



# Workshop on Assimilation, Control and Computational Speedup

June 6-7, 2023  
Villetaneuse, France

# Schedule & Venue

- All sessions will take place in the LAGA laboratory of **Université Sorbonne Paris-Nord** in **Room B405, Bâtiment B-D** ([Directions to LAGA](#)).
- Talks will be broadcast live on BBB via this link: <https://bbb.math.univ-paris13.fr/b/ber-fjh-dms-797>
- Each block consists of a 30-minute presentation followed by 10 minutes of discussions.

Time	Tuesday	Wednesday
09:00 – 09:40	Gabriele Ciaramella	Katia Aït-Ameur
09:40 – 10:20	Duc Quang Bui	Norbert Tognon
10:20 – 10:50	<i>Coffee Break</i>	
10:50 – 11:30	Thanh Vuong Dang	Sever Hirstoaga
11:30 – 12:10	Nicole Spillane	Felix Kwok
12:10 – 12:50	<i>Lunch</i>	Martin Gander
12:50 – 13:45		
13:45 – 14:25		Hongyu Liu
14:25 – 15:05		Tiphaine Delaunay
15:05 – 15:45		Didier Auroux
15:45 – 16:15	<i>Coffee break</i>	
16:15 – 16:55	Lucas Perrin	
16:55 – 17:35	Damiano Lombardi	

# Abstracts

**Tuesday 9:00-9:40**

**On space-time RAS methods for wave-type equations**

*Gabriele Ciaramella (MOX Lab, Politecnico di Milano, Italy)*

**Abstract:**

The parallel-in-time integration of wave-type equations is well known to be a difficult task. When applying classical waveform-relaxation and parareal type methods, one generally experiences rapid error growths before converging in a finite number of iterations. This unfortunate behavior prevents the successful application of these domain decomposition methods for long time intervals. However, we can precisely characterize this convergence behavior by an analysis based on the d'Alembert principle. This analysis allows us to present new space-time restricted additive Schwarz (XT-RAS) frameworks capable of treating wave problems defined on long time intervals. Moreover, we also show that XT-RAS methods can be modified following tent-pitching strategies in order to be adapted to the specific wave propagation problem. Numerical experiments support the presented theoretical findings and show the efficiency of the new proposed frameworks.

This is a joint work with Martin J. Gander and Ilario Mazzieri.

**Tuesday 9:40-10:20**

**Optimized Schwarz method in Time direction for Control problem of discrete transport equation**

*Duc Quang Bui (Université Sorbonne Paris-Nord)*

**Abstract:**

We investigate optimized Schwarz domain decomposition methods in time for the control of transport equation. In the case of an internal control all over the domain, the optimization problem can be transformed into a system of two coupled PDEs. We then apply the time-domain decomposition strategy on this PDE system. Under Fourier analysis, we find optimized parameters for both one-sided and two-sided cases, in the latter case we get a much better convergence factor. We illustrate our results by numerical tests.

## Tuesday 10:50-11:30

### **An internal controlled wave problem: Schwarz and Dirichlet-Neumann parallel in time methods**

*Thanh Vuong Dang (Université Sorbonne Paris-Nord)*

#### **Abstract:**

Optimal control deals with the problem of finding a control law for a given system such that a certain optimal criterion is achieved. A control problem includes a cost function that is a function of state and control variables. In this talk, we will present the minimization problem of a control which is satisfied the wave constraint over interior domain with some given initial data. With optimality system, we propose Schwarz and Dirichlet Neumann algorithm in time for computing the discrete control. Furthermore, we point out several strategies to optimize the convergence speed of two algorithms.

## Tuesday 11:30-12:10

### **Some remarks on the convergence of GMRES with weighted norms, preconditioning and deflation**

*Nicole Spillane (École Polytechnique)*

#### **Abstract:**

GMRES is a very well-established iterative solver for general linear systems. At iteration  $k$ , it computes the approximate solution  $x_k$  to the linear system  $Ax = b$  that minimizes the norm of the residual in a certain space called the Krylov subspace. It is also well known that convergence can be accelerated by means of:

- preconditioning, i.e., providing the solver with an (easier to compute) approximate of the inverse of  $A$ ,
- deflating, i.e., pre-solving a projected version of the problem,
- weighting, i.e., changing the norm that is minimized at each iteration.

I will show how convergence of GMRES can be analyzed theoretically. I will focus particularly on problems whose symmetric part  $(A+A^*)/2$  is positive definite and on symmetric positive definite preconditioners.

## Tuesday 13:45-14:25

### **Inverse problems for nonlinear waves**

*Hongyu Liu (City University of Hong Kong)*

#### **Abstract:**

In this talk, I shall discuss our recent study on inverse problems for nonlinear waves as well as the corresponding applications.

**Tuesday 14:25-15:05**

**Solving inverse source wave problem: from observability to observer design**

*Tiphaine Delaunay (Inria — LMS, Ecole Polytechnique, CNRS — Institut Polytechnique de Paris)*

**Abstract:**

The objective of this work is to propose a practical method using observers to estimate a source term of a wave equation, from internal measurements in a subdomain  $\omega$ . The first part of the work consists in proving an identifiability result from classical observability conditions for wave equations. We deduce that the source reconstruction is an ill-posed inverse problem of order 2. This inverse problem is solved using a sequential strategy that is proven to be equivalent to a minimization of a cost functional with Tikhonov regularization.

**Tuesday 15:05-15:45**

**Observers for data assimilation**

*Didier Auroux (Université Côte-d'Azur)*

**Abstract:**

Nudging is a data assimilation method that uses dynamical relaxation to adjust a model towards observations. The standard nudging algorithm consists in adding to the model equations a feedback term, proportional to the difference between the observations and the corresponding model state. Also known as the Luenberger (or asymptotic) observer, it theoretically requires an infinite time window to converge. The Back and Forth Nudging (BFN) algorithm has been introduced in order to extend the efficiency of nudging to finite/small time windows. It consists in alternately solving the model forwards and backwards in time, with a nudging term in both cases, over the assimilation window. These approaches can be extended to more complex observers, for which non-observed variables can also be corrected with observed ones. We will give in this talk an overview of nudging, observers, and backward-forward algorithms, with applications to oceanography and fluid dynamics.

**Tuesday 16:15-16:55**

**TBD**

*Lucas Perrin (Inria Paris)*

**Abstract:**

TBD

**Tuesday 16:55-17:35**

**Parametric PDE solvers for parameter estimation and uncertainty quantification**

*Damiano Lombardi (Inria Paris)*

**Abstract:**

In the present work we present a parametric solver for parametric partial differential equations. Given a system of parametric Partial Differential Equations, we consider the parameters as extra variables and approximate the problem solution by using tensor methods. The possibility of constructing an efficient interpolator based on the solution approximation enables the use of Markov Chain Monte Carlo methods for parameter estimation. Several examples on fluid-structure interaction problems will be presented.

This is a joint work with Sébastien Riffaud and Miguel Fernández.

**Wednesday 9:00-9:40**

**Un algorithme parallèle adapté aux schémas en temps multi-pas**

*Katia Aït-Ameur (École polytechnique)*

**Abstract:**

Dans cet exposé, je présenterai une stratégie de parallélisation en temps pour réduire le temps de résolution d'un système d'EDO discrétisé à l'aide d'un schéma en temps multi-pas. Cette stratégie se base sur une méthode de décomposition de domaine en temps: l'algorithme parallèle. Un schéma en temps multi-pas permet d'obtenir une approximation d'ordre élevé, cependant, l'initialisation de la propagation en temps en chaque fenêtre doit être définie avec rigueur. Autrement, l'erreur commise à l'initialisation sera propagée sur l'intervalle de temps et empêchera l'algorithme parallèle de converger vers la solution avec la précision souhaitée. Dans l'objectif d'aborder ce problème, nous avons proposé une nouvelle variante de la méthode parallèle, adaptée à ce type de discrétisation et qui permet de conserver les mêmes propriétés que la méthode originelle. Un effort particulier a été réalisé afin de construire une méthode non intrusive dans les solveurs grossier et fin. Nous montrons à l'aide de résultats théoriques et numériques que les propriétés de précision et de convergence de l'algorithme parallèle multi-pas sont préservées lorsque l'on initialise rigoureusement chaque fenêtre en temps.

**Wednesday 9:40-10:20**

**Paraopt algorithm and Runge-Kutta Methods**

*Norbert Tognon (Inria Paris)*

**Abstract:**

In this talk, we present the convergence analysis of Paraopt algorithm applied to a linear optimal control problem. The Paraopt algorithm is a parallel in time method for solving a optimality system arising in partial differential equations (PDEs) constrained optimization. This solving required two time integration methods and depends in crucial way on a quadrature formula and a Runge-Kutta method. We use an operator norm analysis to show that the convergence rate depends on the

discretization parameters and the smaller of the orders of the time integration methods. We illustrate this by some numerical examples.

### **Wednesday 10:50-11:30**

#### **Efficiency of a parareal algorithm for highly oscillatory Vlasov-Poisson systems**

*Sever Hirstoaga (Inria Paris)*

##### **Abstract:**

We present a specific version of the parareal algorithm for solving highly oscillatory two-dimensional Vlasov-Poisson systems. We use reduced models, obtained from the two-scale convergence theory, for the coarse solving. These models are useful to approximate the original Vlasov-Poisson model at a low computational cost since they are free of high oscillations. The equations are numerically solved with a particle-in-cell method. We provide a thorough analysis of the efficiency of the parareal algorithm on the basis of the ideal speedup.

### **Wednesday 11:30-12:10**

#### **Analysis of Schwarz waveform relaxation methods by exponential weighting**

*Felix Kwok (Université Laval)*

##### **Abstract:**

In Schwarz waveform relaxation (SWR) methods, each iteration consists of the parallel solution of time-dependent subdomain problems, followed by an exchange of interface data over the whole time interval; the process is then repeated until convergence. SWR methods are known to converge superlinearly for bounded time intervals. However, to obtain sharp convergence estimates, one often needs to compute complicated inverse Laplace transforms, and for some interface conditions, there are no known formulas for the inverse transforms. In this talk, we show how exponential weighting techniques can be used to produce superlinear estimates without having to compute the inverse Laplace transforms explicitly. We will use this technique to analyze the convergence of SWR with Dirichlet and Robin transmission conditions.

This is joint work with Alejandro Alfonso Rodriguez (U. Laval).

### **Wednesday 12:10-12:50**

#### **ParaDiag methods for time parallelization**

*Martin J. Gander (University of Geneva)*

##### **Abstract:**

Time parallelization by diagonalization was proposed by Maday and Ronquist in 2008 under the name tensor-product space-time solvers. The idea is to diagonalize the time stepping matrix, so that all time steps can then be solved simultaneously in parallel. When using constant time steps, the time stepping matrix for classical time stepping schemes is unfortunately a Jordan block and cannot be diagonalized. A first approach is to use variable time steps, which lead to the first class

of ParaDiag-I algorithms. In these algorithms there is a delicate balance between the discretization error when using variable time steps and the round-off error during diagonalization which limits the number of time steps that can be efficiently parallelized in double precision.

A different idea for obtaining a diagonalizable time stepping matrix is to approximate it by a circulant matrix that is then used as preconditioner, as proposed by McDonald, Pestana and Wathen in 2018. An even more efficient technique is to use an alpha-circulant matrix, as suggested by Wu and collaborators in 2019 and 2020. This leads to the ParaDiag-II family of Parallel in Time (PinT) algorithms, which is less sensitive to round-off error than the ParaDiag-I methods, and permits large scale parallel simulations in time of both parabolic and hyperbolic problems. ParaDiag-II methods have also successfully been applied within other PinT algorithms as components for further parallelization.

I will give an introduction to this exciting class of ParaDiag algorithms, and illustrate their performance with numerical experiments.