TD 2

ex du cours :

$$PGL_2(\mathbb{C}) \longrightarrow \{\text{Homographies}\}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto z \mapsto \frac{az+b}{cz+d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} aa'+bc' & ab'+bd' \\ ca'+dc' & cb'+dd' \end{pmatrix}$$

$$z \longmapsto \frac{az+b}{cz+d} \longmapsto \frac{a'\left(\frac{az+b}{cz+d}\right)+b'}{c'\left(\frac{az+b}{cz+d}\right)+d'}$$

$$\frac{z(aa'+cb')+(a'b+db')}{z(c'a+cd')+(bc'+dd')}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} aa'+bc' & — \\ — & — \end{pmatrix}$$

$\boxed{\text{surj :}}$ OK $\quad z \mapsto \dfrac{az+b}{cz+d} \quad ad-bc\neq 0$

$$\uparrow$$

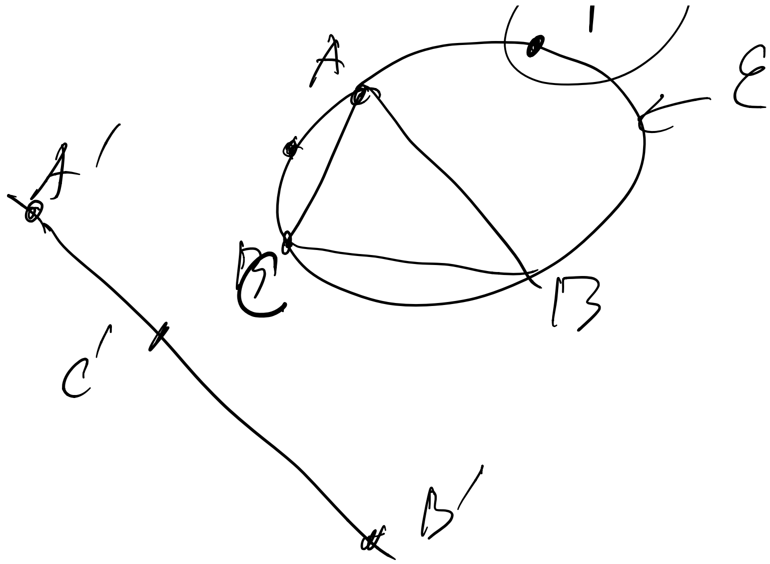
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\boxed{\text{inj :}}$ $\quad \dfrac{az+b}{cz+d} = z \qquad \underbrace{cz^2 + z(d-a) + b}_{=0}$

$$\Leftrightarrow c = 0 = b$$
$$a = d$$

$$\leadsto a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in \text{Centre de } GL_2(\mathbb{C})$$
$$= \bar{0} \text{ dans } PGL_2(\mathbb{C})$$

<u>Ptolémée</u> (exo 2 feuille 4)



si $P \in \mathcal{C}$

$\Updownarrow$

A', C', B' alignés dans cet ordre

$$\overline{A'B'} \leq \overline{A'C} + \overline{C'B'}$$

$=$

ex 1 : $A'B' = \ell^2 \dfrac{AB}{PA\cdot PB} = \ell^2 \dfrac{AC}{PA\cdot PC} + \ell^2 \dfrac{BC}{PB\cdot PC}$

$$\times \frac{PA\cdot PB \cdot PC}{\ell^2}$$

$$AB\cdot PC = AC\cdot PB + BC\cdot PA$$

Peaucellier

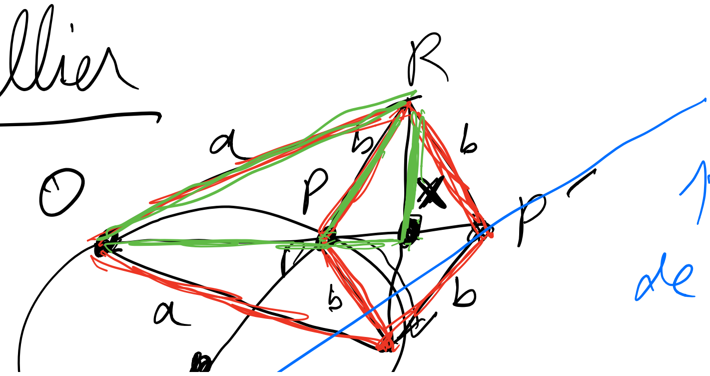
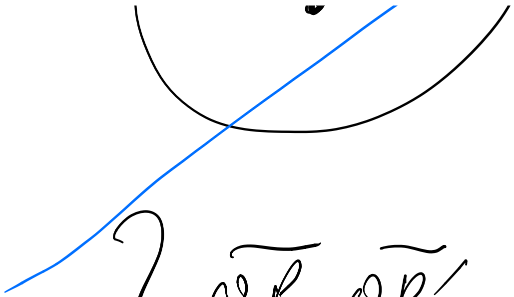


image de $\mathcal{C}$ par l'inversion de centre O



rapport a^2-b^2

? $\vec{OP}\cdot\vec{OP'} = a^2-b^2$?

$$\vec{OP}\cdot\vec{OP'} = (\vec{OX}+\vec{XP})(\vec{OX}-\vec{XP})$$

$$= OX^2 - PX^2$$

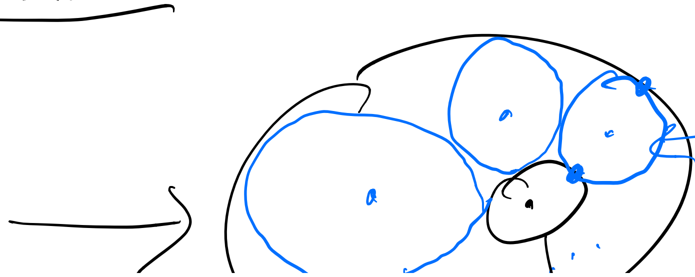
$$= OX^2 + (RX^2 - RX^2) - PX^2$$

$$= \underbrace{(OX^2+RX^2)}_{OR^2} - (\underbrace{RX^2+PX^2}_{PR^2})$$

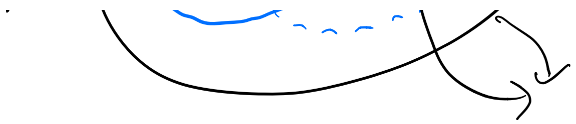
$$= a^2 - b^2$$

Exo 3

Périodique ?

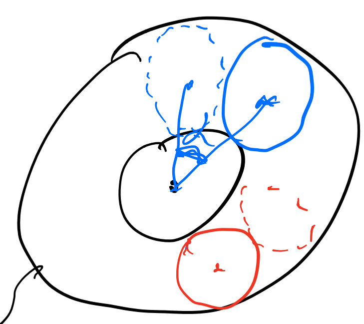


indépendant



$$c(\mathcal{E},\mathcal{E}') = \frac{1+\sin^2(\pi p/k)}{\cos^2(\pi p/k)}$$

<u>idée</u> : $\exists$ inversion $\underbrace{\mathcal{E} \text{ et } \mathcal{E}'}_{\text{2 cercles m\^{e}me centre}}$



$\cancel{\alpha = 2\pi}$

$\frac{\alpha}{2\pi} \in \mathbb{Q}$

$c(\mathcal{E},\mathcal{E}') = \quad \alpha = \pi \frac{p}{k}$