Alpine Algebraic and Applied Topology (Saas 2016) Abstracts

Dave Barnes: *Rational S^1-equivariant ring spectra*

Rational S^1-spectra are the representing object for S^1-equivariant cohomology theories which take values in rational vector spaces. Working rationally removes much of the topological complexity of stable homotopy theory, but leaves interesting equivariant phenomena. An algebraic category describing the homotopy category was constructed by Greenlees along with a Adams short exact sequence.

Recent work improves this description to the level of a monoidal Quillen equivalence and hence gives a classification of rational S^1-equivariant ring spectra in terms of the algebraic model. The generality of the methods used can be extended to the case of $O(2)$, $SO(3)$ and product of copies of S^1. This is joint work with Greenlees, Kędziorek, and Shipley.

Kristine Bauer: *A combinatorial model for certain Taylor towers*

In the mid 1990's, Goodwillie developed a tower of polynomial n-excisive approximations for homotopy functors which is by now very well known. Shortly afterwards, Johnson and McCarthy developed a related "discrete" tower of polynomial degree n approximations, rather than n-excisive approximations. The original Johnson-McCarthy tower was limited in its scope; one could only define the tower for functors whose domain category had a basepoint (an object which is both initial and terminal). Goodwillie's tower, on the other hand, did not have this limitation. In recent work with Eldred, Johnson and McCarthy, we have expanded the scope of the discrete tower in two ways. The first expansion allow us to provide a tower of polynomial degree n approximations for functors whose domain has an initial object which differs from the terminal object. In many cases, this polynomial degree n tower agrees with Goodwillie's tower.

The second expansion allows us to handle functors whose domain category needn't have a terminal object at all. This latter tower resembles the Taylor tower for a function f(x), but considered as a function of its center of expansion, rather than of than x. For this reason, I will call it the tower with a varying center. The purpose of this talk is to explain a concrete, combinatorial model for the tower with a varying center. This model was motivated by a particular example, due to Goodwillie: the identity functor for rational simplicial rings, whose Taylor tower resembles the De Rham complex. In this talk I will explain the importance of the tower with a varying center, its combinatorial model, and the motivating example of the De Rham complex.

Tilman Bauer: *Chromatic unstable homotopy, plethories, and the Dieudonné correspondence*

Chromatic unstable homotopy theory is the study of the $K(n)$-local category of spaces, where $K(n)$ is a Morava K-theory. As in the stable setting, we have a fairly good understanding of this category when $n = 0$ or 1, but not for $n > 1$. Of particular interest is the space of maps between two $K(n)$-local spaces X and Y. There is an unstable Adams (or Bousfield-Kan) spectral sequence based on $K(n)$-homology to compute the homotopy groups of such mapping spaces, but its E^2-term is a monadic derived functor which is not easily accessible. It can also be described as an Ext term of modules over an algebraic structure called a plethory, which is a kind of Hopf algebroid in the category of coalgebras, which is still quite unwieldy. In this talk, I will explain how Dieudonné theory can be used to understand this structure in much simpler terms, and to do explicit computations.

Agnès Beaudry: *The $K(2)$-local Picard group at $p = 2$*

The Picard group is an important invariant of a symmetric monoidal category. In the homotopy category of spectra, these are precisely the isomorphism classes of the n-spheres and the Picard group is a copy of the integers. However, after $K(n)$-localization, the Picard group can become more complicated. The $K(n)$-local categories thus provide examples of interesting Picard groups. Their importance in chromatic homotopy theory is highlighted by the fact that the dualizing object for Brown-Commenetz duality comes from an invertible element.

The $K(n)$-local Picard groups have been computed at all primes when $n = 1$ and all odd primes when $n = 2$. Mahowald predicted that the torsion in the $K(2)$-local Picard group at the prime 2 would be very large compared to the torsion in the $K(2)$-local Picard groups at odd primes. In this talk, I will explain why he was right and explain our current understanding of the structure of this group.

Pedro Boavida: *Configuration categories and embedding spaces*

I will explain a connection between spaces of smooth embeddings and operad theory via certain categories called configuration categories. In some cases, e.g. when the manifolds are disks, this connection can be pushed further and in some respects improves on related results of Arone-Turchin and Dwyer-Hess. This is joint work with Michael Weiss.

Wojciech Chachólski : *Multi-persistence and topological data analysis*

In this talk I will present a new method for extracting persistence information out of multi-graded modules. This method builds on understanding mathematical properties of what one might regard as noise. Denoising can be then thought as the localisation away from the noise. In the talk I will focus mainly on stability aspects of these new invariants and give several illustrating examples. This is a report on a join work of the topological data analysis group at KTH in Stockholm that consists of Anders Lundman, Ryan Ramanujam, Martina Scolamiero, Sebastian Öberg and myself. I will also take this opportunity to describe our experience of collaborating with scientist from out side of the math department particularly from Immunology and Clinical Neuroscience departments at the medical school Karolinska.

David Chataur: *Intersection homotopy theory*

In this talk, we will survey some recent works on algebraic topology of singular spaces in collaboration with Joana Cirici and with Martin Saralegui and Daniel Tanr. We will describe a homotopical background for intersection cohomology and give some applications of this homotopical treatment to the topology of singular complex algebraic varieties.

Cristina Costoya: *Kahn's realizability problem*

Kahn's Realizability Problem asks for groups that occur as the group of self-homotopy equivalences of some space X. Raised in the late 1960's, the general case is still an open question.

In this talk we present recent progress on a long term project, joint with A. Viruel, tackling Kahn's problem. First, we start by presenting a general method, based on techniques from rational homotopy theory, that allows us to prove that every finite group is realizable. Afterwards, we change our perspective by looking into Kahn's problem through the eyes of Invariant Theory. As a result of that, we can realize a larger family of groups (including finite ones) that can be described as the rational points of certain algebraic groups. Finally, since our techniques force spaces X to be *rational*, we end this talk with a new insight inspired by Toric Topology in order to realize finite groups through *integral* spaces.

Emanuele Dotto: *Equivariant calculus and the tower of the identity on pointed G-spaces*

To any functor between equivariant homotopy theories, the equivariant excisive approximations provide a tower which under suitable conditions converges back to the original functor. In the case of endofunctors on pointed G-spaces the layers of this tower are the infinite loop spaces of genuine G-equivariant spectra. In the talk I will explain how to construct this tower and describe the layers for the identity functor on pointed G-spaces in terms of partition complexes of finite G-sets.

Vincent Franjou: *Lannes' T vs Harish-Chandra restriction*

In the 1980's, Adams, Gunawardena and Miller computed Steenrod-algebra maps between elementary abelian group cohomologies. As a consequence, their decomposition is governed by the modular representations of the semi-groups of square matrices. Indeed, given $V_n = (\mathbb{Z}/p)^n$, and for a summand P in $\mathbb{F}_p[M_n(\mathbb{F}_p)]$, $L_P := \mathrm{Hom}_{M_n}(P, H^*V_n)$ is a summand in H^*V_n. Applying Lannes' T functor on these defines an intriguing construction for representation theorists. We show that $T(L_P) \cong L_P \oplus H^*V_1 \otimes L_{\delta(P)}$, defining a functor δ from $\mathbb{F}_p[M_n(\mathbb{F}_p)]$-projectives to $\mathbb{F}_p[M_{n-1}(\mathbb{F}_p)]$-projectives, and we relate this new functor δ to classical constructions in the representation theory of the general linear groups.

This is joint work with Nguyen Dang Ho Hai and Lionel Schwartz.

Mark Grant: *The Poincaré-Hopf theorem for line fields*

A line field on a manifold is a section of the projectivized tangent bundle. These objects find applications in soft matter physics, where they may be used to model ordered media made up of rod-shaped molecules, such as nematic liquid crystals.

In this talk I will present an analogue of the Poincaré-Hopf theorem for line fields. For a line field with finitely many isolated topological defects on a closed manifold, this relates the sum of the local indices of the defects with the Euler characteristic of the manifold. (A version of this result for orientable surfaces was known to H. Hopf.)

If time permits I will discuss analogues for other types of ordered media, such as biaxial nematics.

This is joint work with D. Crowley.

John Greenlees: *Rational equivariant cohomology theories and affine formal covers of the sphere*

For a compact Lie group G, rational G-cohomology theories are represented by rational G-spectra. Rational G-spectra are modules over the sphere spectrum. If we can construct the sphere (as a ring spectrum) from pieces which are determined by their coefficient rings, this gives the basis of an algebraic model for G-spectra. The talk will focus on a Hasse-Tate type decomposition for the sphere when G is a torus, and view the decomposition from isotropical, algebraic and geometric points of view.

This is joint work with D. Barnes, M. Kędziorek, and B. Shipley)

Lars Hesselholt: *Topological Hochschild homology and the Hasse-Weil zeta function*

In the nineties, Deninger gave a detailed description of a conjectural cohomological interpretation of the (completed) Hasse-Weil zeta function of a regular scheme proper over the ring of rational integers. He envisioned the cohomology theory to take values in countably infinite dimensional complex vector spaces and the zeta function to emerge as the regularized determinant of the infinitesimal generator of a Frobenius flow. In this talk, I will explain that for a scheme smooth and proper over a finite field, the desired cohomology theory naturally appears from the Tate cohomology of the action by the circle group on the topological Hochschild homology of the scheme in question.

Marc Hoyois: *On the vanishing of negative equivariant K-theory*

Weibel conjectured that the negative K-theory of a singular scheme vanishes below minus its dimension. This is now known in a number of cases, in particular up to p-torsion in positive characteristic p. In joint work with A. Krishna, using techniques from stable equivariant motivic homotopy theory, we prove an analogous vanishing result for the negative K-theory of G-schemes in positive characteristic, where G is a finite or diagonalizable group scheme.

Adeel Khan: *Homotopy invariant cohomology theories of E_∞-ring spectra*

Spectral algebraic geometry, as developed by Lurie and Toen-Vezzosi, is a framework where E_∞-ring spectra can be studied using the language of schemes. We will explain how, using a spectral version of motivic homotopy theory, one can prove that motivic cohomology and homotopy invariant K-theory of connective E_∞-ring spectra are insensitive to higher homotopy groups.

Nitu Kitchloo: *Real Johnson-Wilson theories and Landweber flat real pairs*

The Real Johnson-Wilson theories, $ER(n)$, were first introduced by Hu-Kriz and further developed by Kitchloo-Wilson. They are a family of cohomology theories generalizing (2-local) real K-theory, KO, which is $ER(1)$. They are constructed by taking fixed points of an involution corresponding to complex-conjugation acting on the complex-oriented Johnson-Wilson theories, $E(n)$. These fixed-point theories however, are not complex orientable (indeed, they detect some stable stems).

Real Johnson-Wilson theories have proven to be remarkably powerful, as well as computationally amenable. For example, their properties were exploited to demonstrate some new nonimmersions of real projective spaces. The main tool for computation is a (Bockstein-type) spectral sequence that begins with $E(n)$-cohomology and converges to $ER(n)$-cohomology.

We take advantage of the internal algebraic structure of this spectral sequence converging to $ER(n)^*(pt)$, to prove that for certain spaces Z with Landweber-flat $E(n)$-cohomology, the cohomology ring $ER(n)^*(Z)$ can be obtained from $E(n)^*(Z)$ by a somewhat subtle base change. In particular, our results allow us to compute the Real Johnson-Wilson cohomology of the Eilenberg-MacLane spaces $Z = K(Z, 2m+1), K(Z/2, m), K(Z/2^q, 2m)$ for any integers m and q, as well as connective covers of BO: BO, BSO, $BSpin$, and $BO < 8 >$.

This is joint work with Stephen W. Wilson and Vitaly Lorman.

Anssi Lahtinen: *Modular characteristic classes for representations over finite fields*

While the cohomology groups of the general linear group $GL_n(F)$ over a finite field F of characteristic p have been completely computed by Quillen with coefficients in a field of characteristic different from p, little is known about the cohomology groups of $GL_n(F)$ in the modular case where the coefficient field is also of characteristic p. In this talk, I will report on joint work in progress with David Sprehn which sheds light on these modular cohomology groups by constructing a new system of characteristic classes for representations over finite fields. In particular, our work provides explicit nontrivial elements in the cohomology of $GL_n(F)$ in degrees linear in n, in contrast with the lowest-dimensional non-trivial elements known so far, which reside in degrees exponential in n.

Ran Levi: *Topological analysis of neural networks*

A standard way to schematically represent networks in general and neural network in particular is as a graph. Depending on context, graphs representing networks can be directed or undirected, and in some cases carry labels or weights on their vertices and edges. With any graph one can associate a variety of combinatorial and topological objects, as well as certain algebraic invariants. Much work has been done in recent years along these lines in the context of random undirected graphs.

In this talk I will survey an on-going collaborative project where we apply topological techniques and ideas to the study of the brain. The project was motivated by the creation of a biologically accurate digital reconstruction of a small part of the cortex of a young rat by the Blue Brain Project. This digital model provided a way of extracting very accurate structural and functional information. I will describe some of our methods and the results obtained by applying them to the Blue Brain reconstruction. Some other applications will also be discussed, as well as future directions.

Vidit Nanda: *Discrete Morse theory and classifying spaces*

Large-scale homology computations are often rendered tractable by discrete Morse theory. Every discrete Morse function on a given cell complex

X produces a Morse chain complex whose chain groups are spanned by critical cells and whose homology is isomorphic to that of X. However, the space-level information is typically lost because very little is known about how critical cells are attached to each other. In this talk, we discretize a beautiful construction of Cohen, Jones and Segal in order to completely recover the homotopy type of X from an overlaid discrete Morse function.

Wolfgang Pitsch: *Relative resolutions via truncations*

Our aim is to present a framework to do relative homological algebra. By this we mean that, if homological algebra is understood as a way to study objects in abelian categories through invariants determined by their resolutions, than we want to construct resolutions using more general objects that just injective modules and more bluntly we would like to be allowed to use a priori any object as an injective. This idea we borrowed from homotopy theory, where the closely related idea of cellularization and A-homotopy theory (with an a priori space A and its suspensions taking the place usually devoted to the spheres) developed for instance by Farjoun has proved to be extremely fruitful. A convenient tool to actually compute resolutions of a complex is to built out of it a tower of truncations approximating the initial complex, resolve each truncation and glue back the truncations; this procedure fits well into the notion of a relative model category. The main problem arises as we try to glue back the resolutions of the truncations, and is related to an extension of Grothendiecks axiom AB4* due to Roos. This is joint work with W. Chacholski and J. Scherer.

Sune Precht Reeh: *Saturated fusion systems as stable retracts of groups*

A saturated fusion system associated to a finite group G encodes the p-structure of the group as the Sylow p-subgroup enriched with additional conjugation. The fusion system contains just the right amount of algebraic information to for instance reconstruct the$p$-completion of BG, but not BG itself. Abstract saturated fusion systems $\mathfrak{F}$ without ambient groups exist, and these have (p-completed) classifying spaces $B\mathfrak{F}$ as well. The suspension spectrum of $B\mathfrak{F}$ is a retract of the suspension spectrum of BS, for the Sylow p-subgroup S, and this retract is encoded as a characteristic idempotent in the double Burnside ring of S. In joint work with Tomer Schlank and Nat Stapleton, we make use of these retracts to do Hopkins-Kuhn-Ravenel character theory for all saturated fusion systems by building on the theorems for finite p-groups. This involves studying free loop spaces for $B\mathfrak{F}$, and constructing transfer maps for these.

Steffen Sagave: *Logarithmic topological Hochschild homology*

The notion of a logarithmic ring spectrum is an extension of the usual notion of a ring spectrum that allows to form intermediate localizations between connective and periodic ring spectra. In this talk I will explain the definition of logarithmic ring spectra and the construction of their logarithmic topological Hochschild homology. Topological K-theory spectra give rise to logarithmic ring spectra that sit between the corresponding connective and periodic K-theory spectra, and I will present both structural and computational results about the logarithmic topological Hochschild homology of these logarithmic ring spectra. I will also outline how logarithmic topological Hochschild homology can be refined to a logarithmic version of topological cyclic homology.

This is joint work with John Rognes and Christian Schlichtkrull.

Tse Leung So: *Homotopy theory of gauge groups*

Gauge theory arises from field theory in physics and it has many applications in physics and mathematics. The gauge group associated to a principal G-bundles $P \to M$ is defined to be a group of G-equivariant automorphisms of P fixing M. When G is a simply-connected, simply compact Lie group, Theriault shows that if M is an orientable simply-connected closed 4 manifold, its gauge groups can be decomposed into products of gauge groups of S^4 and double loop spaces of G. In this talk, I will discuss my work talk on homotopy decompositions of gauge groups when M is certain non simply-connected closed 4 manifold.

Marc Stephan: *Categorical suspension and stable Postnikov data*

This is joint work with Nick Gurski, Niles Johnson and Anglica Osorno about categorical models for spectra. Any symmetric monoidal category C can be considered as a symmetric monoidal bicategory $\Sigma(C)$ with only one object. We prove that the K-theory spectrum of $\Sigma(C)$ has the same stable homotopy type as the suspension of the ordinary K-theory spectrum of C.

As an application, we model part of the Postnikov data of a stable 2-type categorically. A stable 2-type is a connective spectrum whose homotopy groups vanish above the second dimension. By a recent result of the three above, every stable 2-type is modeled by a Picard 2-category, i.e., a symmetric monoidal bicategory in which each cell is invertible. We deduce from the categorical description of the Postnikov data that there is no strict, skeletal model of the 2-type of the sphere spectrum.

Vesna Stojanoska: *Galois extensions of motivic ring spectra*

Rognes introduced the notion of Galois extensions of rings in homotopy theory, and showed that a number of interesting ring spectra form Galois extensions. I will briefly discuss a general formal framework in which homotopical Galois theory can be studied, and elaborate on applications of this theory to motivic homotopy. In particular, I will present motivic examples which showcase the parallels and differences between ordinary and motivic homotopy theory.

This is joint work with Agnès Beaudry, Kathryn Hess, Magdalena Kędziorek, and Mona Merling.

Christian Wimmer: *Rational global homotopy theory*

In global homotopy theory (with respect to finite groups) one considers homotopy types that encode compatible actions of all finite groups at the same time. This can be made precise in the following way: An orthogonal spectrum may be regarded as a G-equivariant spectrum with trivial G-action for every finite group G. The equivariant homotopy types arising in this way need not be trivial. For example equivariant K-theory can be represented by a single orthogonal spectrum. The global stable homotopy category is obtained as the localization of the category of orthogonal spectra at those maps which induce G-equivalences for all G.

There is also a Real refinement of global homotopy theory. Here one considers groups $G \to C_2$ augmented over the cyclic group of order two and takes twisted actions into account. The Real global homotopy category can be modeled by Real spectra, a variation of unitary spectra. As in the classical case, simplifications occur upon inverting the rational equivalences in the sense that the rational global homotopy category admits an "algebraic" description. I will discuss a multiplicative equivalence with the derived category of rational presheaves on Out, where Out denotes the category of finite groups with morphisms the conjugacy classes of surjections. This equivalence is implemented by a rather explicit "geometric fixed points" construction. There is also an analogous story in the Real case. One has to replace Out by a suitable category with objects the finite groups augmented over C_2.