High Performance Computing
USN-HCMV , Paris 13
Joint Master 2017

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Worksheet 1

Exercise 1

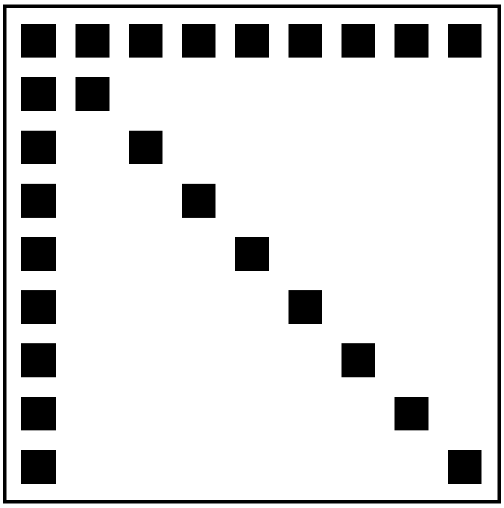

FIGURE 1 – Sparse Matrix A

- Write the Graph of A
- Reverse the order or the elements and visualize the profile of the reversed matrix B with the matrix command spy.
  (Create an array R ={i1, i2,..., in} ,$B = (b_{ij}) = (b_{R(i)R(j)})$), R is called the permutation array . This is a symmetric permutation of A i.e. rows and columns are permutated, then define R(i) = n-i+1))
- Write the Graph of B

What can you say about the results.

Exercise 2

Consider the mesh of a discretized PDE by Finite differences or Finite Elements. In which situations is the graph representing this mesh the same as the adjacency graph of the matrix ?

Exercise 3

Let A and B be two sparse (square) matrices of the same dimension. How can the graph of A + B be characterized with respect to the graphs of A and B ?

Exercise 4

Write a function that performs the Reversed Cuthill McKee algorithm on a matrix

Exercise 5

Consider a matrix which has the pattern

$$A = \begin{pmatrix} \star & \star & & & \star & & & \star \\ \star & \star & \star & & & \star & & \\ & \star & \star & \star & & & \star & \\ & & \star & \star & \star & & & \star \\ \star & & & \star & \star & \star & & \\ & \star & & & \star & \star & \star & \\ & & \star & & & \star & \star & \star \\ \star & & & \star & & & \star & \star \end{pmatrix}.$$

FIGURE 2 – Sparse Matrix A

- Show the adjacency graph of A (Place the 8 vertices on a circle.)
- Consider the permutation R = { 1,3,5,7,2,4,6,8}.
  Show the adjacency graph and new pattern for the matrix obtained from a symmetric permutation of A based on the permutation array R
- Show the adjacency graph and new pattern for the matrix obtained from a reverse Cuthill- McKee ordering of A starting with the node 1. (Assume the vertices adjacent to a given vertex are always listed in increasing order in the data structure that describes the graph.)

Exercise 6

Apply the Reverse Cuthill McKee (RCM) algorithm to the matrix A in exercise 1 starting with the node 1. What remarks can be made . Why is step 1 important in the RCM algorithm.

Exercise 7

The most economical storage scheme in terms of memory usage is the following : Store all Nz nonzero values aij in a real array A, A(1 :Nz) and the corresponding "linear array address" (i-1)*n + i in an array JA (1 :Nz).
The order in which these corresponding entries are stored is unimportant as long as they are both in the same position as their respective arrays. What are the advantages and disadvantages of this data structure ?
Write a short routine for performing a matrix-by-vector product in this format.