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ON ZERO-SUM SUBSEQUENCES OF RESTRICTED SIZE. IV

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Abstract. For a finite abelian group G, we investigate the invariant $\mathsf{s}(G)$ (resp. the invariant $\mathsf{s}_0(G)$) which is defined as the smallest integer $l \in \mathbf{N}$ such that every sequence S in G of length $|S| \geqq l$ has a subsequence T with sum zero and length $|T| = \exp(G)$ (resp. length $|T| \equiv 0 \mod \exp(G)$).

1. Introduction

Let $\mathbf{N}$ denote the set of positive integers and let $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. For integers $a, b \in \mathbf{Z}$ set $[a, b] = \{x \in \mathbf{Z} \mid a \leqq x \leqq b\}$, and for $c \in \mathbf{N}$ let $\mathbf{N}_{\geqq c} = \mathbf{N} \setminus [1, c-1]$. Let G be a finite abelian group with $\exp(G) = n \geqq 2$. Let $\mathsf{s}(G)$ (resp. $\mathsf{s}_0(G)$) denote the smallest integer $l \in \mathbf{N}$ such that every sequence S in G with length $|S| \geqq l$ contains a zero-sum subsequence T with length $|T| = n$ (resp. with length $|T| \equiv 0 \mod n$).

The invariant $\mathsf{s}(G)$ was first studied for cyclic groups by Erdős, Ginzburg and Ziv. For every $n \in \mathbf{N}$ denote by C_n a cyclic group with n elements. In [3], Erdős et al proved that $\mathsf{s}(C_n) = 2n - 1$. In 1983, A. Kemnitz conjectured that $\mathsf{s}(C_p^2) = 4p - 3$ for every prime $p \in \mathbf{N}$. This conjecture is still open and a positive answer would imply immediately that $\mathsf{s}(C_n^2) = 4n - 3$ for every $n \in \mathbf{N}$. The best result known so far states that $\mathsf{s}(C_q \oplus C_q) \leqq 4q - 2$ for every prime power $q \in \mathbf{N}$. For further results on $\mathsf{s}(G)$, also for groups with higher rank, we refer to [11], [1], [4], [14], [6], [7], [2].

The invariant $\mathsf{s}_0(G)$ was introduced recently in [9]. It was studied in groups of the form $G = C_n \oplus C_n$, and it turned out to be an important tool for a detailed investigation of sequences in $C_n \oplus C_n$. By definition, we have $\mathsf{s}_0(G) \leqq \mathsf{s}(G)$, and it is easy to see that equality holds for cyclic groups and

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for elementary 2-groups, for which we have $\mathsf{s}(C_2^r) = \mathsf{s}_0(C_2^r) = 2^r + 1$. The situation is different for groups G with rank two. We conjecture that $\mathsf{s}_0(C_n^2) = 3n - 2$ for all $n \geqq 2$. This conjecture holds true if n is either a product of at most two distinct prime powers or $\mathsf{s}(C_p^2) = 4p - 3$ for all primes p dividing n (cf. [9, Theorem 3.7]).

The *Davenport constant* $\mathsf{D}(G)$ of G is defined as the smallest integer $l \in \mathbf{N}$ such that every sequence S in G with length $|S| \geqq l$ contains a zero-sum subsequence. A simple argument shows that $3n - 2 \leqq \mathsf{s}_0(C_n^2) \leqq \mathsf{D}(C_n^3)$ (see [9, Lemma 3.5]). It is well known that equality holds if n is a prime power. However, it is still unknown whether $\mathsf{D}(C_n^3) = 3n - 2$ holds for every $n \in \mathbf{N}$.

The aim of this paper is to derive some unconditional results on $\mathsf{s}_0(C_n \oplus C_n)$ (i.e., results which do not rest on any unproved assumptions on $\mathsf{s}(\cdot)$ or $\mathsf{D}(\cdot)$). We formulate the main result.

THEOREM 1.1. *Let $m, n \in \mathbf{N}_{\geqq 2}$ with $n \geqq \frac{m^2-m+1}{3}$. If $\mathsf{s}_0(C_m^2) = 3m - 2$ and $\mathsf{D}(C_n^3) = 3n - 2$, then $\mathsf{s}_0(C_{mn}^2) = 3mn - 2$.*

The following corollary is known for $l \in \{1, 2\}$ (cf. [9, Theorem 3.7]).

COROLLARY 1.2. *Let $n = \prod_{i=1}^{l} q_i \in \mathbf{N}_{\geqq 2}$ where $l \in \mathbf{N}$ and $q_1, \ldots, q_l \in \mathbf{N}$ are pairwise distinct prime powers. If $3q_{i+1} \geqq q_1^2 \cdot \ldots \cdot q_i^2 - q_1 \cdot \ldots \cdot q_i + 1$ for every $i \in [2, l-1]$, then $\mathsf{s}_0(C_n \oplus C_n) = 3n - 2$.*

The proof of Theorem 1.1 rests on the recent result that $\mathsf{s}(C_q \oplus C_q) \leqq 4q - 2$ for every prime power $q \in \mathbf{N}$ (see [5]) and a suitable multiplication formula giving an upper bound for $\mathsf{s}(C_n \oplus C_n)$ for every $n \in \mathbf{N}$, which may be of its own interest.

2. Preliminaries

Throughout, all abelian groups will be written additively and for $n \in \mathbf{N}$ let C_n denote a cyclic group with n elements. Let $\mathcal{F}(G)$ denote the (multiplicatively written) free abelian monoid with basis G. An element $S \in \mathcal{F}(G)$ is called a *sequence in* G and will be written in the form

$$S = \prod_{g \in G} g^{\mathsf{v}_g(S)} = \prod_{i=1}^{l} g_i \in \mathcal{F}(G).$$

A sequence $S' \in \mathcal{F}(G)$ is called a *subsequence of* S, if there exists some $S'' \in \mathcal{F}(G)$ such that $S = S' \cdot S''$ (equivalently, $S' \mid S$ or $\mathsf{v}_g(S') \leqq \mathsf{v}_g(S)$ for every $g \in G$). If this holds, then $S'' = S'^{-1} \cdot S$. Subsequences $S_1, \dots, S_k$ of S are said to be *pairwise disjoint* if their product $\prod_{i=1}^k S_i$ is a subsequence of S. For a sequence $T \in \mathcal{F}(G)$ we set

$$\gcd(S,T) = \prod_{g \in G} g^{\min\{\mathsf{v}_g(S), \mathsf{v}_g(T)\}} \in \mathcal{F}(G).$$

As usual

$$\sigma(S) = \sum_{g \in G} \mathsf{v}_g(S) g = \sum_{i=1}^{l} g_i \in G$$

denotes the *sum of* S,

$$|S| = \sum_{g \in G} \mathsf{v}_g(S) = l \in \mathbf{N}_0$$

denotes the *length of* S and

$$\Sigma(S) = \Big\{ \sum_{i \in I} g_i \mid \emptyset \ne I \subset [1, l] \Big\} \subset G$$

is the set of all possible subsums of S. Clearly, $|S| = 0$ if and only if $S = 1$ is the empty sequence. We say that the sequence S is

- *zero-sumfree,* if $0 \notin \Sigma(S)$,
- *a zero-sum sequence* (resp. *has sum zero*), if $\sigma(S) = 0$,
- *a minimal zero-sum sequence,* if it is a non-empty zero-sum sequence and each proper subsequence is zero-sumfree.

For a finite abelian group H and a map $f: G \to H$, set $f(S) = \prod_{i=1}^l f(g_i) \in \mathcal{F}(H)$. If f is a homomorphism, then $f(S)$ has sum zero if and only if $\sigma(S) \in \ker(f)$.

Suppose that $G = C_{n_1} \oplus \ldots \oplus C_{n_r}$ with $1 < n_1 \mid \cdots \mid n_r$. It is well known that

$$1 + \sum_{i=1}^{r} (n_i - 1) \leqq \mathsf{D}(G) = \max\big\{ |S| \mid S \text{ is a minimal zero-sum sequence in } G \big\}$$

(e.g., [8, Section 3]). If G is a p-group or $r \leqq 2$, then $1 + \sum_{i=1}^r (n_i - 1) = \mathsf{D}(G)$ (cf. [12] and [13]).

3. Proof of Theorem 1.1 and Corollary 1.2

We start with the announced multiplication formula, which generalizes an old result of Harborth (see [10, Hilfssatz 2]).

PROPOSITION 3.1. *Let G be a finite abelian group, $H < G$ a subgroup and $S \in \mathcal{F}(G)$ a sequence with length $|S| \geqq \big(\mathsf{s}(H)-1\big)\exp(G/H) + \mathsf{s}(G/H)$. Then S has a zero-sum subsequence with length $\exp(H)\exp(G/H)$. In particular, if $\exp(G) = \exp(H)\exp(G/H)$, then*

$$\mathsf{s}(G) \leqq \big(\mathsf{s}(H)-1\big)\exp(G/H) + \mathsf{s}(G/H).$$

PROOF. Let $\varphi : G \to G/H$ denote the canonical epimorphism. Then S has pairwise disjoint subsequences $S_1, \ldots, S_{\mathsf{s}(H)}$ with length $|S_i| = \exp(G/H)$ such that $\varphi(S_i)$ has sum zero for every $i \in \big[1, \mathsf{s}(H)\big]$. Then the sequence

$$\prod_{i=1}^{\mathsf{s}(H)} \sigma(S_i) \in \mathcal{F}\big(\ker(\varphi)\big)$$

contains a zero-sum subsequence S' with length $|S'| = \exp(H)$, say $S' = \prod_{i \in I}\sigma(S_i)$ where $I \subset \big[1, \mathsf{s}(H)\big]$ with $|I| = \exp(H)$. Thus $\prod_{i\in I} S_i$ is a zero-sum subsequence of S with length $|I|\exp(G/H) = \exp(H)\exp(G/H)$. □

COROLLARY 3.2. *Let $n_1, n_2 \in \mathbf{N}_{\geqq 2}$ with $n_1 \mid n_2$ and $G = C_{n_1} \oplus C_{n_2}$.*

(1) *Let $l \in \mathbf{N}, q_1, \ldots, q_l \in \mathbf{N}_{\geqq 2}$, $n_1 = \prod_{i=1}^{l} q_i$ and $a, b \in \mathbf{N}_0$ such that $\mathsf{s}(C_{q_i}^2) \leqq aq_i - b$ for every $i \in [1, l]$. Then*

$$\mathsf{s}(G) \leqq 2n_2 + (a-2)n_1 - b + (a-b-1)\sum_{i=1}^{l-1}\prod_{j=1}^{i} q_j.$$

(2) *If $n_1 = \prod_{i=1}^{l} q_i$ with pairwise distinct prime powers $q_1 \leqq \ldots \leqq q_l$, then*

$$\mathsf{s}(G) \leqq 2n_1 + 2n_2 - 2 + \sum_{i=1}^{l-1}\prod_{j=1}^{i} q_j.$$

PROOF. (1) Set $H = \{q_1 g \mid g \in G\}$ whence $H \cong C_{\frac{n_1}{q_1}} \oplus C_{\frac{n_2}{q_1}}$ and $G/H \cong C_{q_1} \oplus C_{q_1}$. We proceed by induction on l. If $l = 1$, then the Theorem of Erdős–Ginzburg–Ziv and Proposition 3.1 imply that

$$\mathsf{s}(G) \leqq \left(\mathsf{s}\left(C_{\frac{n_2}{q_1}}\right) - 1\right) q_1 + \mathsf{s}(C_{q_1} \oplus C_{q_1})$$

$$\leqq \left(2\frac{n_2}{q_1} - 2\right) q_1 + (aq_1 - b) = 2n_2 + (a-2)n_1 - b.$$

If $l \geqq 2$, then induction hypothesis and Proposition 3.1 imply that

$$\mathsf{s}(G) \leqq \left(\mathsf{s}\left(C_{\frac{n_1}{q_1}} \oplus C_{\frac{n_2}{q_1}}\right) - 1\right) q_1 + \mathsf{s}(C_{q_1} \oplus C_{q_1})$$

$$\leqq \left(2\frac{n_2}{q_1} + (a-2)\frac{n_1}{q_1} - b + (a-b-1)\sum_{i=1}^{l-2}\prod_{j=1}^{i} q_{j+1} - 1\right) q_1 + (aq_1 - b)$$

$$= 2n_2 + (a-2)n_1 - b + (a-b-1)\sum_{i=1}^{l-1}\prod_{j=1}^{i} q_j.$$

(2) For every prime power $q \in \mathbf{N}$ we have $\mathsf{s}(C_q^2) \leqq 4q - 2$ by [5]. Thus the assertion follows from (1) with $a = 4$ and $b = 2$. □

Proposition 3.3. *Let $m \in \mathbf{N}_{\geqq 2}$ and $S \in \mathcal{F}(C_m \oplus C_m)$ with length $|S| \geqq 4m - 3$. If S contains some element g with multiplicity $\mathsf{v}_g(S) \geqq m - \left\lfloor \frac{m}{2} \right\rfloor - 1$, then S contains a zero-sum subsequence with length m.*

Proof. This is a special case of [7, Proposition 2.7]. □

Proof of Theorem 1.1. Let $m, n \in \mathbf{N}_{\geqq 2}$ with $n \geqq \frac{m^2 - m + 1}{3}$, $\mathsf{s}_0(C_m^2) = 3m - 2$ and $\mathsf{D}(C_n^3) = 3n - 2$. Set $G = C_{mn} \oplus C_{mn}$ and show that $\mathsf{s}_0(G) \leqq 3mn - 2$.

Let $S \in \mathcal{F}(G)$ be a sequence with length $|S| = 3mn - 2$, $H = G \oplus \langle e \rangle \cong C_{mn}^3$ a group containing G and let $f : G \to H$ be defined by $f(g) = g + e$ for every $g \in G$. Let $\varphi : H \to H$ denote the multiplication by n. Then $\ker(\varphi) \cong C_n^3$, $\varphi(G) \cong C_m^2$ and $\varphi(H) \cong C_m^3$. If $U' \in \mathcal{F}(G)$ with length $|U'| \equiv 0 \bmod m$ such that $\varphi(U')$ has sum zero, then $\sigma(U') \in \ker(\varphi)$ and $\sigma\big(f(U')\big) \in \ker(\varphi)$. Obviously, it suffices to verify that $f(S)$ contains a zero-sum subsequence. We proceed in three steps.

1. For every $h' \in \varphi(G)$ let

$$S_{h'} = \prod_{\substack{g \in G \\ \varphi(g) = h'}} g^{\mathsf{v}_g(S)},$$

and let $h \in \varphi(G)$ be such that $|S_h| = \max\big\{ |S_{h'}| \mid h' \in \varphi(G) \big\}$. Since $3n \geqq m^2 - m + 1$, we obtain that

$$|S_h| \geqq \frac{|S|}{\big|\varphi(G)\big|} = \frac{3mn - 2}{m^2} \geqq 2\big(m - \lfloor m/2 \rfloor - 1\big).$$

Let $U_1, \dots, U_{l_1}$ be pairwise disjoint subsequences of $S_h^{-1} \cdot S$ with length $|U_1| = \dots = |U_{l_1}| = m$ such that $\varphi(U_1), \dots, \varphi(U_{l_1})$ have sum zero and $W = \big(\prod_{i=1}^{l_1} U_i \cdot S_h \big)^{-1} \cdot S$ contains no subsequence U' with length $|U'| = m$ such that $\varphi(U')$ has sum zero. Then $S = U_1 \cdot \ldots \cdot U_{l_1} \cdot S_h \cdot W$, and if $m = \prod_{i=1}^{l} q_i$ with pairwise distinct prime powers $q_1 \leqq \dots \leqq q_l$, then Corollary 3.2 implies that

$$|W| \leqq 4m - 2 + \sum_{i=1}^{l-1} \prod_{j=1}^{i} q_j \leqq 4m - 2 + \lfloor m/2 \rfloor .$$

2. If $|W| \geqq 4m - 3 - \big(m - \lfloor m/2 \rfloor - 1 \big)$, then by Proposition 3.3 there exists a subsequence U_{l_1+1} of $S_h \cdot W$ with length $|U_{l_1+1}| = m$ such that $\varphi(U_{l_1+1})$ has sum zero, $\big| \gcd (U_{l_1+1}, S_h) \big| \leqq \big(m - \lfloor m/2 \rfloor - 1 \big)$ and $\big| \gcd (U_{l_1+1}, W) \big| \geqq \lfloor m/2 \rfloor + 1$.

We iterate this argument: if $\big| \gcd \, (U_{l_1+1}, W)^{-1} \cdot W \big| \geqq 4m - 3 - \big(m - \lfloor m/2 \rfloor - 1 \big)$, then by Proposition 3.3 there exists a subsequence U_{l_1+2} of $U_{l_1+1}^{-1} \cdot S_h \cdot W$ with length $|U_{l_1+2}| = m$ such that $\varphi(U_{l_1+2})$ has sum zero, $\big| \gcd (U_{l_1+2}, S_h) \big| \leqq \big(m - \lfloor m/2 \rfloor - 1 \big)$ and $\big| \gcd (U_{l_1+2}, W) \big| \geqq \lfloor m/2 \rfloor + 1$.

Since

$$\begin{aligned} |W| - 2\big(\lfloor m/2 \rfloor + 1 \big) &\leqq 4m - 2 + \lfloor m/2 \rfloor - 2\big(\lfloor m/2 \rfloor + 1 \big) \\ &\leqq 4m - 4 - \big(m - \lfloor m/2 \rfloor - 1 \big), \end{aligned}$$

there exist some $l_2 \in [0, 2]$ and pairwise disjoint subsequences $U_{l_1+1}, \dots, U_{l_1+l_2}$ of $S_h \cdot W$ with length $|U_{l_1+1}| = \dots = |U_{l_1+l_2}| = m$ such that $\varphi(U_{l_1+1}), \dots, \varphi(U_{l_1+l_2})$ have sum zero and

$$(*) \qquad \big| \gcd \, (U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2}, W)^{-1} \cdot W \big| \leqq 4m - 4 - \big(m - \lfloor m/2 \rfloor - 1 \big).$$

3. Let $U_{l_1+l_2+1}, \dots, U_{l_1+l_2+l_3}$ be pairwise disjoint subsequences of

$$\gcd \, (S_h, U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2})^{-1} \cdot S_h$$

such that $|U_{l_1+l_2+1}| = \dots = |U_{l_1+l_2+l_3}| = m$ and

$$\big| \gcd \, (S_h, U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2})^{-1} \cdot (U_{l_1+l_2+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \big| \leqq m - 1.$$

By construction of S_h, the sequence $\varphi(U_{l_1+l_2+i})$ has sum zero for every $i \in [1, l_3]$. Thus we obtain that

$$\big| (U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W \big|$$

$$\leqq 4m - 4 - \big(m - \lfloor m/2 \rfloor - 1\big) + (m-1) = 4m - 4 + \lfloor m/2 \rfloor .$$

We distinguish two cases.

Case 1: $\big|(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W\big| \leqq 4m - 4$. Then it follows that

$$l_1 + l_2 + l_3 = \frac{\big|\big((U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W\big)^{-1} \cdot S\big|}{m}$$

$$\geqq \frac{3nm - 2 - (4m-4)}{m} > 3n - 4$$

whence $l_1 + l_2 + l_3 \geqq 3n - 3$. If $l_1 + l_2 + l_3 = 3n - 3$, then

$$\big|(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W\big| = 3m - 2,$$

and since $\mathsf{s}_0\big(\varphi(G)\big) = 3m - 2$, the sequence $(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W$ contains a subsequence U_{3n-2} with length $|U_{3n-2}| \equiv 0 \bmod m$ such that $\varphi(U_{3n-2})$ has sum zero. Thus S has pairwise disjoint subsequences $U_1, \ldots, U_{3n-2}$ with length $|U_i| \equiv 0 \bmod m$ and such that $\varphi(U_i)$ has sum zero for every $i \in [1, 3n-2]$. Since $\prod_{i=1}^{3n-2} \sigma\big(f(U_i)\big) \in \ker(\varphi) \cong C_n^3$ and $\mathsf{D}(C_n^3) = 3n - 2$, the sequence $\prod_{i=1}^{3n-2} \sigma\big(f(U_i)\big)$ contains a zero-sum subsequence whence $\prod_{i=1}^{3n-2} f(U_i) = f\big(\prod_{i=1}^{3n-2} U_i\big)$ and $f(S)$ contain a zero-sum subsequence.

Case 2: $\big|(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W\big| \geqq 4m - 3$. Then $(*)$ implies that

$$\Big|\gcd\big(S_h, (U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W\big)\Big| \geqq m - \lfloor m/2 \rfloor - 1.$$

Therefore, by Proposition 3.3, the sequence $(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3})^{-1} \cdot S_h \cdot W$ has a subsequence $U_{l_1+l_2+l_3+1}$ with length $|U_{l_1+l_2+l_3+1}| = m$ such that $\varphi(U_{l_1+l_2+l_3+1})$ has sum zero. Then

$$\big|(U_{l_1+1} \cdot \ldots \cdot U_{l_1+l_2+l_3+1})^{-1} \cdot S_h \cdot W\big| \leqq 4m - 4 + \lfloor m/2 \rfloor - m < 4m - 4$$

and we continue as in Case 1. □

Proof of Corollary 1.2. We proceed by induction on l. If $l \in [1,2]$, then the assertion follows from [9, Theorem 3.7]. Suppose that $l \geqq 3$ and that for $m = \prod_{i=1}^{l-1} q_i$ we have $\mathsf{s}_0(C_m \oplus C_m) = 3m - 2$. Since $\mathsf{D}(C_{q_l}^3) = 3q_l - 2$, the assertion follows from Theorem 1.1. □

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