NUMHYP 2011: main ideas evoqued in the discussion session

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The audience made several contributions: the main ones were

- F. Bouchut proposed to quantify precisely "how better well-balanced schemes are"; one simple (and maybe too elementary) is to study the residues decay for boundary value problems which are known to stabilize correctly onto an asymptotic regime.
- F. Coquel emphasized the importance of "well-prepared initial/boundary data" when it comes to consider ambitious AP strategies, like for instance from Navier-Stokes toward simplified shallow water problems in order to avoid boundary or internal layers.
- J. Sainte-Marie also advocated a program for numerically passing from compressible Navier-Stokes to shallow water models.
- L. Gosse raised the question about defining the correct central notion (assuming it exists) between Well-Balanced and Asymptotic-Preserving: WB can be recast as a special AP process where time t is rescaled like t/ε and AP schemes can be straightforwardly derived starting from rigorous WB Godunov discretizations including non-conservative products. Care must be taken for infinite domains as travelling waves are not asymptotic profiles which enter naturally these numerical frameworks; the natural setup is probably the boundary value problem (which connects also to F. Coquel's remark).

The panel (D. George for modelling, P. Noble for analysis, C. Pares for numerical analysis and computation, S. Jin for general expertise) stressed several directions of investigation under the supervision of S. Noelle.

They are reported in the following table:

David George	Pascal Noble	Carlos Parés	Shi Jin
Confinement inside hyperbolic theory: too restrictive or not ? (cf. dispersive terms as raised by P. Noble)	Handling complex steady-states (with general pressure laws or Coriolis force) and 2-D problems	Non-hydrostatic effects in shallow water (modelling and computational viewpoints)	Implicit schemes with preconditioning (cf. BGK penalization of Boltzmann eqn)
Stiffness and relative importance of different terms (balancing issues ?)	Super-characteristic relaxations actually appear and must be handled correctly	Undestanding nonlinear resonance (relevance ?) and non-conservative products	Problems with $\neq$ time scales (e.g. combustion)
Modelling of multiphase flows: terra incognita ? Generally leading to non-hyperbolic pbs	Numerical challenge of handling high order (dispersive) partial differential terms	Noise in the data (continuity w.r.t. data), stochastic terms	singular limits of Vlasov eqn: for instance high-field limit
	Asymptotic-Preserving (AP) from Navier-Stokes toward St-Venant system		**are Well-Balanced and AP both sides of the same coin ?** (writing $\partial/\partial(t/\varepsilon)$, $t \to 0$, for large time consistency)