} rS + \frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial V}{\partial S}\sigma S \Delta Wt$$

Thus after substitution, we get:

$$\Delta \Pi = \left(-\frac{\partial V}{\partial t} - \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t$$

We also have that :

$$\Delta \Pi = r\Pi \Delta t$$

Then,

$$\left(\frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(V - \frac{\partial V}{\partial S} S\right)\Delta t$$

Finally, we get the famous Black Scholes PDE:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 + \frac{\partial V}{\partial S} S = rV$$

With the assumptions of the BlackScholes model, this second order partial differential equation holds for any type of option as long as its price function V is twice differentiable with respect to S and once with respect to t.

Vanilla option's pricing

We note : C the price of a call option and P the price of a put option.

Call

As introduced previously, we have that the payoff of a call is :

$$\text{Payoff}(S_T) = \text{Max}(S_T - K, 0) = (S_T - K)_+$$

The price of a call follows the following formula:

$$C = E(\text{Payoff} \times e^{-rT})$$

$\Rightarrow$

$$C = e^{-rT} \times E(\text{Payoff})$$

$\Rightarrow$

$$C = e^{-rT} \times E((S_T - K)_+)$$

$\Rightarrow$

$$C = e^{-rT} \times \int_{-\infty}^{+\infty} \left(S_0 \times e^{(r - \frac{\sigma^2}{2}T + \sigma W_T)} - K \right)_+ \times \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

Then ,

$$C = S_0 \times N(d_1) - Ke^{-rT} N(d_2)$$

With $d_1 = \frac{ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

In the following picture , we show the behavior of the Call price regarding to the spot price and the time to maturity:

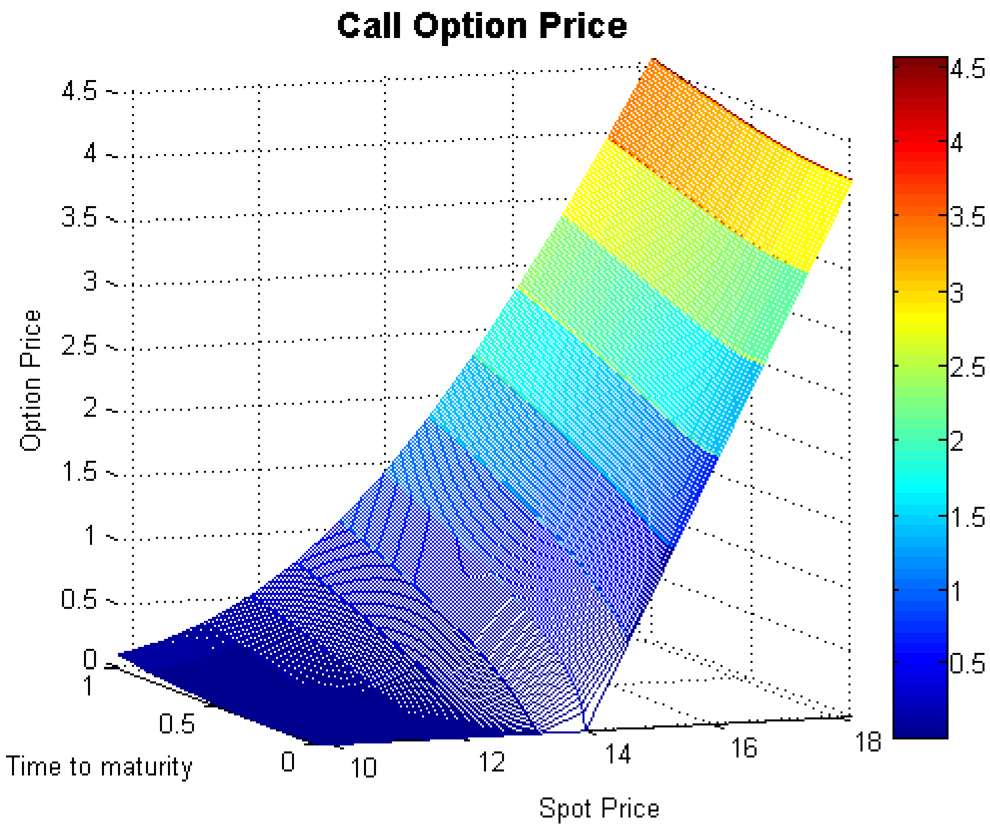
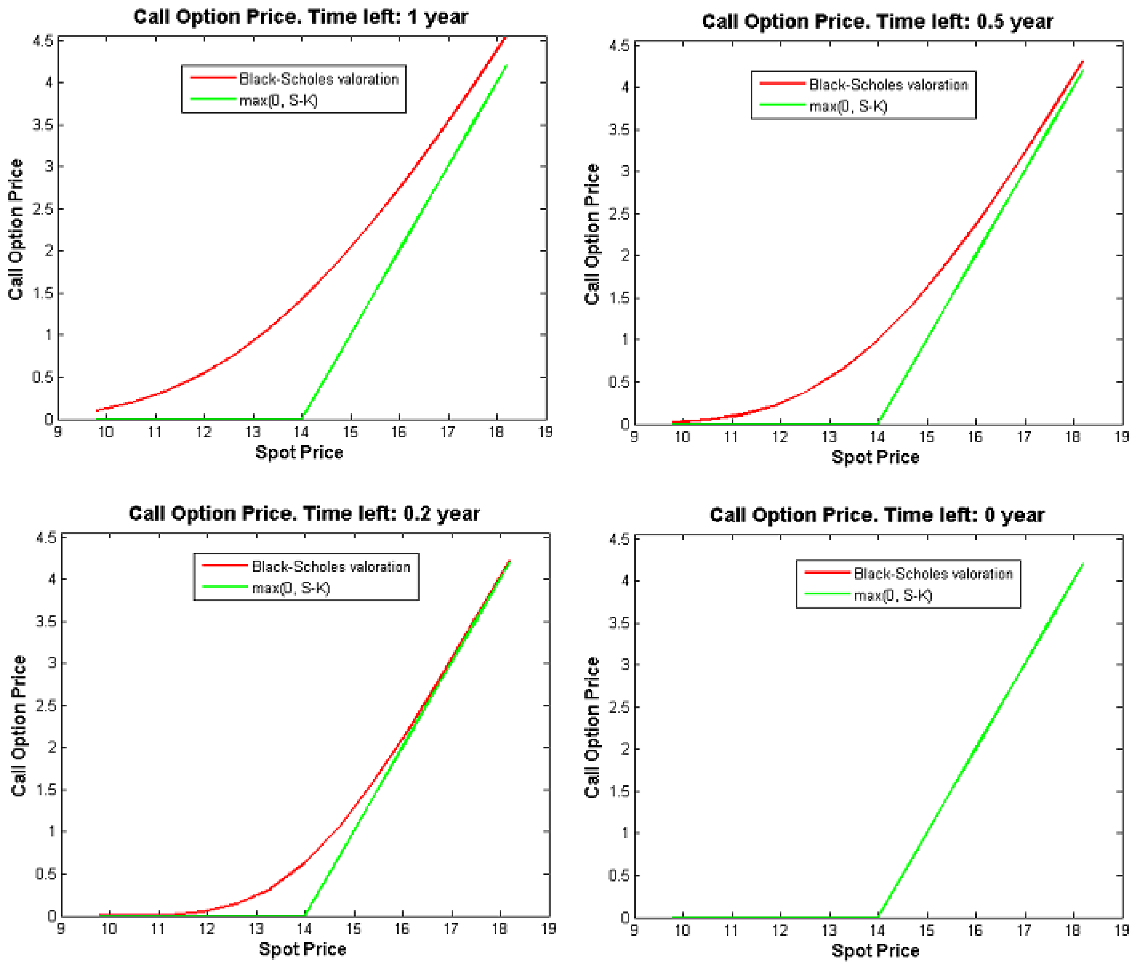


Figure 8. Call price in function of Spot and Maturity

The parameters used in this call pricing are :

- The maturity T= 1 year
- The Strike K= 14 euros
- The volatility $\sigma = \frac{25}{100}$
- The risk free rate r=$\frac{0.5}{100}$

Call convergence study :



We can see that call price converges to the prefixed payoff.

Put

The payoff of a put is :

$$\text{Payoff}(S_T) = \text{Max}(K - S_T, 0) = (K - S_T)_+$$

The price of a put follows the following formula:

$$P = E(\text{Payoff} \times e^{-rT})$$

$\Rightarrow$

$$P = e^{-rT} \times E(\text{Payoff})$$

$\Rightarrow$

$$P = e^{-rT} \times E((K - S_T)_+)$$

$\Rightarrow$

$$P = e^{-rT} \times \int_{-\infty}^{+\infty} \left(K - S_0 \times e^{(r - \frac{\sigma^2}{2} T + \sigma W_T)} \right)_+ \times \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

Then ,

$$P = K e^{-rT} N(-d_2) - S_0 \times N(-d_1)$$

With $d_1 = \frac{ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

In the following picture , we show the behavior of the Put price regarding to the spot price and the time to maturity:

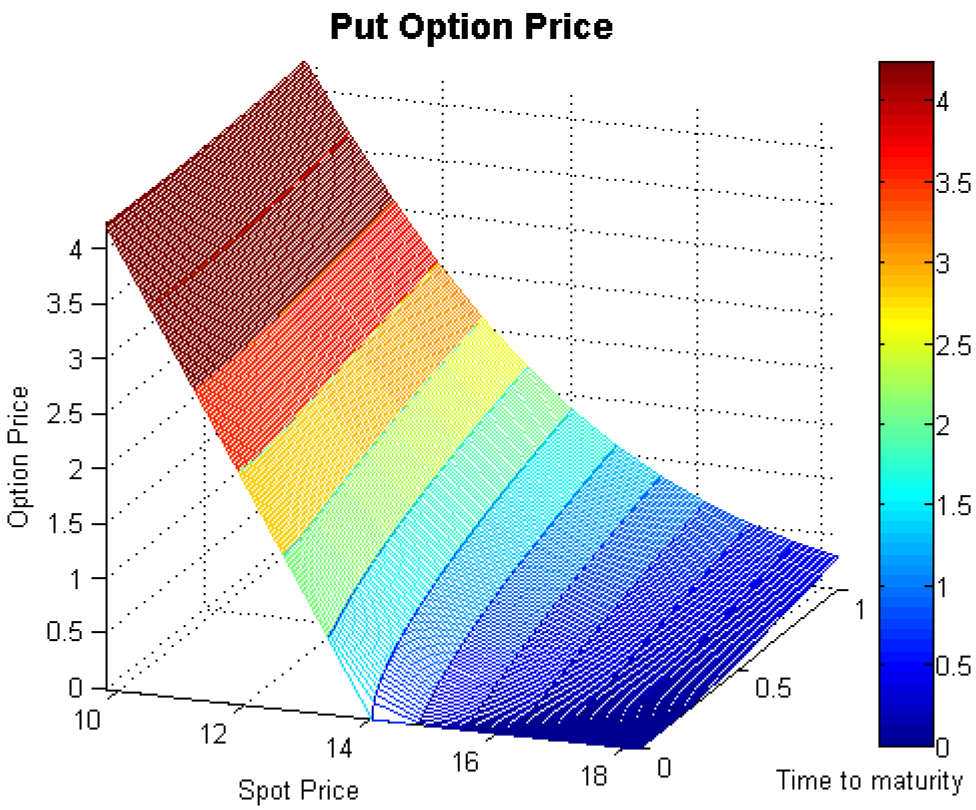
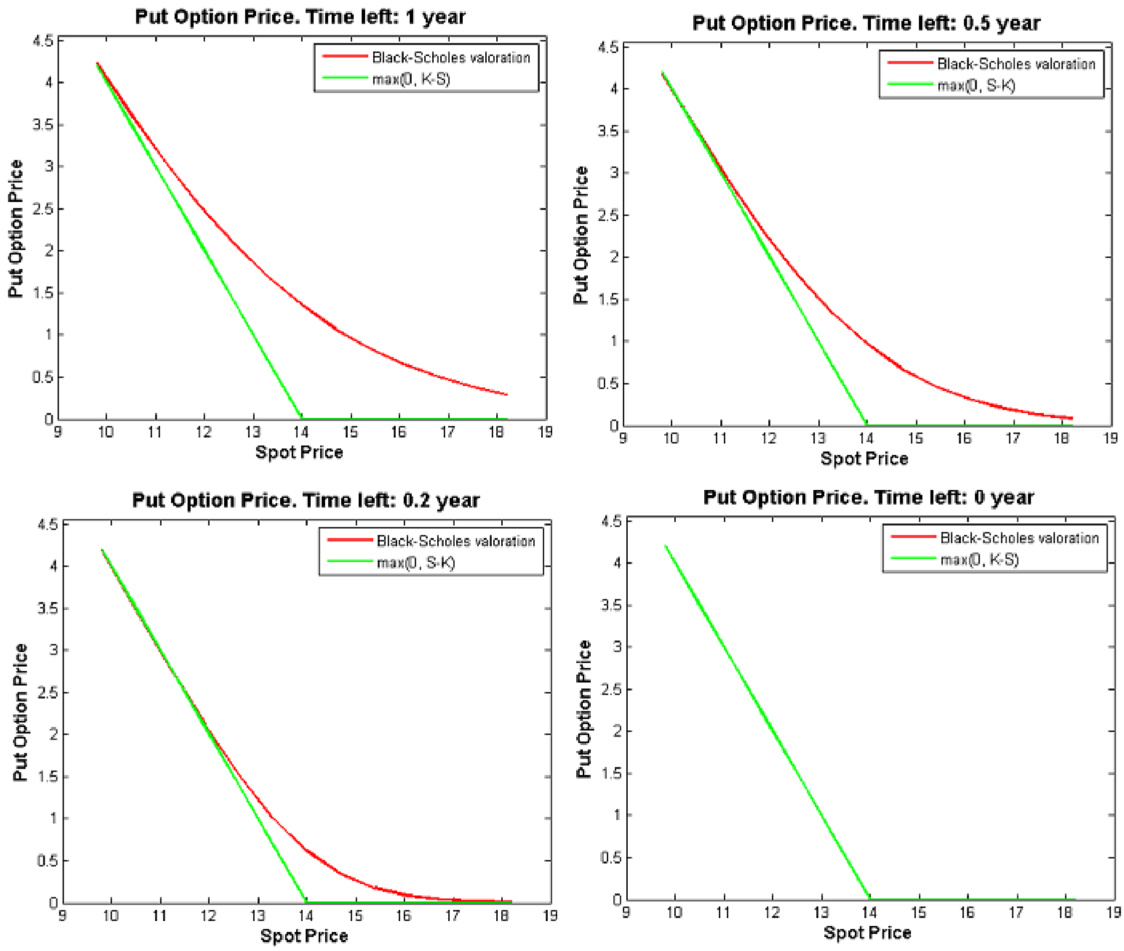


Figure 9. Put price in function of Spot and Maturity

Put convergence study :



We can see that put price converges to the prefixed payoff.

5.3 Modeling Autocallable structured products with PDE

It was recently presented by Geng Deng, Joshua Mallett and Craig McCann [3] that it is possible to price an Autocallable structure by using a flexible PDE. Their valuation model follows the Black-Scholes framework presented previously with risk-neutral assumptions.

This method is useful only in the single underlying asset case. In general, the PDE is a very efficient and useful method. It is rewritten as an ordinary heat equation and solved by well known method as the finite difference for example. Indeed, the finite difference allows to determine the expected return for such a product without having to simulate any scenarios.

By the way, in this work we have choosen to study an alternative approach to valuing discrete autocallables. We renamed it the closed form solution and it consists in calculating the probability of the autocall being exercised on each call dates.

5.4 The Closed Form Solution(theory)

Another way to estimate the value of an autocallable structured product is by calculating the probability of the autocall on each observation dates. Let p_i for $i = 1, ..., n$ be the probability of the autocall being exercised at time t_i^c. Then, the probability of the autocall never being exercised is then $1 - \sum_{i=1}^{n} p_i$ where each p_i is conditional on the autocall not being exercised at any previous autocall date $(t_1^c, ..., t_{i-1}^c)$.

We recall that an asset price $S_{t_i^c}$ follows a lognormal distribution [4]:

$$S_{t_i^c} = S_{t_{i-1}^c} e^{(r-\frac{1}{2}\sigma^2)\Delta t_i^c + \sigma\Delta\sqrt{t_i^c} W_i}$$

Where Δt_i^c is the time between two autocall dates $\Delta t_i^c = t_i^c - t_{i-1}^c$ and W_i where $i = 1, ..., n$ are i.i.d standard normal variables. To simplify notation, we use $X_i = (r - \frac{1}{2}\sigma^2)\Delta t_i^c + \sigma\Delta\sqrt{t_i^c} W_i$ to represent the continuously compounded return from t_{i-1}^c to t_i^c. This means that the final stock price is written as :

$$S_T = S_0 e^{\sum_{i=1}^{n}(r-\frac{1}{2}\sigma^2)\Delta t_i^c + \sigma\Delta\sqrt{t_i^c} W_i}$$

$$S_T = S_0 e^{\sum_{i=1}^{n} X_i}$$

Because of the price's Markov property the X_i's are pairwise independent. Furthermore, if Δt_i^c is a constant, the X_i's are i.i.d normal variables. Then, the probability of the autocall being exercised at time t_i^c can now be written as:

$$p_i = \text{Prob}\Bigg(S_{t_j}^c < C, \quad j = 1, 2, ..., i-1, \quad \text{and} \quad S_{t_i}^c \geq C\Bigg)$$

$$p_i = \text{Prob}\Bigg(\sum_{k=1}^{j} X_k < log\bigg(\frac{C}{S_0}\bigg), \quad j = 1, 2, ..., i-1, \quad and \sum_{k=1}^{i} X_k \geq log\bigg(\frac{C}{S_0}\bigg)\Bigg)$$

$$p_i = \int_{\sum_{k=1}^{j} x_k < log(\frac{C}{S_0}), j=1,2,...,i-1, \sum_{k=1}^{j} x_k \leq log(\frac{C}{S_0})} .... \int g(x_1, ..., x_n) dx_1 dx_2 ... dx_n$$

Where $g(x_1, ..., x_n)$ is the joint probability density function of $X_1, ..., X_n$. Because the X_i's are independent. Then we can estimate the product's present value as :

$$V = \sum_{i=1}^{Number\ of\ autocall\ Dates} p_i \times \text{Rebate}_i \times e^{-rT_i}$$

5.5 The Monte Carlo Simulation (theory)

We use the Monte Carlo method to price products when a closed formula is not available or depends on too many inputs (and is therefore less robust). Monte Carlo method's principle is to sample random paths for a given underlying, calculate the payoff of the product for the underlying path obtained and repeat the process until it becomes robust enough. For each simulation, we evaluate the final payoff of the product by discounting the cash flow with the corresponding CDS and risk free rate. Lastly, we calculate the mean of all payoffs to get the final value of the structured product. Monte Carlo is based on the Law of Large Numbers and is very useful as all products can be priced with the right script.

The following inputs are used to simulate the underlying(s) paths,

- Implied Volatility of the underlying(s).
- Last price of the underlying(s).
- Risk Free Interest Rate to simulate the drift of the underlying(s).
- Dividend Yield to simulate the drift of the underlying(s) when relevant.
- Correlation Matrix of the underlyings if it is a multiple underlying product.
- Expiry and Settlement Date of the product.
- The step used is 1-trading day, i.e. dt = 1=252.
- Number of simulations is 1000.

Since there is only one possible implied volatility for the Monte Carlo simulation under the Black- and-Scholes model, we are using ATM (At-The-Money) implied volatility of the corresponding expiry. Each underlying asset is considered as a FOR/DOM pair. The drift of each asset is calculated as the difference on returns between FOR and DOM as follows,

$$r = R_{\text{DOM}} - R_{\text{FOR}}$$

Then, for each step of each simulation,we generate a multivariate normal distribution matrix W with a zero mean and variance-covariance matrix implied by the correlation matrix and simulate each underlying as follows,

$$S_{j,i,k} = S_{j,i-1,k} \times e^{(r-\frac{\sigma^2}{2})\times dt+\sqrt{dt}\times W_j}$$

Where,

- dt is the time step
- i is the step number
- j is the asset number
- k is the simulation number

As already mentioned, once all the underlyings' paths have been simulated, we set the specifications of the product's payoff (script) and calculate the payoff value for each simulation. We finally take the mean in order to get the final structured product price.

In the next part we will expose the numerical results we got from applying those two methods to a real autocallable contract.

Part III

Application to the autocallable exotic contract:

5.6 General description of our autocallable structure

Below, an example of contract on Telefonica equity on which we will base our numerical results and compare the two methods:

FINAL TERMS

1. **Issuer :** Morgan Stanley
2. **Serie Number :** 4016
3. **Specified currency :** EUR
4. **Specified denomination :** 1000 EUR
5. **Calculation amount :** 1000 EUR
6. **Issue Date :** 03/09/2013
7. **Maturity Date :** 03/09/2016

8. **Obligatory Redemption Date :**
Each of the dates specified as such in item 10 below; provided that if the corresponding scheduled observation date is postponed in accordance with these final terms, the relevant Obligatory Redemption date shall be such date .

10.

Observation Dates	Obligatory Redemption Amount
03/09/2014	110 per cent of the specified denomination
03/09/2015	120 per cent of the specified denomination
03/09/2016	130 per cent of the specified denomination

9. **Obligatory Redemption amount:**
If, on any observation date, the reference price of the equity is , equal to or greater than its autocall level, the obligatory redemption amount payable shall be an amount in EUR equal to the amount specified in the table below.

11.

Issuer Equity	Bloomberg Code	Strike
Telefonica	TEF SM	9.86

For the purpose of the final terms: **Autocall level** means 90 per cent of the strike level.

12. Maturity :
If on the 03/09/2016 the autocall Condition is not fulfilled, you get the 100 per cent of the specified denomination

In order to make the understanding of this contract easier, we will sum it up.

- If at $\underline{T_1 : 03/09/2014}$

$$S_{T_1} > 90\% \quad \Rightarrow \quad \text{Payoff} = \text{Nominal} \times 110\%$$

- If at $\underline{T_2 : 03/09/2015}$

$$S_{T_1} > 90\% \quad \Rightarrow \quad \text{Payoff} = \text{Nominal} \times 120\%$$

- If at $\underline{T_3 : 03/09/2016}$

$$S_{T_1} > 90\% \quad \Rightarrow \quad \text{Payoff} = \text{Nominal} \times 130\%$$

$$S_{T_1} < 90\% \quad \Rightarrow \quad \text{Payoff} = \text{Nominal} \times 100\%$$

As the third date is at the same time an autocallable date and the maturity date, if the contract doesn't autocall it matures.

5.7 The Closed Form Solution(numerical results)

We consider the previous autocallable structured product with three rebates. We want to modelise it with the closed form solution. We set, T1=1, T2=2, T3=3;

We have that :

$$S_{T1} = S_0 \times e^{(W1 \times \sigma + (r - \frac{\sigma^2}{2}))}$$
$$S_{T2} = S_0 \times e^{(W2 \times \sigma \times \sqrt{2} + (r - \frac{\sigma^2}{2}) \times 2)}$$
$$S_{T3} = S_0 \times e^{(W3 \times \sigma \times \sqrt{3} + (r - \frac{\sigma^2}{2}) \times 3)}$$

and our aim is to calculate the following probability: $P(S_{T1} < K_1, S_{T2} < K_2, S_{T3} > K_3)$

Where K_1: condition 1, K_2 : condition 2, K_3 condition 3.

Using : $S_{T1} < K_1$ we get: $W_1 < (\frac{ln(K_1/S0) - (r - \frac{\sigma^2}{2})}{\sigma})$

Using : $S_{T2} < K_2$ we get: $W_2 < (\frac{ln(K_2/S0) - 2 \times (r - \frac{\sigma^2}{2})}{\sqrt{2} \times \sigma})$

Using : $S_{T3} > K_g$ we get: $W_3 > (\frac{ln(K_3/S0) - 3 \times (r - \frac{\sigma^2}{2})}{\sqrt{3} \times \sigma})$

we recall

$$(\frac{ln(K_1/S0) - (r - \frac{\sigma^2}{2})}{\sigma}) = A$$

$$(\frac{ln(K_2/S0) - 2 \times (r - \frac{\sigma^2}{2})}{\sqrt{2} \times \sigma}) = B$$

$$(\frac{ln(K_3/S0) - 3 \times (r - \frac{\sigma^2}{2})}{\sqrt{3} \times \sigma}) = C$$

Considering that $W_1, W_2, W_3 \backsimeq Z_1, Z_2, Z_3 \sim N(0,1)$ we compute this probability :

$$P(Z_1 < A, Z_2 < B, Z_3 > C)$$

Below some details of the correlation calculus :

$$E(W_1W_2) = E(W_1(W_1 + (W_2 - W_1))) = E(W_1^2) = 1$$

$$E(W_2W_3) = E(W_2(W_2 + (W_3 - W_2))) = E(W_2^2) + E(W_2(W_3 - W_2)) = 2$$

$$E(W_1W_3) = E(W_1(W_1 + (W_3 - W_1))) = E(W_1^2) = 1$$

$$corr(W_1, W_2) = \frac{E(W_1W_2) - E(W_1)E(W_2)}{\sqrt{V(W_1)} \times \sqrt{V(W_2)}} = \frac{1}{\sqrt{2}}$$

$$corr(W_1, W_3) = \frac{E(W_1W_3) - E(W_1)E(W_3)}{\sqrt{V(W_1)} \times \sqrt{V(W_3)}} = \frac{1}{\sqrt{3}}$$

$$corr(W_2, W_3) = \frac{E(W_2W_3) - E(W_2)E(W_3)}{\sqrt{V(W_2)} \times \sqrt{V(W_3)}} = \frac{\sqrt{2}}{\sqrt{3}}$$

Then we get the following correlation Matrix :

$$\Sigma = \begin{pmatrix} 1 & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1 & \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} & \sqrt{2}/\sqrt{3} & 1 \end{pmatrix}$$

We then rewrite the probability we have to calculate as :

$$P(Z_1 < A, Z_2 < B, Z_3 > C) = \int_{-\infty}^{A}\int_{-\infty}^{B}\int_{C}^{+\infty} \frac{1}{\sqrt{|\Sigma|(2\Pi)^3}} \times e^{\left(\frac{-1}{2}(x-\mu)\Sigma^{-1}(x-\mu)'\right)}$$

To finish with, the closed form price we get will be:

Product Value $= P(Z_1 > A) \times \text{Rebate}_1 \times e^{-RT_1} + P(Z_1 < A, Z_2 > B) \times \text{Rebate}_2 \times e^{-RT_2} + P(Z_1 < A, Z_2 < B, Z_3 > C) \times \text{Rebate}_3 \times e^{-RT_3} + P(Z_1 < A, Z_2 < B, Z_3 < C) \times \text{Rebate}_4 \times e^{-RT_3}$

Numerical Result:

After running the programm (6.2) in the appendix, which is the closed form solution script for the final terms exposed beyond, we get a price of 1056€. It means that if someone wants to buy this contract, he will have to paye 1056€.

To get this exact price we have used the following parameters and values :

- Strike= 9.8€
- CDS= 175/10000
- Risk free rate = 0.20/100
- Volatility = 25/100

This price is considered as high because the product participate in the underlying's upside performance. In other words, it is a secure contract(Capital protected) and you don't loose any money.

Adding a short put

If we add a short put to the contract, it is obvious that the price we will obtain will be lower. Indeed, by adding the put at the maturity, we increase chances to loose money at the expiration of the contract, that's why the price of the contract falls considerably. Below, thanks to the Matlab code in the appendix (6.4), we experienced what we said above. After running the code, we get a price of 624€.

5.8 The Monte Carlo generation path (numerical results)

After running the programm (6.3) in the appendix, which is the Monte Carlo script for the final terms exposed beyond with , we get with the same parameters values than previously by changing the number of simulations the following prices:

Number of simulations	**Product Value**
50	1075€
500	1070€
5000	1069€
10000	1065€

We can see that the more we increase the number of simulations, the more we converge to the price obtained with the closed form solution. The error we get is about 0.8 %.

Usualy, for the Monte Carlo we use 10 000 simulations to get high precision. However, the velocity of the convergence is in $\frac{1}{\sqrt{n}}$, hence to double the precision, we have to quadruple the number of simulations.

Adding a short put

After running the Monte carlo code with a put for 5000 simulations, we get a price of 855€. Once again, the price has fallen which is logical as explained above.

Part IV

Risk study:

In this part our aim is to study the risk of an autocallable contract. For doing that, we will use two models. The first model we will use is the historical model and the second model will be the normal model.

<u>The historical model</u>

For this model, we started by taking the historical prices for a Telefonica share from the 27/06/2012 until 17/05/2013 [1]. We set the start price of the Telefonica share at 17.4€.

From this data base we calculated the returns "R" (daily variations of S in percentage) and we used the following basic fomula to compute the values of S_1. The formule we used to calculate

S_1 in this model is the following :

$$S_i = S_0(1 + R_i) \qquad \forall i = 1, ..., 226$$

We then noted that the probability is constant in this case, equal to $\frac{1}{227}$.

The next step in our risk study with the historical model, is using the closed form solution introduced previously, to compute the value of the autocall for each value of S_1. We will still use the previous example given above on Telefonica.

We then work out the 5% Var and TailVar.

Results:

HISTORICAL MODEL			
Dénomination Put	624,00 €		
Dénomination No Put	1 056,00 €		
Value Put	89,59%		
Value No Put	104,47%		
Percentile	5%		
Var 5%, Put	87,59%		546,58 €
Var 5%, No Put	104,28%		1 101,22 €
TailVar	5%		
tailvar 5%, Put	86,03%		536,84 €
tailvar 5%, No Put	103,86%		1 096,76 €

Comments :

We see in those results that the value of the no put autocall is higher than the value of the contract which is in line with our previous explanations. Moreover the prices we got with this risk study are more or less the same than the ones calculated previously with both pricers we created.

Concerning the percentile, which consists in finding x such that $P(\text{Autocall} < x) = 5\%$, we found that we have 5% of chances for my autocall contract with put to be lower than the 87.59% of the price found with our pricer (which is around 546,58€) against 5% of chances that the my autocall contract without put be lower than the 104.28% of the price we found (which is around 1101,22€).

Finally for the TAILVAR, we took the average of all the prices below 87.59% with 5% of chances and we found around 86.03% for the autocall contract with put whereas we got the mean of all the cases where we were below 104.28% with 5% of chances around 103.80% .

The normal model

For this model, we started by a discretization of the N values (with N the normal law). We calculated the probability for each N value for example for N=4.8 we computed the following probability $P(-4.9 < N < -4.7)$

Then, in the normal model, we know that:

$$S_1 = S_0 \times e^{\sigma \times \sqrt{\frac{1}{252}} N}$$

and thanks to this equation, we completed the column with the S_1 values.

As before, our next step is to compute thanks to the closed form solution script, the autocall values in function of every S_1. We did this calculation once again for the previous contract with and without put.

Then we calculate VAR and Tailvar:

Results:

Normal Model			
Dénomination Put	624,00 €		
Dénomination No Put	1 056,00 €		
Value Put	93,25%		
Value No Put	105,78%		
Percentile	5%		
Put	91,48%		570,86 €
No Put	105,33%		1 112,31 €
TailVar	5%		
Put	89,47%		558,30 €
No Put	104,79%		1 106,59 €

Comments :

In the previous results , we got almost the same conclusions than with the historical model.

Indeed, we can see that the value of the no put autocall is higher than the value of the contract.

Concerning the percentile, which consists in find x such that $P(\text{Autocall} < x) = 5\%$, we found that we have 5% of chances that my autocall contract with put be lower than the 91.48% against 5% of chances that the my autocall contract without put be lower than the 105.33%.

Finally for the TAILVAR, we took the average of all the prices below 91.48% with 5% of chances and we got around 89.47% for the autocall contract with put whereas we got the mean of all the cases where we were below 105.33% with 5% of chances around 104.79% .

Part V

Conclusion

This report introduces a two main algorithms of pricing autocallable structured products which are the closed form solution and the Monte Carlo simulations. The strengh of the Monte Carlo solution is it's availability to price all very complex options whereas the closed form solution doesn't allow us for instance to price an autocall of a basket of underlyings. However the Monte Carlo simulation is time consumming, especially if the number of simulation is high for instance more than 2 hours to price an autocallable product with 50 000 simulations. This is obviously not the case with the closed form solution, which gives an instantaneous result. We finally mentionned that there is a convergence of the Monte Carlo simulation towards the closed form solution as the number of simulation goes to infinity which is implied by central limit theorem. However, it worths mentionning that we only assumed that the volatility is a constant parameter. We then would suggest to complete, this work, by studying the different models of volatility such as the local volatility model, the stochastic volatility model in order to get more realistic price in line with the market volatility surface.

6 Appendix

6.1 Matlab Code for the call/put price in function of spot and maturity

```
%% OPTION ANALYSIS
clear all
clc

%% INPUT DATA
disp(' ');
disp('Time of maturity (years): ');
T=input(' ');
disp(' ');
disp('Strike: ');
Strike=input(' ');
disp(' ');
disp('Volatility: ');
Sigma=input(' ');
disp(' ');
disp('Risk-free rate: ');
r=input(' ');
disp(' ');
disp('Call or Put: ');
disp('1-> Call');
disp('2-> Put');
CoP=input(' ');

T=T:-0.01*T:0;
Spot=Strike-Strike*0.3:0.05*Strike:Strike+Strike*0.3;

if CoP==1
    aux='Call';
else
    aux='Put';
end
```

```
%% CALCULATION OF THE PRICE
h=waitbar(0,'Please wait');
for i=1:length(T)
    waitbar(i/(length(T)),h);
    for j=1:length(Spot)
        [Call(j,i),Put(j,i)] = blsprice(Spot(j), Strike, r, T(i), Sigma);
    end
end
close(h)

figure(1)
if CoP==1
    mesh(T(1:end),Spot,Call)
else
    mesh(T(1:end),Spot,Put)
end

%% 3D GRAPH (OPTION PRICE IN FUNCTION OF SPOT PRICE AND TIME TO MATURITY)
set(gca,'xdir','reverse');
xlabel('Time to maturity');
ylabel('Spot Price');
zlabel('Option Price');
title([aux ' Option Price'],'FontWeight','bold','FontSize',14);
axis tight;
colorbar('vert')
```

```
%% MOVIE: OPTION PRICE vs SPOT PRICE IN FUNCTION OF TIME TO MATURITY
for i=1:length(T)

figure(2)
if CoP==1
    plot(Spot,Call(:,i),'r','LineWidth',2)
    hold on
    plot(Spot,max(0,Spot-Strike),'g','LineWidth',2)
    axis on
    hold off
else
    plot(Spot,Put(:,i),'r','LineWidth',2)
    hold on
    plot(Spot,max(0,Strike-Spot),'g','LineWidth',2)
    axis on
    hold off
end

ylim([0,max(max(Call))]);

title([aux ' Option Price. Time left: ' num2str(T(i)) ' year'],'FontWeight
xlabel('Spot Price','FontWeight','bold','FontSize',12)
ylabel([aux ' Option Price'],'FontWeight','bold','FontSize',12)
legend('Black-Scholes valoration','max(0, K-S)')
pause;
end
```

6.2 Closed form solution Matlab script without put

```
% Closed form solution for an autocallable product

clear all;close all;


Strike=9.8;
Condition_level_1 =  90/100*Strike;% condition 1
Condition_level_2 =  90/100*Strike;% condition 2
Condition_level_3 =  90/100*Strike;% condition 3

Rebate_1          =  110/100;
Rebate_2          =  120/100;
Rebate_3          =  130/100;

Cds               =  175/10000;          % CDS of the issuer in
ReturnDOM         =  0.25/100;             bps divided by 10000
r                 =  ReturnDOM+Cds     ;% taux d'interet
Vols              =  25/100;  % volatility
```

```
%Create the discount factors
DfAutoCallDate_1 = exp(-(r)* 1);
DfAutoCallDate_2 = exp(-(r)* 2);
DfAutoCallDate_3 = exp(-(r)* 3);


A  = (log(Condition_level_1/Strike)-(r-(Vols^2/2)))/Vols;
B  = (log(Condition_level_2/Strike)-(r-(Vols^2/2))*2)/(sqrt(2)*Vols);
C  = (log(Condition_level_3/Strike)-(r-(Vols^2/2))*3)/(sqrt(3)*Vols);

%Probability calculus DO NOT TOUCH
%-----------------------------------------------------------------
mu                 = [0 0 0];
Sigma              = [1 1/sqrt(2) 1/sqrt(3); 1/sqrt(2) 1 sqrt(2)/sqrt(3)
                                            ; 1/sqrt(3) sqrt(2)/sqrt(3) 1];

x1 = -3:.2:3; x2 = -3:.2:3; x3 = -3:.2:3;
i=find(abs(x1-A)==min(abs(x1-A))) ;
x1(i);
j=find(abs(x2-B)==min(abs(x2-B))) ;
x2(j);
k=find(abs(x3-C)==min(abs(x3-C))) ;
x3(k);
[X1,X2,X3] = meshgrid(x1,x2,x3);
F = mvncdf([X1(:) X2(:) X3(:)],mu,Sigma);
F = reshape(F,length(x2),length(x1),length(x3));

prob_triv        =F(i,j,31) - F(i,j,k); % Trivariate probability
prob_biv         =F(i,31,31)-F(i,j,31); % Bivariate probability
prob_univ        =F(31,31,31)-F(i,31,31);% Univariate probability
%-----------------------------------------------------------------
ProductValue     =prob_univ*Rebate_1*DfAutoCallDate_1+prob_biv*Rebate_2*DfAutoCallDate_2
                  +prob_triv*Rebate_3*DfAutoCallDate_3+F(i,j,k)*DfAutoCallDate_3
```

6.3 Monte Carlo Matlab script without put

```
clear all;close all;

%PUT Global Inputs for Simulation here
%-------------------------------------------------------------

NSim              = 10000;
EvaluationDate    = today;
ExpirationDate    = '09-03-2016';
SettlementDate    = '09-05-2016';
StartPrices      =  10.86;      % Current Values of the underlyings
Vols              = 25/100;
Cds               = 175/10000;            % CDS of the issuer
ReturnDOM         = 0.25/100;

ReturnFOR         = 0/100;


Correls           = 100/100;

dt                =  1/252;             % one trading day
%---------------------------------------------------------------------------


%Price Simulation DO NOT TOUCH
%----------------------------------------------------------------
Drift = ReturnDOM - ReturnFOR;
Vols = Vols;
Sigma = corr2cov(Vols, Correls);
NWorkdays=wrkdydif(EvaluationDate, ExpirationDate, 1);
NWorkdaysAnn = NWorkdays /252;
NObs = round(NWorkdaysAnn/dt);
[P,R,Vols,FinalVal,CorrPrice,CorrReturn] =
GMC(EvaluationDate,ExpirationDate,StartPrices,Drift,Sigma,NSim,dt);
```

```
%Inputs for payoff calculation
Strike = 9.8;
Condition_level=(90/100)*Strike;



AutocallableValuationDate = ['09-03-2014';'03-03-2015'];
AutocallableSettlementDate = ['09-05-2014';'09-05-2015'];
AutocallablePrice = [110/100;120/100];
RedemptionPriceUp = 130/100;
[Nauto,m] = size(AutocallableValuationDate);

AutoCallIndex = wrkdydif(EvaluationDate, AutocallableValuationDate, 1);
%Create the discount factors
DfAutoCallDate =
exp(-(mean(ReturnDOM)/100+Cds)*((datenum(AutocallableSettlementDate)
-datenum(EvaluationDate))/360));
DfSettlementDate =
exp(-(mean(ReturnDOM)/100+Cds)*((datenum(SettlementDate)
-datenum(EvaluationDate))/360));

%Create payof from prices simulation

for k=1:NSim

    TestAtAutoCall = ones(1,Nauto);
    for l=1:Nauto
    TestAtAutoCall(1,l) = P(:,AutoCallIndex(1,1),k)>Condition_level;
    end

    TestAtExpiry = P(:,end,k)>Condition_level;

    if sum(TestAtAutoCall)>0
```

```
        i = 1;
        while (TestAtAutoCall(1,i) == 0)
            i = i +1;
        end
        Payoff(k)= AutocallablePrice(i,1) * DfAutoCallDate(i,1);
    else

    if sum(TestAtExpiry)>0
    Payoff(k)= RedemptionPriceUp * DfSettlementDate;

    else
    Payoff(k)= DfSettlementDate;
    end
    end

end

ProductValue = mean(Payoff)
```

6.4 Closed form solution Matlab script with put

```
% Closed form solution for an autocallable product

clear all;close all;


Strike=9.8;
Condition_level_1 =  90/100*Strike;% condition 1
Condition_level_2 =  90/100*Strike;% condition 2
Condition_level_3 =  90/100*Strike;% condition 3

Rebate_1          =  110/100;
Rebate_2          =  120/100;
Rebate_3          =  130/100;

Cds               =  175/10000;           % CDS of the issuer in
ReturnDOM         =  0.25/100;              bps divided by 10000
r                 =  ReturnDOM+Cds     ;% taux d'interet
Vols              =  25/100;  % volatility
```

```
%Create the discount factors
DfAutoCallDate_1 = exp(-(r)* 1);
DfAutoCallDate_2 = exp(-(r)* 2);
DfAutoCallDate_3 = exp(-(r)* 3);


A   = (log(Condition_level_1/Strike)-(r-(Vols^2/2)))/Vols;
B   = (log(Condition_level_2/Strike)-(r-(Vols^2/2))*2)/(sqrt(2)*Vols);
C   = (log(Condition_level_3/Strike)-(r-(Vols^2/2))*3)/(sqrt(3)*Vols);

%Probability calculus DO NOT TOUCH
%---------------------------------------------------------------
mu                  = [0 0 0];
Sigma               = [1 1/sqrt(2) 1/sqrt(3); 1/sqrt(2) 1 sqrt(2)/sqrt(3)
                                            ; 1/sqrt(3) sqrt(2)/sqrt(3) 1];

x1 = -3:.2:3; x2 = -3:.2:3; x3 = -3:.2:3;
i=find(abs(x1-A)==min(abs(x1-A))) ;
x1(i);
j=find(abs(x2-B)==min(abs(x2-B))) ;
x2(j);
k=find(abs(x3-C)==min(abs(x3-C))) ;
x3(k);
[X1,X2,X3] = meshgrid(x1,x2,x3);
F = mvncdf([X1(:) X2(:) X3(:)],mu,Sigma);
F = reshape(F,length(x2),length(x1),length(x3));

prob_triv        =F(i,j,31) - F(i,j,k); % Trivariate probability
prob_biv         =F(i,31,31)-F(i,j,31); % Bivariate probability
prob_univ        =F(31,31,31)-F(i,31,31);% Univariate probability
%---------------------------------------------------------------
[Call, Put] = blsprice(10.86, 9.8, ReturnDOM+Cds, 3, 25/100)
ProductValue     =prob_univ*Rebate_1*DfAutoCallDate_1+prob_biv*Rebate_2*DfAutoCallDate_2
                   +prob_triv*Rebate_3*DfAutoCallDate_3-F(i,j,k)*Put
```

6.5 Monte Carlo Matlab script with put

```
clear all;close all;

%PUT Global Inputs for Simulation here
%------------------------------------------------------------

NSim                = 10000;
EvaluationDate      = today;
ExpirationDate      = '09-03-2016';
SettlementDate      = '09-05-2016';
StartPrices       =  10.86;      % Current Values of the underlyings
Vols                = 25/100;
Cds                 = 175/10000;           % CDS of the issuer
ReturnDOM           = 0.25/100;

ReturnFOR           = 0/100;


Correls             = 100/100;

dt                  =  1/252;              % one trading day
%--------------------------------------------------------------------------


%Price Simulation DO NOT TOUCH
%---------------------------------------------------------------
Drift = ReturnDOM - ReturnFOR;
Vols = Vols;
Sigma = corr2cov(Vols, Correls);
NWorkdays=wrkdydif(EvaluationDate, ExpirationDate, 1);
NWorkdaysAnn = NWorkdays /252;
NObs = round(NWorkdaysAnn/dt);
[P,R,Vols,FinalVal,CorrPrice,CorrReturn] =
GMC(EvaluationDate,ExpirationDate,StartPrices,Drift,Sigma,NSim,dt);
```

```
%Inputs for payoff calculation
Strike = 9.8;
Condition_level=(90/100)*Strike;



AutocallableValuationDate = ['09-03-2014';'03-03-2015'];
AutocallableSettlementDate = ['09-05-2014';'09-05-2015'];
AutocallablePrice = [110/100;120/100];
RedemptionPriceUp = 130/100;
[Nauto,m] = size(AutocallableValuationDate);

AutoCallIndex = wrkdydif(EvaluationDate, AutocallableValuationDate, 1);
%Create the discount factors
DfAutoCallDate =
exp(-(mean(ReturnDOM)/100+Cds)*((datenum(AutocallableSettlementDate)
-datenum(EvaluationDate))/360));
DfSettlementDate =
exp(-(mean(ReturnDOM)/100+Cds)*((datenum(SettlementDate)
-datenum(EvaluationDate))/360));

%Create payof from prices simulation

for k=1:NSim

    TestAtAutoCall = ones(1,Nauto);
    for l=1:Nauto
    TestAtAutoCall(1,l) = P(:,AutoCallIndex(l,1),k)>Condition_level;
    end

    TestAtExpiry = P(:,end,k)>Condition_level;

    if sum(TestAtAutoCall)>0
```

```
            i = 1;
            while (TestAtAutoCall(1,i) == 0)
                i = i +1;
            end
            Payoff(k)= AutocallablePrice(i,1) * DfAutoCallDate(i,1);
        else

        if sum(TestAtExpiry)>0
        Payoff(k)= RedemptionPriceUp * DfSettlementDate;

        else
        Perf = P(1,NObs,k)/Strike;
        Payoff(k)= Perf* DfSettlementDate;
        end
        end

    end

    ProductValue = mean(Payoff)
```

7 References

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