SOLVING THE MULTI-LAYER SHALLOW WATER EQUATIONS USING THE FINITE VOLUME MODIFIED METHOD OF CHARACTERISTICS

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ABSTRACT

Mathematical modelling of tidal flows in water systems is based on the formulation and solution of the appropriate equations of continuity and motion of water. In general, tidal flows represent a three-dimensional turbulent Newtonian flow in complicated geometrical domains. The costs of incorporating three-dimensional data in natural water courses is often excessively high. Computational efforts needed to simulate three-dimensional turbulent flows can also be significant. In view of such considerations, many researchers have tended to use rational approximations in order to develop two-dimensional hydrodynamical models for tidal flows. Indeed, under the influence of gravity, many free-surface water flows can be modelled by the shallow water equations with the assumption that the vertical scale is much smaller than any typical horizontal scale. These equations can be derived from the depth-averaged incompressible Navier-Stokes equations using appropriate free-surface and boundary conditions along with a hydrostatic pressure assumption. The shallow water equations in depth-averaged form have been successfully applied to many engineering problems and their application fields include a wide spectrum of phenomena other than water waves. For instance, the shallow water equations have applications in environmental and hydraulic engineering, for example, for tidal flows in an estuary or coastal regions, rivers, reservoir and open channel flows. Such practical flow problems are not trivial to simulate since the geometry can be complex and the topography irregular. However, single-layer shallow water equations have the drawback of missing some physical dynamics in the vertical motion. Therefore, during the last decades, multi-layer shallow water models have been attracted more attention and have became a very useful tools to solve hydrodynamical flows such as rivers, estuaries, bays and other nearshore regions where water flows interact with the bed geometry and wind shear stresses. The main advantage of these models is the fact that the multi-layer shallow water model avoids the expensive three-dimensional NavierStokes equations and obtains stratified horizontal flow velocities as vertical velocities are relatively small and the flow is still within the shallow water regime.

Numerical treatment of the multi-layer shallow water equations often presents difficulties due to their nonlinear form, presence of the advective term, coupling between the free-surface equation and the equations governing the water flow. In addition, the difficulty in these models comes from the coupling terms involving some derivatives of the unknown physical variables that make the system non-conservative and eventually non-hyperbolic. Due to these terms, a numerical scheme originally designed for single-layer shallow water equations will lead to instabilities when it is applied to each layer separately. In this contribution we propose a new finite volume modified method of characteristics to solve the multi-layer shallow water equations. The method avoids the solution of Riemann problems and belongs to the predictor-corrector type methods. The predictor stage uses the method of characteristics to reconstruct the numerical fluxes whereas, the corrector stage recovers the conservation equations. The proposed method is simple, conservative, non-oscillatory and suitable for multi-layer shallow water equations for which Riemann problems are difficult to solve. Numerical examples are presented to verify the multi-layer shallow water model against analytical solutions. We demonstrate the models capability of calculating lateral and vertical distributions of velocities for wind-driven circulation over complex bathymetry.