A fast finite volume solver for hydrostatic shallow water flows

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Example of a Shallow Water model : Sediment Transport in 1D

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = \frac{E-D}{1-p}$$

$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) = gh(-\frac{\partial Z}{\partial x} - S_f) - \frac{(\rho_s - \rho_w)gh^2}{2\rho}\frac{\partial c}{\partial x} - \frac{(\rho_0 - \rho)(E-D)u}{\rho(1-p)}$$

$$\frac{\partial (hc)}{\partial t} + \frac{\partial (huc)}{\partial x} = E - D$$

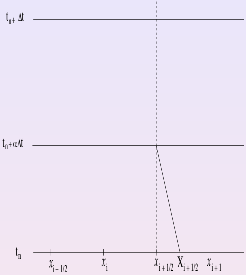
$$\frac{\partial Z}{\partial t} = -\frac{E-D}{1-p}$$

[3] Computational Dam-Break Hydraulics over Erodible Sediment Bed by Cao, (2004)

FVC method

$$\partial_t U + \partial_x F(U) = 0.$$

$$\partial_t U + V \partial_x U = 0, \qquad V = F'(U).$$



$$\begin{aligned} \frac{dX_{i+1/2}(\tau)}{d\tau} &= V_{i+1/2}\Big(\tau, X_{i+1/2}(\tau)\Big), \\ X_{i+1/2}(t_n + \alpha\Delta t) &= x_{i+1/2}, \end{aligned}$$

where $V_{i+1/2} = F'(U_{i+1/2})$.

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\Big(F(U_{i+1/2}^n) - F(U_{i-1/2}^n)\Big).$$

$$U_{i\pm 1/2}^n ?$$

Classical shallow water model

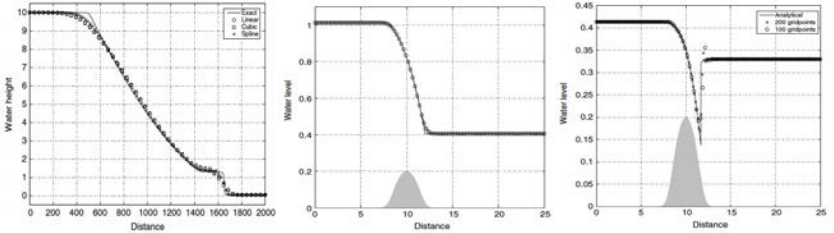


Figure: FVC. (left) Dam-break at $t = 50\ s$ using $h_r/h_l = 0.005$. (center) transcritical flow without shock. (right) transcritical flow with shock.

Gridpoints	Roe method	SRNH method	FVC method
500	8.746	13.193	1.008
1000	34.780	52.655	2.707
2000	134.152	210.620	15.756
4000	534.124	834.055	61.096
8000	2178.701	3378.303	249.209

Table: Computational times in seconds for dam-break on wet bed at $t = 50\ s$ using $h_r/h_l = 0.005$ and different gridpoints.

Multi-layered shallow water model

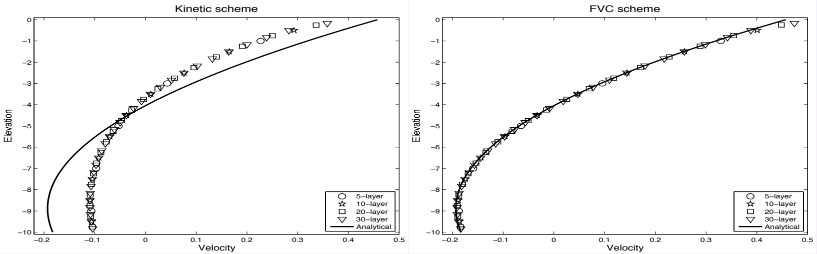


Figure: Comparisons of numerical predictions with the analytical solution for the wind-driven circulation flow without bottom friction : (left) Kinetic scheme, (right) FVC scheme.

	10-layer model		20-layer model	
Gridpoints	Kinetic scheme	FVC scheme	Kinetic scheme	FVC scheme
200	3.2	3.0	5.4	4.8
400	12.5	9.7	20.8	15.5
800	49.1	34.9	81.9	55.4

Table: CPU times for 10-layer and 20-layer models on different meshes for a dam-break at $t = 14$.

Density-driven shallow water flows

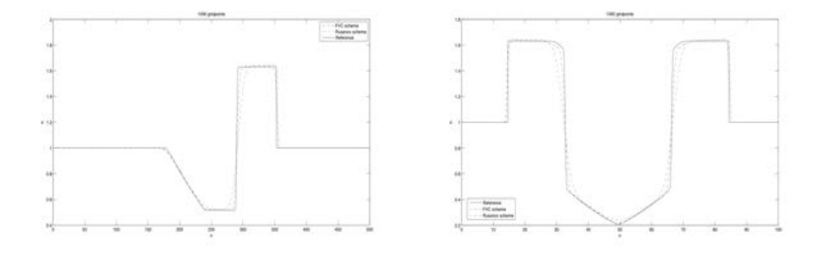


Figure: Density dam-break problem on a flat bottom with : (left) a single initial discontinuity, (right) two initial discontinuities.

	a single initial discontinuity				two initial discontinuities			
Gridpoints	FVC	Rusanov	ROE	SRNH	FVC	Rusanov	ROE	SRNH
100	0.21	0.06	0.78	0.81	0.61	0.17	2.67	2.7
200	0.4	0.19	3.1	3.17	1.33	0.6	10.36	10.6
400	0.94	0.67	12.18	12.58	3.36	2.22	41.37	42.46
800	2.58	2.66	48.83	50.34	9.64	8.73	164.86	172.34
1600	8.48	10.25	193.5	206.58	31.29	34.46	656.72	705.67

Table: CPU times for density dam-break problem at $t = 200\ s$ using different gridpoints.

Rotational shallow water flows

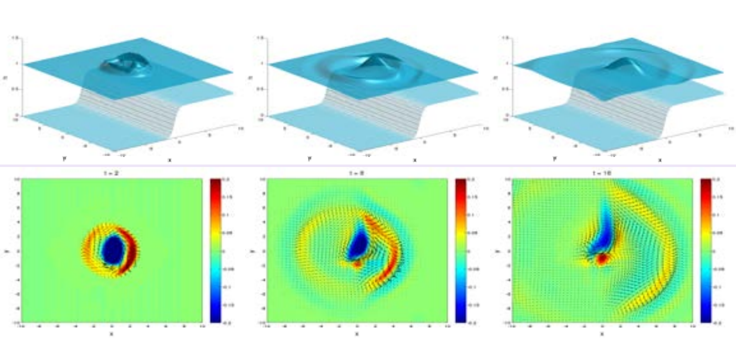


Figure: Water depth and velocity fields for the circular dam-break problem on non-flat bottom at t = 2, 8 and 16.

	4 s			8 s			20 s		
Gridpoints	FVC	ROE	SRNH	FVC	ROE	SRNH	FVC	ROE	SRNH
50×50	3.74	6.03	6.16	10.66	18	18.2	17.64	29.74	30.02
100×100	23.52	44.63	46.27	70.61	137.22	139.81	115.17	230.29	232.54

Table: CPU times on different meshes for circular dam-break on a flat bottom at different times.