High Performance meshfree methods for fluid flows Computation

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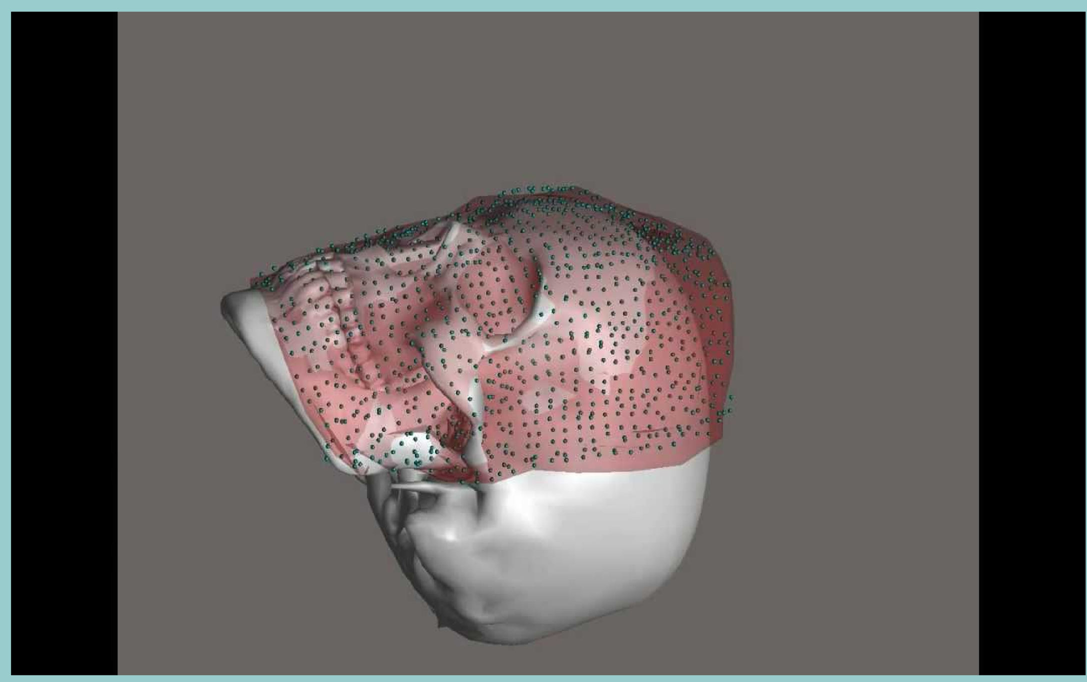
Introduction

Let us consider the conservation law defined by

$$\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{w})}{\partial x} + \frac{\partial \mathbf{g}(\mathbf{w})}{\partial y} = 0, \tag{1}$$

to be solved in a complex shaped domain.



Objectifs

We aim at constructing a general RBF Meshfree method having good qualities of accuracy an robustness, efficient enough and able to deal with convection dominated problems.

Radial basis function formulation

Let us assume that

$$\mathbf{f}(\mathbf{w_i}) = \sum_{j \in I_i} \lambda_j \varphi(r_{ij}, \varepsilon), \tag{2}$$

where $\Lambda = \{\lambda_1, \dots, \lambda_m\}$ are the expansion coefficients to be determined and $\varphi(r, \varepsilon)$ is a radial basis function with r_{ij} the Euclidean distance between the $i$eth and the $j$eth points.

In a compact form, equation (2) can be written as

$$\mathbf{f}(\mathbf{w}) = \Phi \Lambda, \tag{3}$$

which is solved to determine the local expansion coefficients, if the RBF matrix Φ is invertible;

$$\Lambda = \Phi^{-1} \mathbf{f}(\mathbf{w}). \tag{4}$$

By linearity, the partial derivative of the flux $\mathbf{f}(\mathbf{w}_i)$ from equation (1) can be evaluated in its compact form

$$\mathbf{f}_x(\mathbf{w}) = \Phi_x \Lambda = \Phi_x \Phi^{-1} \mathbf{f}(\mathbf{w}) \tag{5}$$

where $\mathbf{D}_x = \Phi_x \Phi^{-1}$ is called the differentiation matrix associated to the partial derivative that respects x and $\mathbf{D}_x = \Phi_y \Phi^{-1}$ is calculated is the same manner. Hence, in (1) :

$$\mathbf{f}_x(\mathbf{w}) = \mathbf{D}_x \mathbf{f}(\mathbf{w}) \quad \text{and} \quad \mathbf{g}_y(\mathbf{w}) = \mathbf{D}_y \mathbf{g}(\mathbf{w}). \tag{6}$$

Inserting the above expression in (1), a semi-discrete formulation of the system (1) can be obtained as

$$\frac{\partial \mathbf{w}}{\partial t} + \mathbf{D}_x \mathbf{f}(\mathbf{w}) + \mathbf{D}_y \mathbf{g}(\mathbf{w}) = 0. \tag{7}$$

The canonical system of ordinary differential equations of (7) is

$$\frac{d\mathbf{w}}{dt} = \mathbf{F}(\mathbf{w}) + \mathbf{G}(\mathbf{w}), \quad t \in (0, T], \tag{8}$$

where $\mathbf{F}(\mathbf{w}) = -\mathbf{D}_x \mathbf{f}(\mathbf{w})$ and $\mathbf{G}(\mathbf{w}) = -\mathbf{D}_y \mathbf{g}(\mathbf{w})$. Therefore,

$$\mathbf{w}^{n+1} = \mathbf{w}^n + \Delta t \left[\mathbf{F}(\mathbf{w}^n) + \mathbf{G}(\mathbf{w}^n) \right]. \tag{9}$$

Stabilized high performance schemes

The proposed meshfree scheme (9) is only first order accurate. Instead of using this formulation, some stable high order schemes are achieved :

- A quasi second order predictor-corrector scheme based on the method of characteristics and midpoints;
- A reconstruction of states using MUSCL slope limiters to obtain high order schemes.

Bibliography

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- **F. Benkhaldoun, A. Halassi, D. Ouazar, M. Seaïd and A. Taik.** A new local radial basis function for solving free surface water flows with topography (2016). *International Journal of Applied and Computational Mathematics*, submitted.
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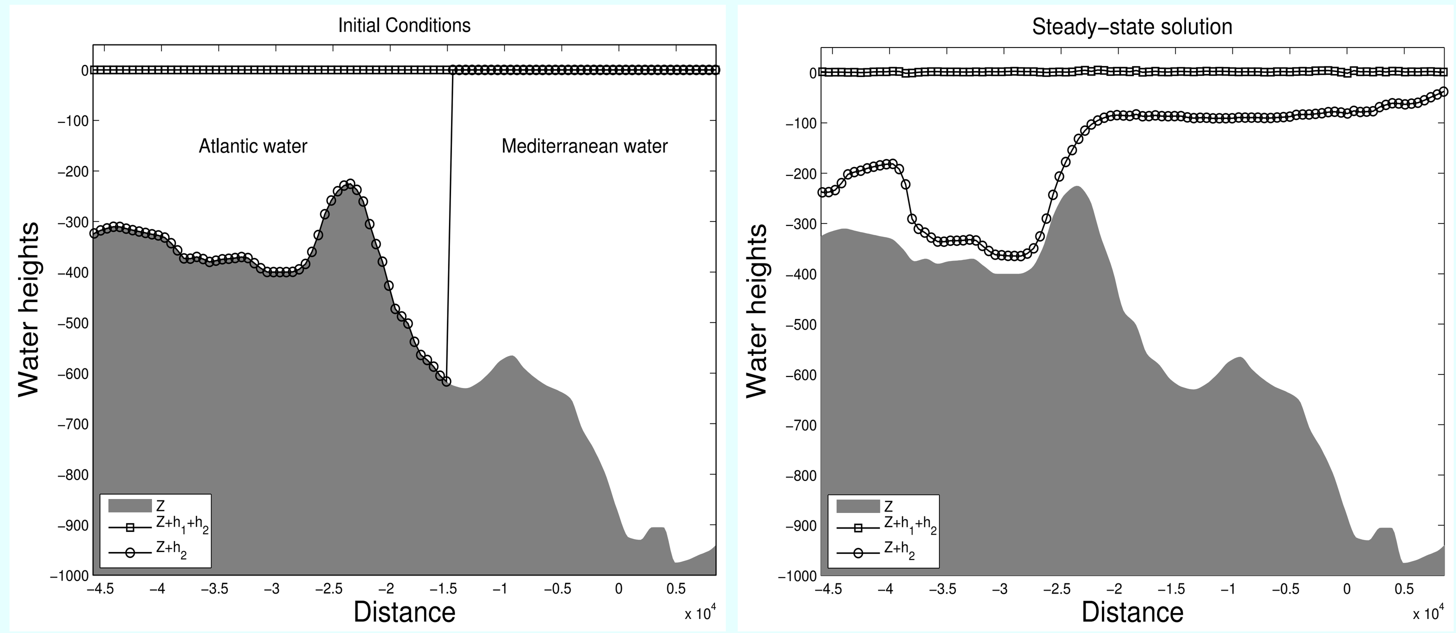
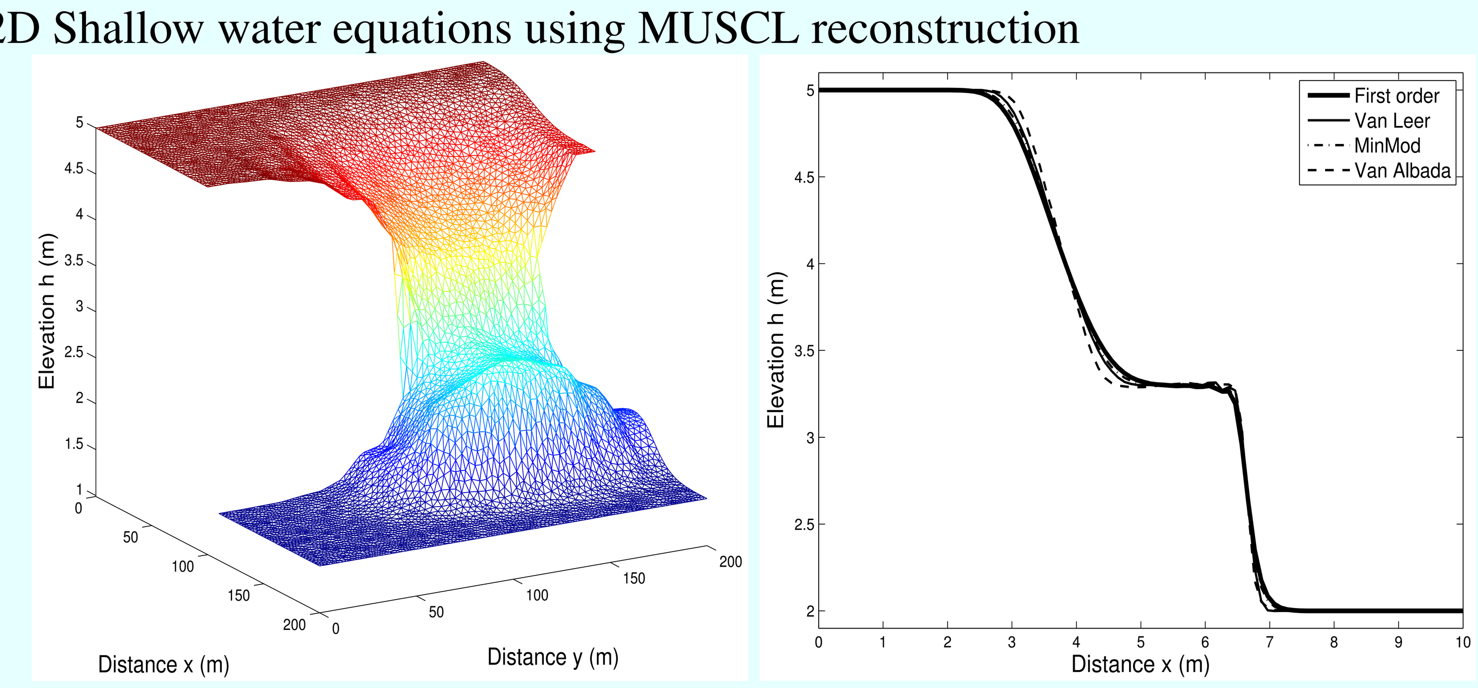
Numerical results

- 1D Shallow water equations using predictor-corrector scheme



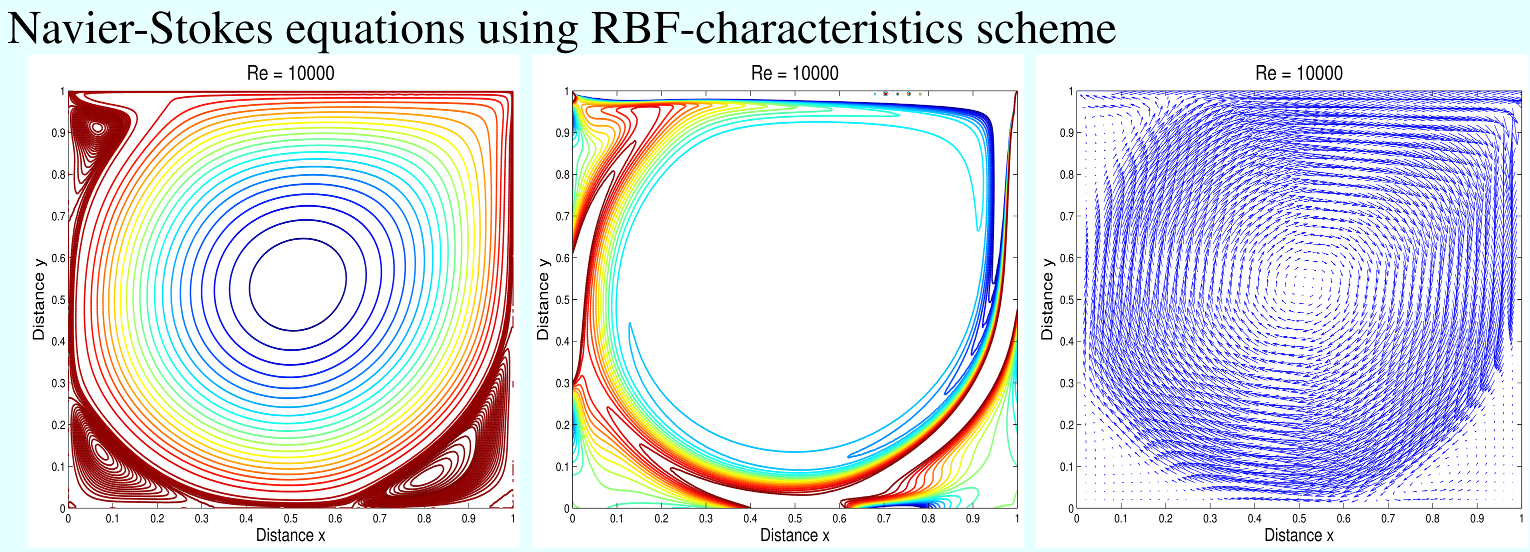
Lock-exchange in the Gibraltar Strait

- 2D Shallow water equations using MUSCL reconstruction



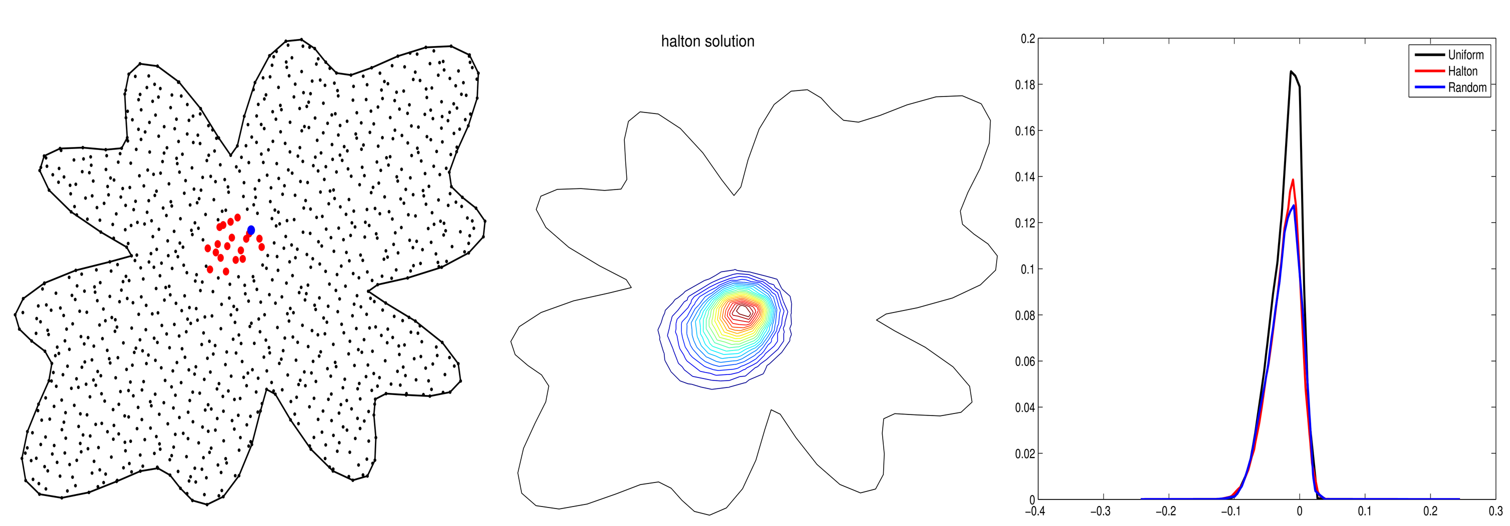
Dambreak problem: results with Van Albada (left) and comparison of profiles using different limiters (right)

- Navier-Stokes equations using RBF-characteristics scheme



Lid-driven cavity flow problem using Re = 10,000.

Other results



2D Burgers problem on comlex geometry using different distributions of collocation points and an adapted upwind local stencil.

Perspectives

- Solving pollutant and sediment transport in two-dimensional shallow water flows
- considering more complex shaped domains such as Mediterranean Sea and North Sea