Master Vietnam-France in HCMC
High Performance computing

TP 6 : Effective coefficient of a composite material and domain decomposition

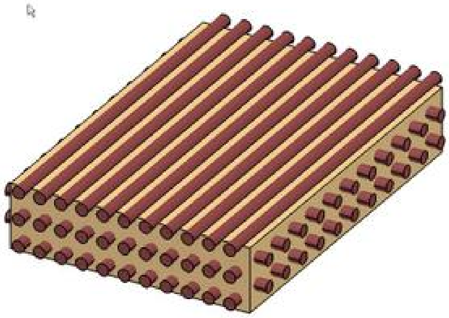

Figure 1 – Composite material

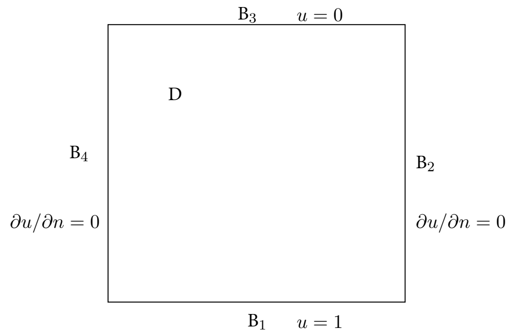

Figure 2 – Computational domain

Composite material are everywhere nowadays, in the building, aircrapfts, chairs, etc.. It is important to understand their mechanical and thermical properties.

Start with a heat conducting material, non homogeneous, occupying the space delimite by a square. $a(x)$ denotes the conductivity at point $\boldsymbol{x}$, and the heat flux is given by the Ficke law, $J = -a(\boldsymbol{x})\nabla u$, where u is the temperature. We are interested in computing the apparent conductivity of the material, defined as follows : The boundaries Γ_1 and Γ_3 are heated to 0 and 1 respectively, and the heat flux across the lateral boundaries Γ_2 and Γ_4 are supposed to be zero. The system is

$$\begin{array}{rcll} -\nabla(a(x)\nabla u) &=& 0 & \text{in } D \\ \partial_n u &=& 0 & \text{on } B_2 \cup B_4 \\ u &=& 1 & \text{on } B_1 \\ u &=& 0 & \text{on } B_3 \end{array} \tag{1}$$

We are searching for a solution $u \in H^1(D)$. The apparent conductivity is the ratio between the mean flux and the temperature gradient, that is

$$a_{app} = -\int_{B_3} a\partial_y u(x,y)dx$$

We will suppose that the square is $(0,3)\times(0,3)$, that the conductivity depends on x only and

$$a(x) = 1 \text{ on } (0,1), \quad a(x) = \epsilon \text{ on } (1,2), \quad a(x) = 1 \text{ on } (2,3)$$

The aim of this work is to compute the apparent conductivity.

1 Global computations

1. -2. Write a variational formulation and show well-posedness through Lax-Milgram theorem.

1. -1. Use the `matlab` codes available, modify the `newmesh` to deal with the different coefficients.Modify the finite element code to take the Neumann condition into account.

1. 0. Write a routine computing the apparent conductivity.

1. 1. Fix $\epsilon = 0.1$. Compute the apparent conductivity for refined values of the step size, and compare with the solution obtained by keeping N modes in the expansion above.

1. 2. Choose $\epsilon = 10^{-4}$. Compute the apparent conductivity for refined values of the step size, draw the solution, and see what the mesh size must be to be accurate. What is the computational time in this case ?

2 Domain decomposition in the y direction

An approximation of the solution is given by dividing the domain into two subdomains $D_1 = (0,3) \times (0,2)$ and $(D_2 = (0,3) \times (2+L,3)$ with $L > 0$. A sequence of problems is introduced, u_j^n is the solution at step n in D_j,

$$\begin{array}{c|c} -\nabla(a(x)\nabla u_1^n) = 0 \text{ in } D_1 & -\nabla(a(x)\nabla u_2^{n+1}) = 0 \text{ in } D_2 \\ \partial_x u_1^n(0,y) = 0 \quad u_1^n(x,2+L) = u_2^{n-1}(x,2+L) & u_2^{n+1}(x,2) = u_1^n(x,2) \qquad \partial_x u_2^{n+1}(3,y) = 0 \end{array}$$

With data at step $n = 0$, $u_2^0(x,L) \equiv g_1$ and $u_1^0(x,0) \equiv g_2$.

2. 1. Write a solver computing the sequence by finite elements.

2. 2. Take for initalization of the algorithm $g_1 = \sin k\pi x$ and $g_2 = 0$, and show the iterates for various values of k and ϵ. Draw the curve of convergence of $\|u - u_j^n\|_{L_2(D_j)}$ for each k and various relevant values of h and ϵ.

3 Improvement for domain decomposition in one dimension

3. 1. Compute numerically the eigenmodes of the one-dimensional operator in x, that is (λ, v) solution of

$$(a(x)v')' + \lambda a v = 0 \text{ in } (0,3), \qquad \text{with } v'(0) = v'(3) = 0.$$

3. 2. What happens in the previous algorithm if the initial guess is the first mode ?

3. 3. (optional) Replace the transmission conditions above by

$$(d_y + p)u_1^n(x,L) = (d_y + p)u_2^{n-1}(L), \quad (d_y - p)u_2^{n+1}(0) = (d_y - p)u_1^n(0).$$