

INTAS PROPOSAL FOR  
South Caucasian Republics 2006 - Research Project

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- 1.1 **TITLE:**  
**K-theory, non-commutative geometry, homology theories, homotopy theory, operator and normed algebras**
- 1.1.1 **Keyword 1 : Algebra**  
**Keyword 2 : Geometry, Algebraic Geometry**  
**Keyword 3 : Topology & Manifolds**
- 1.1.2 **Free word 1 : K-theory and cyclic (co)homology**  
**Free word 2 : homotopical algebra**  
**Free word 3 : Operator and Banach algebras**
- 1.1.3 **Intended Start Date: March 2007**
- 1.1.4 **Duration: 24 Months**
- 1.2 **CONSORTIUM**  
University of Copenhagen - Denmark  
A. Razmadze Mathematical Institute - Georgia  
Institute of Mathematics and Mechanics - Azerbaijan  
University of Muenster - Germany  
Universidad de Valladolid - Spain  
Universite Paris 13, Institut Galilee - France
- 1.3 **SUMMARY**  
The project is centered around simplicial algebra and topology, (co)homology theories, algebraic K-theory and cobordism, operator and normed algebras. It is intended to make progress on the Karoubi conjecture, to develop further the interrelation between algebraic K-theory, bivariant K-theory and equivariant homology of groups and extension of these methods to the case of quantum groups. One of the main tools will be investigation of locally convex algebras by homotopical and homological methods, construction and development of the non-abelian cohomology of crossed structures, construction and study the n-fold Cech cohomology of open covers and the n-fold Cech derived functors of group valued functors.  
The second aspect of the program is to produce new information on the multiplicative structure of the symplectic cobordism ring, to give presentations of Morava K-theory and Brown-Peterson cohomology of p-groups in terms of transferred Chern classes.  
The third aspect of the program is to produce new information on the spectral characteristics of weighted type composition operators on closed subspaces of uniform algebras and structure of their eigensubspaces, on new topological radicals of normed algebras and on algebras of functions.

## 2 TEAM INFORMATION

### 2.1 Team : Copenhagen

#### 2.1.1 Team Description

The expertise areas of the Copenhagen team are non-commutative geometry (R. Nest), operator algebras (E. Christensen and S. Eilers) and algebraic topology (J. Grodal and N. Wahl). It has strong relations and ongoing cooperation both with the Tbilisi team and the three other INTAS teams.

#### 2.1.2 List of publications

- 1 The Baum-Connes Conjecture via Localization of Categories, R.Meyer, R.Nest, Topology 45 (2006) 2009-2059.
- 2 Propagating sharp group homology decompositions, J.Grodal, S.D.Smith, Advances in Math. 200 (2006), 525--538 .
- 3 From mapping class groups to automorphism groups of free groups, N.Wahl, J. London Math. Soc. (2) 72 (2005) no. 2, 510-524.
- 4 Remarks on Modules over Deformation Quantization Algebras, R.Nest, B.Tsygan, Moscow Mathematical Journal 4 (2004).
- 5 Homological stability for the mapping class groups of non-orientable surfaces, N.Wahl, preprint (2006).

#### 2.1.3 Team Leader and address

Title	Prof.
Position	Senior Scientist
Sex	Male

Date Of Birth	25/04/1950
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First Name	Ryszard
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Patronic Name	
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Family Name	Nest
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Organisation Type	Public
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Organisation Registration Nr	
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Academy / Branch	
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Organisation / University / Institute	University of Copenhagen
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Department	Department of Mathematics
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#### 2.1.4 List of Senior Scientists in the team

- 1) First Name Erik  
Patronic Name  
Family Name Christensen  
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Insitute Mathematics Institute
- 2) First Name Soren  
Patronic Name  
Family Name Eilers  
Year Of Birth 1969  
Insitute Mathematics Institute
- 3) First Name Jesper  
Patronic Name  
Family Name Grodal  
Year Of Birth 1972  
Insitute Mathematics Institute
- 4) First Name Nathalie  
Patronic Name  
Family Name Wahl  
Year Of Birth 1976  
Insitute Mathematics Institute

#### 2.1.5 Statistics

Number of Team Members involved in this project: 5  
Number of Team Members under 35: 2  
Number of Team Members who have individually received grants in INTAS projects: 0

#### 2.2 Team : Tbilisi

##### 2.2.1 Team Description

Tbilisi team is experienced in homological and homotogical algebra, non-abelian (co)homology of groups and Lie algebras (H.and N.Inassaridze, E.Khmaladze), K-theory of normed algebras and locally convex algebras (H.Inassaridze, T.Kandelaki), cyclic homology (N.Inassaridze, T.Kandelaki, G.Donadze), generalized co(homology) theories, characteristic classes, formal group laws, transfers (M.Bakuradze).  
It has strong scientific relations with the INTAS teams and new developing relation with Baku team.

##### 2.2.2 List of publications

1 Algebraic K-theory of normed algebras, H.Inassaridze, K-Theory 21(1) (2000), 25-56.

2 Equivariant homology and cohomology of groups, H. Inassaridze, Topology and its Applications 153 (2005), 66-89.

3 Transferred Chern classes in Morava K-theory, M.Bakuradze, S.Priddy, Proc. Amer. Math. Soc. 132(2004), 1855-1860.

4 KK-theory as the K-theory of C\*-categories, T.Kandelaki, Homology Homotopy Appl. 2(2000), 127-145, <http://intlpress.com/hha/>

5 N-Fold Cech derived functors and generalised Hopf type formulas, G. Donadze, N. Inassaridze, T. Porter, K-Theory (2005), in press. K-theory Preprint Archives, <http://www.math.uiuc.edu/K-theory/0624/>

### 2.2.3 Team Leader and address

Title	Prof.
Position	Head of Department
Sex	Male

Date Of Birth	06/12/1932
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First Name	Hvedri
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Patronic Name	
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Family Name	Inassaridze
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Organisation Type	Public
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Organisation Registration Nr	
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### 2.2.4 List of Senior Scientists in the team

- 1) First Name     Malkhaz  
    Patronic Name  
    Family Name   Bakuradze  
    Year Of Birth   1961  
    Insitute        A. Razmadze Mathematical Institute
- 2) First Name     Guram  
    Patronic Name  
    Family Name   Donadze  
    Year Of Birth   1972  
    Insitute        A. Razmadze Mathematical Institute
- 3) First Name     Nick  
    Patronic Name  
    Family Name   Inassaridze  
    Year Of Birth   1971  
    Insitute        A. Razmadze Mathematical Institute
- 4) First Name     Zurab  
    Patronic Name  
    Family Name   Janelidze  
    Year Of Birth   1981  
    Insitute        A. Razmadze Mathematical Institute
- 5) First Name     Tamaz  
    Patronic Name  
    Family Name   Kandelaki  
    Year Of Birth   1951  
    Insitute        A. Razmadze Mathematical Institute
- 6) First Name     Emzar  
    Patronic Name  
    Family Name   Khmaladze  
    Year Of Birth   1970  
    Insitute        A. Razmadze Mathematical Institute
- 7) First Name     Shota  
    Patronic Name  
    Family Name   Meladze  
    Year Of Birth   1981  
    Insitute        A. Razmadze Mathematical Institute

#### 2.2.5     **Statistics**

Number of Team Members involved in this project: 8  
 Number of Team Members under 35: 3  
 Number of Team Members who have individually received grants in INTAS projects: 5

#### 2.3        **Team : Baku**

##### 2.3.1     **Team Description**

Baku team consists of scientists of Institute of Mathematics and Mechanics of NAS of Azerbaijan. The team mainly applies the methods of functional analysis in the theory of Banach and normed algebras. The main interests are weighted endomorphisms of Banach algebras, weighted composition operators on uniform algebras and subspaces of function algebras; topological radicals of normed algebras; algebras of bounded functions.

### 2.3.2 List of publications

- 1 Nuclear weighted composition operators on space of analytic functions, A.I.Shahbazov, Trans. Nat. Acad. of Sciences of Azerb.25 (7) (2005), 119-124.
- 2 Extensions of the Stone-Weierstrass and Kakutani-Krein theorem to piecewise continuous functions, A.Yu.Turovskii, Transaction of NASA, XXV, N4, Baku, 2005, 5 pp.
- 3 Topological radicals I. Basic properties, tensor products and joint quasiniptence, V.S.Shulman, Yu.V.Turovskii, Banach Center Publications, 67 (2005), 293-333.
- 4 Invariant subspaces of operator Lie algebras and Lie algebras with compact adjoint action, V.S.Shulman, Yu.V.Turovskii, J. Funct. Analysis 223 (2005), 425-508.
- 5 On the eigenvalues of the substitution operators, A.I. Shahbazov, Int. Jour. Differ. Equat. Appl. 4(2) (2002), 217-221.

### 2.3.3 Team Leader and address

Title	Cand. Sc.
Position	Head of Laboratory
Sex	Male
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Patronic Name	Israfil
Family Name	Shahbazov
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Organisation Registration Nr	
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#### 2.3.4 List of Senior Scientists in the team

- 1) First Name Rafiq  
Patronic Name Chafarkhan  
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- 2) First Name Yuri  
Patronic Name Vladimirovich  
Family Name Turovskii  
Year Of Birth 1954  
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- 3) First Name Andrew  
Patronic Name Ur'evich  
Family Name Turovskii  
Year Of Birth 1980  
Insitute Institute of Mathematics and Mechanics

#### 2.3.5 Statistics

Number of Team Members involved in this project: 4  
Number of Team Members under 35: 1  
Number of Team Members who have individually received grants in INTAS projects: 0

#### 2.4 Team : Muenster

##### 2.4.1 Team Description

The Muenster team studies operator algebras and more general noncommutative algebras. It develops different versions of cyclic theory, K-theory and bivariant K-theory. We also study a broad range of related topics, such as the Baum-Connes conjecture, duality theory for Hopf algebras and groupoids or structure theory for  $C^*$ -algebras.

##### 2.4.2 List of publications

- 1 Algebraic K-theory and locally convex algebras, J.Cuntz, A.Thom, Math. Ann. 334 (2006), 339-371.
- 2 Pseudo-multiplicative unitaries and pseudo-Kac systems on  $C^*$ -modules, T.Timmermann, PhD thesis, Westf. Wilhelms-Universität Münster (2005).
- 3 Going-down functors, the Künneth formula, and the Baum-Connes conjecture, S.Echterhoff, J.Chabert, H.Oyono-Oyono, Geom. Funct. Anal. 14, (2004), 491-528.
- 4 On topologically finite-dimensional simple  $C^*$ -algebras, W.Winter, Math. Ann. 332 (2005), 843-878.

5 Excision in bivariant periodic cyclic cohomology, J.Cuntz, D.Quillen, Invent. Math. 127 (1997), 67-98.

**2.4.3 Team Leader and address**

Title	Prof.
Position	Head of Department
Sex	Male
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First Name	Joachim
Patronic Name	
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Organisation Registration Nr	
Academy / Branch	
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**2.4.4 List of Senior Scientists in the team**

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Patronic Name	
Family Name	Echterhoff
Year Of Birth	1960
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2) First Name	Thomas
Patronic Name	
Family Name	Timmermann
Year Of Birth	1977

Insitute Department of Mathematics  
 3) First Name Christian  
 Patronic Name  
 Family Name Voigt  
 Year Of Birth 1980  
 Insitute Department of Mathematics  
 4) First Name Wilhelm  
 Patronic Name  
 Family Name Winter  
 Year Of Birth 1968  
 Insitute Department of Mathematics

**2.4.5 Statistics**

Number of Team Members involved in this project: 5  
 Number of Team Members under 35: 2  
 Number of Team Members who have individually received grants in INTAS projects: 0

**2.5 Team : Valladolid**

**2.5.1 Team Description**

The group gathers together researchers from several universities in Spain. It has a strong research profile in algebraic homotopy, K-theory, cyclic homology, singularity theory, and both commutative and noncommutative algebra and geometry.

**2.5.2 List of publications**

- 1 The obstruction to excision in K-theory and in cyclic homology, G.Cortiñas, Invent. Math. 454 (2006), 143-173.
- 2 Homology of  $(n+1)$ -types and Hopf type formulas, J.M.Casas, N.Inassaridze, E.Khmaladze, M.Ladra, J. Pure Appl. Algebra 200 (2005), 267-289.
- 3 Ganea term for the homology of precrossed modules, D.Arias, M.Ladra, Comm. Algebra, to appear.
- 4 Isolated invariant curves of a foliation, N.Corrál, P.Fernández-Sánchez, Proc. Amer. Math. Soc. 134 (2006), 1125-1132.
- 5 Leibniz  $n$ -algebras, M.Casas, J.-L.Loday, T.Pirashvili, J.Forum Math. 14 (2002), 189-207.

**2.5.3 Team Leader and address**

Title	Prof.
Position	Academician
Sex	Male
Date Of Birth	15/01/1962
First Name	Guillermo
Patronic Name	
Family Name	Cortiñas
Organisation Type	Public

Organisation Registration Nr

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**2.5.4 List of Senior Scientists in the team**

- 1) First Name Daniel  
Patronic Name  
Family Name Arias Mosquera  
Year Of Birth 1976  
Insitute UNIVERSIDAD DE LEÓN
- 2) First Name José Manuel  
Patronic Name  
Family Name Casas Mirás  
Year Of Birth 1960  
Insitute Universidad de Vigo
- 3) First Name Nuria  
Patronic Name  
Family Name Corral Pérez  
Year Of Birth 1974  
Insitute Universidad de Vigo
- 4) First Name María Cristina  
Patronic Name  
Family Name Costoya Ramos  
Year Of Birth 1973  
Insitute Universidad de Santiago de Compostela
- 5) First Name Manuel  
Patronic Name

Family Name Ladra González  
Year Of Birth 1956  
Insitute Universidade Santiago de Compostela

#### 2.5.5 **Statistics**

Number of Team Members involved in this project: 6  
Number of Team Members under 35: 3  
Number of Team Members who have individually received grants in INTAS projects: 0

#### 2.6 **Team : Paris**

##### 2.6.1 **Team Description**

The Paris team involves people of University Paris 7, University Paris 13, University of Montpellier, University of Lille, as well as two persons in Great Britain (university of Bristol and University of Glasgow. The team mainly covers three directions : algebraic K-theory (Max Karoubi), cobordism theory and characteristic classes (W. Vershinin, A. Baker, A. Lazarev) and also Nigel ray in Manchester who is a great expert in symplectic cobordism; and functor categories, homological algebra and operads(B. Fresse, B. Richter, L. Schwartz). Most of the people involved have had some relations in the past with the group of Tbilisi. It is not only composed of people of one institution because it does not make sense to create different teams, all the people in the team are either close geographically and/or scientifically and in any case meet very often. It should also be said that there is in this team many doctoral students, as well as post-doctoral working in the various fields under consideration. For Paris in a strict sense let us mention A. Djament, Vespa, S. Rairat. It has not pertinent to list them here for they may (and will) travel in the 2 next years, also new students may arrive (N. Karamanov e.g.) who could be interested. Thus a list would restrictive.

##### 2.6.2 **List of publications**

1 Un theoreme d'annulation en cohomologie de Mac Lane, G. Gaudens, L. Schwartz, CRAS Serie A (2005).  
2 Combinatorial operad actions on cochains, C.Berger, B.Fresse, J. Math. Proc. Cambridge Philos. Soc. 137 (2004), 135-174.  
3 Dieudonné modules and p-divisible groups associated with Morava K-theory of Eilenberg-MacLane spaces, V. Buchstaber, A. Lazarev, math.AT/0507036.  
  
The Stasheff model of a simply-connected manifold and the string bracket, A. Lazarev, math.AT/0512596.  
4 Invertible modules for commutative S-algebras with residue fields, A.Baker, B.Richter, Manuscripta Mathematica 118 (1), 2005, 99-119.  
5 Morava K-theory rings for the dihedral, semi-dihedral and generalized quaternion groups in Chern Classes, M.Bakuradze, V.V.Vershinin, Proc. AMS (2005), in press.

##### 2.6.3 **Team Leader and address**

Title	Prof.
Position	Senior Scientist
Sex	Male
Date Of Birth	07/05/1953

First Name	Lionel
Patronic Name	
Family Name	Schwartz
Organisation Type	Public
Organisation Registration Nr	
Academy / Branch	
Organisation / University / Institute	Universite Paris 13, Institut Galilee
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Laboratory	Laboratoire Analyse Geometrie et Applications LAGA UMR /%#) CNRS
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#### 2.6.4 List of Senior Scientists in the team

- 1) First Name Andrew  
Patronic Name  
Family Name Baker  
Year Of Birth 1953  
Insitute University of Glasgow
- 2) First Name Benoit  
Patronic Name  
Family Name Fresse  
Year Of Birth 1970  
Insitute Universite de Lille 1
- 3) First Name Max  
Patronic Name  
Family Name Karoubi  
Year Of Birth 1938  
Insitute Universite de Paris 7

- 4) First Name Andrey  
Patronic Name  
Family Name Lazarev  
Year Of Birth 1968  
Insitute University of Bristol
- 5) First Name Nigel  
Patronic Name  
Family Name Ray  
Year Of Birth 1945  
Insitute University of Manchester
- 6) First Name Birgit  
Patronic Name  
Family Name Richter  
Year Of Birth 1971  
Insitute University of Hamburg
- 7) First Name Vladimir  
Patronic Name  
Family Name Vershinin  
Year Of Birth 1952  
Insitute Universite de Montpellier 2

2.6.5 **Statistics**

Number of Team Members involved in this project: 8

Number of Team Members under 35: 0

Number of Team Members who have individually received grants in INTAS projects: 0

- (1) To develop the theory of smooth K-functors on the category of locally convex algebras over the field of real (or complex) numbers with applications to K-theory.
- (2) To prove the smooth Karoubi's conjecture on the isomorphism of algebraic and topological K-functors in the category of Frechet algebras over the real (or complex) numbers for the negative and mod q cases.
- (3) To study the complex oriented cohomology rings for finite groups.
- (4) To obtain Hopf type formulas for cyclic homology of associative algebras and relevant calculations.
- (5) To introduce and investigate cyclic derived functors of functors in abelian categories.
- (6) To continue the study of H.Inassaridze's equivariant (co)homology of groups, its computations and applications in algebraic K-theory and equivariant homotopy theory.
- (7) To investigate n-fold Cech derived functors, their relation with non-abelian derived functors and to apply to various homology theories and K-theory.
- (8) To study bornological algebras from K-theory point of view.
- (9) To construct and study spectra characterising KK-theory.
- (10) To describe the cohomology of Leibniz n-algebras by crossed extensions.
- (11) To study the compact and nuclear operators on uniform algebras with analytically structure and describe their eigenvalues, eigensubspaces.
- (12) To study ingredients of the theory of topological radicals and investigate the new topological radicals.
- (13) To study algebras of bounded functions on compact spaces.

### 3.2 Background and Justification

All teams of the consortium have close relationship and similar scientific interests in homotopy theory, K-theory, simplicial algebra, locally convex algebras and homology theories. This project will help to continue and intensify previous scientific cooperation between these teams. For instance this project will intensify already existing collaboration between Paris 13 and Tbilisi teams in homological algebra, cobordism, K-theory, characteristic classes and functor categories. This project will intensify scientific cooperation of Tbilisi team with teams of Muenster, Copenhagen and Valladolid in K-theory and cyclic homology. This project will promote the scientific collaboration of Tbilisi team with Baku team in structural and homotopical investigation of Banach algebras.

One of the leading centers of homological and homotopical algebra, homology theory and K-theory in the former Soviet Union was in Tbilisi at A.Razmadze Mathematical Institute which has intensive scientific collaborations to the leading centers in all over the world and particularly in Europe. Members of the Tbilisi team participated in many joint scientific projects, workshops and conferences and were invited as visiting professors. They have close scientific collaboration with most participants.

Members of the participating teams have joint papers on topics related to the project.

- H. and N. Inassaridze visited the University of Paris 13 to give talks at the algebraic topology seminar in 2006.
- Bakuradze and Kandelaki visited the Universities of Paris 13 and Montpellier to attend workshops and to give talks;
- Bakuradze and Vershinin (with Jibladze) wrote 2 joint papers;
- Leaders of Paris 13, Valladolid and Tbilisi teams are collaborating and developing of a new international electronic journal “Journal of Homotopy and Related Structures”.
- H. Inassaridze visited the University of Santiago de Compostela for one month in 2001 and 2004 and gave talks at the algebra seminar.
- N. Inassaridze and Khmaladze visited several times the University of Santiago de Compostela during 2001–2006 to give talks at the algebra seminar and to work on joint papers. N. Inassaridze and Khmaladze wrote 7 joint papers with Ladra and Casas.
- H. and N. Inassaridze, Kandelaki and Bakuradze are invited to attend a workshop at the University of Copenhagen in September, 2006.

All team leaders and most of participants of the consortium are well known experts in current fields of mathematics and particularly in the above mentioned common interests. H.Inassaridze is a leading expert in homological algebra and K-theory [1–3]. He recently developed a new theory of equivariant (co)homology of groups with applications in algebraic K-theory [4]. H.Inassaridze and Kandelaki made substantial contributions in the K-theory of normed algebras and in the investigation of Karoubi’s conjecture on the isomorphism of algebraic and topological K-theories by introducing smooth K-functors and in Kasparov’s bivariant K-theory [5–8]. N.Inassaridze (with Porter and Donadze)

gave substantial contributions to establishing Hopf type formulas for integral homology of groups and algebraic K-functors [9]. H. and N.Inassaridze, Khmaladze and Ladra made important contributions to non-abelian (co)homology of groups and Lie algebras [10–12, 19]. Casas, N.Inassaridze, Khmaladze and Ladra made substantial contributions to the homotopy of  $(n+1)$ -types [13]. Bakuradze and Vershinin (with Jibladze) have derived transfer methods for the investigation of the multiplicative structure in symplectic and self-conjugate cobordism [14, 15]. Bakuradze (with Priddy) has studied the interaction of transfer and characteristic classes in complex oriented cohomology theories [16, 17].

The Baku team is working successfully in the structural theory of operator and Banach algebras [22-34].

Karoubi, Cuntz, Nest and Cortinas are leading experts in K-theory, cyclic homology and non-commutative geometry. Schwartz is a great expert in cohomology theory of functors and Mac Lane cohomology.

The other point of the project is the study the eigenvalues, eigensubspaces of compact, nuclear weighted composition operators on uniform algebras, topological properties of ideals of normed algebras, and algebras of bounded functions. In particular, topological properties of ideals of normed algebras are reflected in the theory of topological radicals of normed algebras, the new branch of theory of normed algebras. On the other hand, the problem of compactness of weighted composition operators has the rich history that begins with the works of E.A.Nordgren, G.C.Schwartz, J.Shapiro, H. Kamowitz, E.A.Gorin, R.K.Singh, G.Takagi etc.

The aim of the project is the study of homological and homotopical properties of algebraic and topological structures, and also topological properties of operator and normed algebras. Working around topics related to the project will greatly benefit the research activity of all teams and will lead to stimulating productive exchanges of ideas and information among team participants through scientific visits, workshops, conferences and internet.

## References

1. H.Inassaridze, *Algebraic K-theory*, Kluwer Academic Publishers, Amsterdam, 1995.
2. H.Inassaridze, *Non-abelian homological algebra and its applications*, Kluwer Academic publishers, Amsterdam, 1997.
3. H.Inassaridze, *Algebraic K-theory of normed algebras*, K-Theory 21, No1 (2001), 25-56.
4. H.Inassaridze, *Equivariant homology and cohomology of groups*, Topology and its Applications 153 (2005), 66-89.
5. H.Inassaridze and T.Kandelaki, *K-theory of stable generalized operator algebras*, K-Theory 27 (2002), 103-110.
6. H.Inassaridze and T.Kandelaki, *Smooth K-theory of locally convex algebras*, 2006, arXiv: math.KT/0603095.

7. T.Kandelaki, KK-theory as the K-theory of C\*-categories, *Homology, Homotopy and Applications*, Vol.2, No2 (2000), 127-145.
8. T.Kandelaki, Algebraic K-theory view on KK-theory, Preprint, 2003.
9. N.Inassaridze (with G.Donadze and T.Porter), *N-fold Cech derived functors and generalized Hopf type formulas*, K-Theory, 2005 (in press).
10. H.Inassaridze, *Higher non-abelian cohomology of groups*, Glasgow Math. J. 44 (2002), 497-520.
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12. N.Inassaridze, E.Khmaladze and M.Ladra, *Non-abelian homology of Lie algebras*, Glasgow Math. J. 46 (2004), 417-429.
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**4**                    **SCIENTIFIC / TECHNICAL DESCRIPTION**  
**4.1**                  **Research Programme**

As said in the document giving the backgrounds of the network the consortium is formed by teams having broad scientific interests, but most teams use simplicial algebra and/or homotopy theory and homological algebra either as central means, either as objects of study themselves. Particular central in the network are certain techniques concerning equivariant homology, locally convex algebras, non-abelian cohomology, cyclic homology, operator algebras. These topics have their own internal interest, however they are, to a very large extent, motivated by their present or potential applications in other fields in of K-theory or algebraic topology. As examples are the applications to algebraic K-theory, cohomology of algebras, structure of ring spectra. The principal aim of the consortium is to promote cooperation between the various nodes so as the nodes can share their expertise.

Precise tasks and scientific objectives are described in details elsewhere, main titles are as follows:

1. Algebraic and topological K-theories (including bivariant K-theory)
2. Cyclic homology
3. Homotopical investigation of locally convex algebras
4. Generalized cohomology, cobordism
5. Operator algebras and algebras of functions,
6. Topological radicals of normed algebras
7. Non-abelian, equivariant and mod  $q$  (co)homologies
8. Derived functors

Items 7 and 8 are strongly using related techniques. These techniques have potential applications in items 1 and 2.

The following results are expected:

Main functorial properties of smooth K-functors in the category of locally convex algebras over the field of real (or complex) numbers. Proof of the isomorphism between algebraic and smooth K-functors for quasi-stable Frechet algebras (including the negative and mod  $q$  cases). Explicite computations of equivariant Tor-functors for finite cyclic groups. Higher Hopf formulas for equivariant integral homology. Construction of K-theory based on the bornological structure and relationship with homology of bornological algebras. Construction of spectra characterizing KK-theory with applications to problems related to Novikov and Baum-Connes conjecture. Continuation of the investigation of the mod  $q$  Tate-Farell-Vogel cohomology of finite groups. Relationship of Brown-Peterson cohomologies' generating relations with transferred Chern classes in Morava K-theory and complex oriented cohomology rings of classifying spaces of finite groups. Hopf type formulas for cyclic homology of associative algebras and low dimensional calculations. Homotopy classification theorems for symmetric group-graded categorical groups. Sufficient conditions for the compatibility of the left derived functors of the projective limit of nilization functors with respect to this projective limit. Construction of cyclic derived functors and applications to

cyclic homology. Connes' exact sequence relating (Cech) derived functors and cyclic (Cech) derived functors. Compactness and nuclearity criterion of weighted composition type operators on different uniform algebras and structure of their eigensubspaces. Topological radicals connected with joint quasinilpotence and Jacobson radical, scattered radical, tensor radicals, radicals connected with multiplication operators, their properties and structure, procedures over radicals. Structure of algebras of bounded functions on a compact space.

There is going to be all along the duration of the network a certain number of congresses in order to enhance discussion and collaboration. Other initiatives will be taken.

## 4.2 Project Structure

### 4.2.1 Task Title : Algebraic and topological K-theories

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi

#### **Objectives :**

To prove Karoubi's conjecture on the isomorphism of algebraic and topological K-functors for a wide class of real or complex topological algebras.

#### **Methodology :**

methods of simplicial algebra, K-theory, category theory and Banach algebras.

#### **Task Input:**

Karoubi's conjecture was confirmed by Higson for Karoubi-Villamayor algebraic K-functors and by Suslin and Wodzicki for Quillen algebraic K-functors. It was proved by H.Inassaridze for polynomial extensions of stable  $C^*$ -algebras and by H.Inassaridze and Kandelaki for stable generalised operator algebras. Recently H.Inassaridze and Kandelaki proved the isomorphism of algebraic and smooth K-functors for real or complex quasi stable Frechet algebras having a uniformly bounded approximate unit. Kandelaki proved that Kasparov  $KK$ \_theory may be expressed by algebraic and topological K-theories.

#### **Result, milestones :**

Isomorphism of algebraic and topological K-functors for a wide class of real or complex locally convex algebras (including the negative and mod  $q$  cases).

### 4.2.2 Task Title : Homotopical and structural investigation of locally convex algebras

Task coordinator : Joachim Cuntz, belonging to team: Muenster

#### **Objectives :**

Study of smooth homotopy functors, particularly smooth K-functors, on the category of real or complex locally convex algebras.

#### **Methodology :**

Methods of K-theory, homotopy theory and Banach algebras.

#### **Task Input:**

The task is depending on : Algebraic and topological K-theories  
The smooth K-functors were recently introduced by H.Inassaridze and Kandelaki to prove the isomorphism of algebraic and topological K-functors for real or complex quasi stable Frechet algebras having a uniformly bounded approximate unit. Recent works of Cuntz on KK-theory for locally convex algebras.

**Result, milestones :**

Continuation of the investigation of functorial properties of smooth K-functors and their relationship with algebraic and topological K-functors.

4.2.3 **Task Title : Cobordism; Complex oriented cohomology rings, Morava K-theories, braid groups**

Task coordinator : Malkhaz Bakuradze, belonging to team: Tbilisi

**Objectives :**

To produce new information on the multiplicative structure of the symplectic cobordism ring. To use transferred characteristic classes for more natural presentation of the complex oriented cohomology rings of finite groups.

To try to extend the Atiyah Segal theorem to more general cohomology oriented theories.

To study the braid groups.

**Methodology :**

Machinery of formal group laws, characteristic classes, transfers and spectral sequences.

The recent and sophisticated techniques concerning spectra may be useful in the context.

Presentation of groups, combinatorics.

**Task Input:**

Various finite groups are good in the sense that their complex oriented cohomology rings are generated by transferred Chern classes. Recently Bakuradze and Priddy completely determined the multiplicative structure for  $\Sigma_p \wr U(1)$ , the universal example, in various complex oriented cohomology theories.

Work of Hoipkins, Kuhn and Ravenel.

**Result, milestones :**

Presentation of Morava K-theory and Brown-Peterson cohomology of finite groups in terms of transferred Chern classes.

To study generalisation of the Atiyah Segal theorem in the context of Morava K-theories.

4.2.3.1 **Task Title : Symplectic cobordism and braid groups**

Task coordinator : Vladimir Vershinin, belonging to team: Paris

**Objectives :**

This sub task involves two aspects of the work of W. Vershinin, partly done in

cooperation with M. Bakuradze.

The main one in the context is the study of relations between Ray's generators for symplectic cobordism. However the second one concerning presentations of the braid group may be of interest also here.

**Methodology :**

As in the task and also group theoretic constructions and geometric ones. Presentation of groups, combinatorics.

**Task Input:**

The task is depending on : Cobordism; Complex oriented cohomology rings, Morava K-theories, braid groups  
Work of Bakuradze and Vershinin using transfer technique and obtaining new information on the multiplicative structure of the symplectic cobordism ring.

**Result, milestones :**

In particular a better understanding of the multiplicative structure of the symplectic cobordism.

**4.2.3.2 Task Title : Complex oriented cohomology rings for finite groups, Morava K-theories**

Task coordinator : Malkhaz Bakuradze, belonging to team: Tbilisi

**Objectives :**

To use transferred characteristic classes for more natural presentation of the complex oriented cohomology rings of finite groups.  
To try to extend the Atiyah Segal theorem to more general cohomology oriented theories; i.e. interpreting the cohomology (for certain theories) of elementary abelian p-groups as a permutation module.  
As well (Lazarev) one is interested in considering the structure of more the Morava k-theories general spaces (see Buschtaber, Lazarevg math.AT/0507036)

**Methodology :**

Machinery of formal group laws, characteristic classes, transfers and spectral sequences.  
The recent and sophisticated techniques concerning spectra may be useful in the context.

Representation theory.

**Task Input:**

The task is depending on : Cobordism; Complex oriented cohomology rings, Morava K-theories, braid groups  
Various finite groups are good in the sense that their complex oriented cohomology rings are generated by transferred Chern classes. Recently Bakuradze and Priddy completely determined the multiplicative structure for  $\Sigma_p$  wr  $U(1)$ , the universal example, in various complex oriented cohomology theories. The principal task being to compute Chern classes of transferred bundle in terms of its transferred Chern roots.  
Work of Hopkins Kuhn and Ravenel.

**Result, milestones :**

Presentation of Morava K-theory and Brown-Peterson cohomology of finite groups in terms of transferred Chern classes.  
Generalization of the Atiyah Segal theorem.

**4.2.4 Task Title : Leibniz n-algebras, crossed modules, foliations, and homotopy types**

Task coordinator : Manuel Ladra, belonging to team: Valladolid

**Objectives :**

The investigation of Leibniz n-algebras will be continued and Poincaré-Birkhoff-Witt theorem for finite dimensional Leibniz n-algebras will be proved. Theory of crossed modules for Leibniz n-algebras will be developed. General theory of universal central extensions for Lie n-algebras, Leibniz n-algebras and catn-groups will be developed and applications in multirelative K-theory will be obtained. Some of the algebraic notions of Lie structures will be applied to the study of foliations.

**Methodology :**

Methods and techniques of homological algebra, algebraic homotopy theory, algebraic K-theory, algebraic Lie theory and noncommutative (computational) algebra (e.g. Gröbner basis).

**Task Input:**

The team has a strong production in all the subjects of the task. Leibniz n-algebras were introduced by J. M. Casas, J.-L. Loday and T. Pirashvili. A joint paper by J. M. Casas, E. Khmaladze y M. Ladra investigated their solvability and nilpotency. The universal central extension via crossed and precrossed modules and its relation with Milnor's singly relative  $K_2$  were studied by Casas and Ladra. N. Corral has succesfully applied Lie theory to the study of the invariant curves of a foliation. Results on the localization of homotopy categories were obtained by M. Costoya.

**Result, milestones :**

See the subtasks descriptions.

**4.2.4.1 Task Title : Poincaré-Birkhoff-Witt theorem and crossed modules for Leibniz n-algebras**

Task coordinator : Manuel Ladra, belonging to team: Valladolid

**Objectives :**

1. To state and prove the Poincaré-Birkhoff-Witt theorem for finite dimensional Leibniz n-algebras.
2. To develop crossed module theory for Leibniz n-algebras and to obtain an interpretation of the second cohomology group of Leibniz n-algebras via crossed extensions.

**Methodology :**

Methods and techniques of algebraic Lie theory, noncommutative (computational) algebra (e.g. Gröbner basis), crossed modules and category theory.

**Task Input:**

Leibniz n-algebras were introduced and their cohomology was constructed by J. M. Casas, J.-L. Loday and T. Pirashvili. A joint paper by J. M. Casas, E. Khmaladze and M. Ladra investigated solvability and nilpotency of Leibniz n-algebras. The interpretation of the third Leibniz cohomology (i.e. the second cohomology of a Leibniz 2-algebra) by crossed modules was obtained by C. Cuvier.

**Result, milestones :**

To develop the theory of Leibniz  $n$ -algebras. To generalize the classical Poincaré-Birkhoff-Witt theorem for Leibniz  $n$ -algebras by using techniques of Gröbner bases. To introduce and investigate crossed modules for Leibniz  $n$ -algebras, to prove their equivalences with 1-cat Leibniz  $n$ -algebras and internal categories. To extend for Leibniz  $n$ -algebras the well known result relating the third cohomology and crossed modules.

**4.2.4.2 Task Title : Homotopy (n+1)-types, Leibniz n-algebras and foliations**

Task coordinator : Jose Manuel Casas, belonging to team: Valladolid

**Objectives :**

1. To study central extensions of crossed squares and catn-groups (homotopy (n+1)-types). To describe the universal central extension of a catn-group via the homology of a catn-group and their relationship with multirelative K-theory.
2. To develop a general theory of universal extensions for Lie and Leibniz  $n$ -algebras. To compare via Liezation functor the universal central extensions of a perfect Lie  $n$ -algebra in the categories of Lie and Leibniz  $n$ -algebras.
3. To apply to the study of foliations some of the algebraic notions of Lie structures.

**Methodology :**

Methods and techniques of homological algebra, algebraic K-theory, algebraic homotopy theory and algebraic Lie theory.

**Task Input:**

The universal central extension via crossed and precrossed modules and its relation with Milnor's singly relative  $K_2$  were studied by J. M. Casas and M. Ladra. These two authors with N. Inassaridze and E. Khmaladze have investigated homology of catn-groups in the context of cotriple homology and obtained Hopf formulas. N. Corral has successfully applied Lie theory to the study of the invariant curves of a foliation. Results on the localization of homotopy categories were obtained by M. Costoya.

**Result, milestones :**

To develop the theories of Lie  $n$ -algebras, Leibniz  $n$ -algebras and catn-groups (homotopy (n+1)-types). Successful application of Lie-algebraic methods to the study of foliations, and of localization of homotopy to rational homotopy types.

**4.2.5 Task Title : Weighted composition operators on unifom algebras**

Task coordinator : A.I.Shahbazov, belonging to team: Baku

**Objectives :**

We will describe eigenvalues, eigenfunctions, dimension of eigen subspaces of compact weighted composition operators on uniform algebras with analytically structures and will investigate nuclearity of weighted composition operators.

**Methodology :**

The methods of Banach algebras, Theory operators and theory of functions.

**Task Input:**

Works of H.Kamowitz, B.Montador, E.A.Gorin..

The problem is to find the compactness, weakly compactness criterion of weighted composition operators on different uniform algebras.

**Result, milestones :**

Compactness, weakly compactness and nuclearity criterion of weighted composition operators on different uniform algebras. Description of the eigenvalues, the eigenfunctions of compact weighted composition operators on uniform algebras with analytic structures.

**4.2.6 Task Title : Topological radicals of normed algebras**

Task coordinator : Turovskii Yu. V., belonging to team: Baku

**Objectives :**

Investigation of topological properties of normed algebras accumulated into closed ideals.

**Methodology :**

Methods of normed algebras, functional analysis

**Task Input:**

Algebraic theory of radicals of rings, the work of P.G. Dixon on topological radicals (1997)

**Result, milestones :**

design of topological radicals by using elementary maps, ingredients of the theory of topological radicals: more simple maps, operations and procedures over topological radicals, introducing new radicals

**4.2.7 Task Title : K-theory, cyclic homology, cdh cohomology**

Task coordinator : Guillermo Cortiñas, belonging to team: Valladolid

**Objectives :**

Various aspects of algebraic K-theory and cyclic homology will be studied, including computations in concrete examples, as well as some theoretical results, including advancing towards solving conjectures such as Vorst's, Bass', Geller's and Berger's, finding a cyclic homology description as cotriple derived functors, and studying cyclic derived functors.

**Methodology :**

Methods of cyclic homology, perturbation and computer algebra techniques (for cyclic homology computations), excision, cdh-cohomology, desingularization (for infinitesimal K-theory computations), triangulated categories (for bivariant  $kk$ ), non-abelian and  $n$ -fold Čech derived functors.

**Task Input:**

The task is depending on : Algebraic and topological K-theories  
The task is depending on : Cyclic homology, K-theory, cdh cohomology, desingularization techniques, perturbation theory, computer algebra methods. This is made precise in the subtasks descriptions.

**Result, milestones :**

See the subtasks descriptions.

**4.2.7.1 Task Title : Chern characters from K-theory to cyclic homology.**  
Task coordinator : Guillermo Cortiñas, belonging to team: Valladolid

**Objectives :**

- 1.To make concrete computations of K-theory in several examples, of both commutative and noncommutative nature. This includes examples coming from (commutative) algebraic geometry, such as hypersurfaces, as well as noncommutative nilpotent extensions of commutative algebras.
- 2.To prove results in the direction of several conjectures (such as Vorst's, Bass', Geller's and Berger's) relating nonsingularity with the vanishing of K, Hochschild and cyclic homology groups.
- 3.To study the interplay between algebraic and topological K-theory. This includes both studying the algebraic K-theory of topological algebras, and applying methods and ideas from topological bivariant K-theory in a completely algebraic setting.

**Methodology :**

Perturbation and computer algebra techniques (for cyclic homology computations), excision, cdh-cohomology, desingularization (for infinitesimal K-theory computations), triangulated categories (for bivariant  $kk$ ).

**Task Input:**

The task is depending on : K-theory, cyclic homology, cdh cohomology  
Through the Chern character, K-theory breaks into two pieces; cyclic homology and infinitesimal K-theory. The first is approachable for direct computations (e.g. via perturbation techniques, as amply shown by work of J.A. and J.J. Guccione and, in some specific cases, computer algebra methods, used by G. Cortiñas and M. de León), and the second (introduced by Cortiñas) is better behaved; it is nilinvariant (Goodwille's theorem), excisive (Cortiñas') has Meyer-Vietoris sequences for blowups (Cortiñas, C. Haesemeyer, M. Schlichting, C. Weibel). V. Voevodsky introduced the cdh-site. Passing from any cohomology theory  $H$  of schemes over a field of characteristic zero to its cdh-fibrant replacement  $H_{cdh}$  obtains a new cohomology theory with many good properties, including Meyer-Vietoris sequences for blowups. There is a map  $H \rightarrow H_{cdh}$ ; if  $H$  is reasonably well-behaved on smooth schemes, it is an isomorphism for such schemes. One hopes to understand  $H$  of a singular scheme by studying first  $H_{cdh}$ , which, via desingularization, is computable in terms of  $H$  applied to smooth schemes, and then the fiber  $FH$  of the map above. For example, Haesemeyer showed that  $K_{cdh}$  is  $KH$ , homotopy K-theory; Cortiñas, Haesemeyer, M. Schlichting and Weibel proved that  $FK = FHC$ .  
J. Cuntz introduced bivariant topological K-theory of locally convex  $C$ -algebras, and showed with A. Thom, that for stable  $L$ ,  $kk_0^{\{top\}}(C, L) = K_0(L)$ . Cortiñas and Thom obtained an algebraic  $kk$  defined for rings and proved  $kk_*(Z, A) = KH_*(A)$ .

**Result, milestones :**

Development of techniques and algorithms for the computation of K and cyclic homology groups in some examples. Results in the direction of vanishing conjectures for nonsingularity. Development of a common ground for algebraic and topological K-theory which allows for a better understanding of their relationship.

**4.2.7.2 Task Title : Cyclic homology and derived functors**  
Task coordinator : Nick Inassaridze, belonging to team: Tbilisi

**Objectives :**

1. To investigate cyclic homology of (non-unital) associative algebras in a Hopf type formula point of view.
2. To introduce and develop cyclic derived and cyclic (n-fold) Cech derived functors of functors in abelian categories.

**Methodology :**

Methods of cyclic (co)homology and non-abelian derived functors; techniques of n-fold Cech derived functors, developed by Donadze, N.Inassaridze and Porter, and spectral sequences.

**Task Input:**

The task is depending on: Works of Barr-Beck, H.Inassaridze and Pirashvili on non-abelian, cotriple derived functors. Initial works of Brown and Ellis on generalized Hopf formulas for higher integral group homology. Recent works of Donadze, N.Inassaridze and Porter on n-fold Cech derived functors and generalised Hopf type formulas. Classical works on cyclic homology.

**Result, milestones :**

To fit the cyclic homology into the context of cotriple homology in the sense of Barr-Beck. In particular, to describe the cyclic homology of (non-unital) associative algebras, in zero characteristic case, as cotriple derived functors of additive abelianization functor. To give the formulas of Hopf type in the sense of Brown-Ellis for cyclic homology of (non-unital) algebras. To introduce and investigate the notions of cyclic derived functors and cyclic Cech derived functors of functors with values in the category of modules, giving applications to cyclic homology. To obtain Connes' exact sequence relating (Cech) derived functors and cyclic (Cech) derived functors.

**4.2.7.3 Task Title : Functor categories, operads, homology theories**

Task coordinator : Lionel Schwartz, belonging to team: Paris

**Objectives :**

To study the structure of certain functor categories, in particular to determine if they are artinian or not (in the context of functors from vector spaces to vector spaces), to determine simple objects in other cases.

To make various computations in homology theories linked to this context (Mac Lane and Hochschild homology).

**Methodology :**

Homological algebra, category theory.  
Operad techniques.

**Task Input:**

The task is depending on : K-theory, cyclic homology, cdh cohomology  
Work of G. Powell, C. Vespa, L. Schwartz concerning the category  $\mathcal{F}$ .  
Work of Lannes, Franjou, Schwartz in Mac Lane homology.  
Work of Kuhn on generic representation theory.

**Result, milestones :**

As said above the main goal is a better understanding of the structure of the categories that are considered. Applications either in representation theory and group cohomology, either in topology would come in a second step.

**4.2.8 Task Title : Spectra and bivariant K-theory**

Task coordinator : Tamaz Kandelaki, belonging to team: Tbilisi

**Objectives :**

To investigate  $C^*$ -categories from K-theory point of view and relationship with KK-theory.

**Methodology :**

Using and generalizing some methods and results of equivariant KK-theory over locally compact second countable group, homological algebra, spectras, algebraic K-theory, KK-theory,  $C^*$ -categories.

**Task Input:**

The task is based on the results of Connes, Cuntz, Higson, Kasparov, Meyer, Pushnigg, Suslin, Wodzicki, Tomsen in KK-theory, cyclic homology, algebraic K-theory and theory of Fredholm modules; also on the results of Kandelaki in KK-theory and on the  $C^*$ -category of Fredholm modules over  $C^*$ -algebras.

**Result, milestones :**

A functor having matrix stability and excision property on the category of  $C^*$ -algebras, will be extended to the category of  $C^*$ -categories and to more general categories.  $C^*$ -categories will be established for which such functors and their stabilization are isomorphic. Applications in cyclic homology and KK-theory will be done.

A  $C^*$ -category of Fredholm Modules over  $C^*$ -algebras with action of a fix locally compact group will be constructed. The relationship between algebraic K-theory of this category and Kasparov's equivariant KK-theory will be established. Applications to problems related to Novikov conjecture will be done.

**4.2.9 Task Title : Equivariant cohomology of groups**

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi

**Objectives :**

Further study of equivariant (co)mology of groups with applications in algebraic K-theory

**Methodology :**

Methods of homological and homotopical algebra, techniques of simplicial algebra and K-theory

**Task Input:**

The task is depending on : Algebraic and topological K-theories  
The study of groups with operators has many important applications in algebra and topology. See, for instance, recent results in equivariant stable homotopy and equivariant algebraic K-theory. The origin of the equivariant investigation in homological algebra goes back to Whitehead. Recently a theory of equivariant homology and cohomology of groups was constructed and developed by H.Inassaridze with applications in algebraic K-theory. Another different theory of

cohomology of groups with operators was given by spanish mathematicians motivated by the graded categorical groups classification problem.

**Result, milestones :**

Equivariant (non-abelian) extensions of groups and relationship with equivariant cohomology groups of H.Inassaridze. Obstruction to the existence of equivariant non-abelian extensions and the third equivariant cohomology of groups. Finite cyclic groups and group actions for which equivariant (co)homology groups could be computed. Relationship of the third Quillen K-functor with equivariant integral homology of groups.

**4.2.9.1 Task Title : Equivariant group cohomology as relative derived functors**

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi

**Objectives :**

Further study of equivariant (co)homology of groups with applications in algebraic K-theory

**Methodology :**

Methods of homological and homotopical algebra, techniques of simplicial algebra and K-theory

**Task Input:**

The task is depending on : Equivariant cohomology of groups  
The study of groups with operators has many important applications in algebra and topology. See, for instance, recent results in equivariant stable homotopy and equivariant algebraic K-theory. The origin of the equivariant investigation in homological algebra goes back to Whitehead. Recently a theory of equivariant homology and cohomology of groups was constructed and developed by H.Inassaridze with applications in algebraic K-theory. Another different theory of cohomology of groups with operators was given by spanish mathematicians motivated by the graded categorical groups classification problem.

**Result, milestones :**

Equivariant (non-abelian) extensions of groups and relationship with equivariant cohomology groups of H.Inassaridze. Obstruction to the existence of equivariant non-abelian extensions and the third equivariant cohomology of groups. Finite cyclic groups and group actions for which equivariant (co)homology groups could be computed. Relationship of the third Quillen K-functor with equivariant integral homology of groups.

**4.2.9.2 Task Title : Genralization of Van Kampen theorems in the equivariant set-up**

Task coordinator : Emzar Khmaladze, belonging to team: Tbilisi

**Objectives :**

To develop non-abelian equivariant homology of groups. To state and prove the generalized Van Kampen theorems in the equivareiant set-up.

**Methodology :**

Methods of (non-abelian) homological algebra and equivariant homotopy theory.

**Task Input:**

The task is depending on : Equivariant cohomology of groups  
The non-abelian homology theory of groups was developed by H.Inassaridze and N.Inassaridze. This theory is based on a commutator theoretic construction in group theory called the non-abelian tensor product, which arose in applications in homotopy theory of generalized Van Kampen theorems for diagrams of spaces and is due to Dennis, Brown, Loday and others.  
The equivariant homology theory of groups was recently constructed and studied by H.Inassaridze.

**Result, milestones :**

To develop and study equivariant non-abelian tensor product and non-abelian version of the equivariant homology theory of H.Inassaridze. To state and prove the generalized Van Kampen theorems in the equivariant set-up.

**4.2.10 Task Title : K-theory of bornological algebras**

Task coordinator : Tamaz Kandelaki, belonging to team: Tbilisi

**Objectives :**

To study bornological algebras from K-theory point of view

**Methodology :**

Using and generalizing methods of homological algebra, topological K-theory, Banach algebras and bornological spaces, C\*-algebras and C\*-categories

**Task Input:**

The task is depending on : Algebraic and topological K-theories  
Cohen-Hewitt factorization theorem, results of Bourbaki, Grothendiek, Hogbe-Nlend and Mayer on bornological spaces and algebras. Results of Karoubi, Villamayor, Quillen, Suslin Wodzicki and H.Inassaridze in K-theory.

The task is also based on the results of Karoubi, Villamayor, Quillen, Inassaridze in K-theory and on the results of Bourbaki, Grothendiek, Hogbe-Nlend and Mayer on bornological spaces and algebras.

**Result, milestones :**

To establish a class of bornological algebras having TF-property Construction of K-theory for bornological algebras. To define a class of bornological algebras for which an analogue of Karoubi's conjecture holds.

**4.2.10.1 Task Title : Bornological algebras and TF-property**

Task coordinator : Zurab Janelidze, belonging to team: Tbilisi

**Objectives :**

To investigate for bornological algebras the excision property in algebraic K-theory

**Methodology :**

Using and generalizing some methods and results in the theory of Banach algebras, C\*-algebras and bornological spaces.

**Task Input:**

The task is depending on : K-theory of bornological algebras

The task is based on the result of Cohen and Hewitt about factorisation of an element in Banach modules, on the results of Suslin and Wodzicki.

**Result, milestones :**

To generalize Cohen-Hewitt theorem for bornological modules over bornological algebras. To establish a class of bornological algebras satisfying TF-property which extends the class of algebras Banach algebras and Frechet algebras satisfying TF-property.

4.2.10.2 **Task Title : An analogue of Karoubi's conjecture for bornological algebras**  
Task coordinator : Tamaz Kandelaki, belonging to team: Tbilisi

**Objectives :**

To investigate the relationship between K-theory of bornological algebras and algebraic K-theory

**Methodology :**

Using and generalizing methods of homological algebra, topological K-theory, Banach algebras and bornological spaces.

**Task Input:**

The task is depending on : K-theory of bornological algebras  
The task is based on results of Karoubi, Villamayor, Quillen, H.Inassaridze in K-theory and on results of Bourbaki, Grothendiek, Hogbe-Nlend and Mayer on bornological spaces and algebras.

**Result, milestones :**

Construction of K-theory for bornological algebras generalizing Karoubi's topological K-theory. To establish a class of bornological algebras where bornological K-theory has the properties like that has topological K-theory. To define stable bornological algebras for which an analogue of Karoubi's conjecture holds.

4.2.11 **Task Title : K-homology of quantum groups**  
Task coordinator : Ryszard Nest, belonging to team: Copenhagen

**Objectives :**

Study of categories of algebras admitting coaction of a Hopf algebra, construction of an analogue of classifying space and study of equivariant theory of quantum groups

**Methodology :**

Methods of equivariant KK- and kk-theory and homological algebra of triangulated categories

**Task Input:**

The triangulated category structure of algebras with action of quantum groups recently introduced and studied by R. Nest and R. Meyer. Adams type spectral sequence for ideals in triangulated categories. General structure of derived functors in triangulated categories. Recent works of Cuntz and A. Thom on KK-theory for locally convex algebras.

**Result, milestones :**

Construction of objects playing the role of classifying spaces for quantum groups

and, more generally, Hopf algebras.  
 Construction of the assembly map and proof of a version of Baum-Connes conjecture for the case of coaction of compact groups and action of  $SU_q(n)$ . Study of the assembly map for general quantum groups.  
 Formulation and study of Künneth formula and universal coefficient spectral sequence for algebras with action of quantum groups

#### 4.3 **Project Management**

##### 4.3.1 **Planning & Task allocation**

##### 4.3.1.1 **List of Task Titles**

1. Algebraic and topological K-theories
2. Homotopical and structural investigation of locally convex algebras
3. Cobordism; Complex oriented cohomology rings, Morava K-theories, braid groups
  - 3.1 Symplectic cobordism and braid groups
  - 3.2 Complex oriented cohomology rings for finite groups, Morava K-theories
4. Leibniz n-algebras, crossed modules, foliations, and homotopy types
  - 4.1 Poincaré-Birkhoff-Witt theorem and crossed modules for Leibniz n-algebras
  - 4.2 Homotopy (n+1)-types, Leibniz n-algebras and foliations
5. Weighted composition operators on uniform algebras
6. Topological radicals of normed algebras
7. K-theory, cyclic homology, cdh cohomology
  - 7.1 Chern characters from K-theory to cyclic homology.
  - 7.2 Cyclic homology and derived functors
  - 7.3 Functor categories, operads, homology theories
8. Spectra and bivariant K-theory
9. Equivariant cohomology of groups
  - 9.1 Equivariant group cohomology as relative derived functors
  - 9.2 Generalization of Van Kampen theorems in the equivariant set-up
10. K-theory of bornological algebras
  - 10.1 Bornological algebras and TF-property
  - 10.2 An analogue of Karoubi's conjecture for bornological algebras
11. K-homology of quantum groups

4.3.1.2 The project will last 24 months with the activities as indicated in the diagram below

Task / SubTasks	Months 1-6	Months 7-12	Months 13-18	Months 19-24	Months 25-30	Months 31-36
Task 1	█	█	█	█		
Task 2	█	█	█	█		
Task 3	█	█	█	█		
SubTask 3.1	█	█	█	█		
SubTask 3.2	█	█	█	█		
Task 4	█	█	█	█		
SubTask 4.1				█		
SubTask 4.2	█					
Task 5	█	█	█	█		
Task 6	█	█	█	█		
Task 7	█	█	█	█		
SubTask 7.1	█	█	█	█		
SubTask 7.2	█	█	█	█		
SubTask 7.3	█	█	█	█		
Task 8	█	█	█	█		
Task 9	█	█	█	█		
SubTask 9.1	█	█	█	█		
SubTask 9.2	█	█	█	█		
Task 10	█	█	█	█		
SubTask 10.1	█	█	█	█		
SubTask 10.2	█	█	█	█		
Task 11	█	█	█	█		

#### 4.3.1.3 Team involvement

Teams	Task 1	Task 2	Task 3		Task 4		Task 5
			Task 3.1	Task 3.2	Task 4.1	Task 4.2	
Copenhagen	■	■					■
Tbilisi	■	■	■	■	■	■	
Baku							■
Muenster	■	■					■
Valladolid	■	■			■	■	
Paris	■		■	■			

Teams	Task 6	Task 7			Task 8	Task 9	
		Task 7.1	Task 7.2	Task 7.3		Task 9.1	Task 9.2
Copenhagen	■	■	■	■	■	■	■
Tbilisi		■	■	■		■	■
Baku	■						
Muenster	■	■	■		■		
Valladolid		■	■	■	■	■	■
Paris				■		■	

Teams	Task 10		Task 11
	Task 10.1	Task 10.2	
Copenhagen	■	■	■
Tbilisi	■	■	■
Baku			
Muenster	■	■	■
Valladolid		■	■
Paris			

#### 4.3.2 Project Management Description

The daily running of the network will be done in cooperation of Ryszard Nest (Copenhagen) and Hvedri Inassaridze (Tbilisi).

H.Inassaridze will be in charge of coordinating the South Caucasian teams.

The scientific committee of the network will consist of the team leaders with addition of Maxim Kontsevich.

In order to organize exchange of ideas and to facilitate scientific cooperation various activities are planned. They are of four different types :

1) Short terms visit of a member of a team in an another team. These visits are supposed to be supported by the network and there should be each year around 8-10 such visits. The aim of such visits will be the developement of a cooperation between members of the network. The priority will be given for advanced projects and well established cooperation. A smaller number of visits can be supported for new cooperation.

2) Long term visits : this will be supported by the network, but also the teams will be encouraged to look for other source of funding. One can expect a few number of such visits over the duration of the network.

Some of the already planned visits are:

N.Inassaridze and Khmaladze to Valladolid and Santiago de Compostela,  
Bakuradze to Paris and Montpellier,  
Kandelaki to Muenster and Copenhagen,  
N.Inassaridze to Muenster and Copenhagen,  
Baku team members to Tbilisi,Muenster and Paris,  
Tbilisi team members to Baku, of H.Inassaridze to Paris, Copenhagen and  
Muenster.

3) Congresses, so that most members of the network could meet. Two large meetings are planned, each of them on two of the subjects:

1. algebraic K-theory
2. functor categories and cohomology, equivariant and non-abelian cohomology,
3. cobordism theory, cyclic homology
4. Bivariant K-theory, locally convex algebras (particularly operator algebras and Banach algebras)

The duration of the meetings would be 5 days, the location of the meeting in 2007 is Tbilisi.

By the nature of the subject, most tasks are expected to extend beyond the 24 months period of the programme.

**4.4 Project costs**

**4.4.1 Cost Table**

The breakdown of costs of the INTAS contribution (in EURO) is given in the tables below.

<b>INTAS MEMBER STATE TEAMS</b>								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
1	Copenhagen	0	500	5500	0	0	0	6000
2	Muenster	0	500	5500	0	0	0	6000
3	Valladolid	0	500	5500	0	0	0	6000
4	Paris	0	500	5500	0	0	0	6000
SUBTOTAL (EURO)		0	2000	22000	0	0	0	24000

<b>NIS TEAMS</b>								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
5	Tbilisi	30960	0	20800	2000	4000	1460	59220
6	Baku	16080	420	8280	2000	4000	0	30780
SUBTOTAL (EURO)		47040	420	29080	4000	8000	1460	90000

<b>TOTAL</b>	<b>(EURO)</b>	47040	2420	51080	4000	8000	1460	114000
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**4.4.2 Justification of Costs**  
**4.4.2.1 Labour costs (only for NIS teams)**

Team name: Tbilisi

Number of individual grants	Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1 180	24	4320
Senior Researcher	4 180	96	17280
Scientist/Engineer	3 130	72	9360
Technical or Other	0 0	0	0
<b>TOTAL</b>			<b>30960</b>

Team name: Baku

Number of individual grants	Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1 180	24	4320
Senior Researcher	2 180	48	8640
Scientist/Engineer	1 130	24	3120
Technical or Other	0 0	0	0
<b>TOTAL</b>			<b>16080</b>

**4.4.2.2 Justification Labour costs**

Team 1 (Copenhagen)

\*

Team 2 (Muenster)

\*

Team 3 (Valladolid)

\*

Team 4 (Paris)

\*

Team 5 (Tbilisi)

\* To work on 8 tasks of the project.

Team 6 (Baku)

\*To work on the project.

**4.4.2.3 Justification Operational costs**

Team 1 (Copenhagen)

\*

Team 2 (Muenster)

\*

Team 3 (Valladolid)

\*

Team 4 (Paris)

\*

#### Team 5 (Tbilisi)

\* Travel & Subsistence -- Visits of N.Inassaridze and Khmaladze to Valladolid and Santiago de Compostela, of Bakuradze to Paris and Montpellier, of Kandelaki to Muenster and Copenhagen, of N.Inassaridze to Muenster and Copenhagen, of Tbilisi team members to Baku, of H.Inassaridze to Paris, Copenhagen and Muenster. Visits to the conferences of world or european level with the corresponding subject.

Consumables-- Costs of materials for maintenance of printer 1000 Euros; Costs of materials for maintenance of photo copier 1000 Euros.

Equipment-- 3 computers (Pentium 4), 800 Euros each; photo copier, 700 Euros; printer, 900 Euros.

Other: organisation of two conferences in Tbilisi, June 2007 and one in 2008, cost of inviting expert speakers from outside of the network; 1460 Euros.

#### Team 6 (Baku)

\*\* Travel & Subsistence -- Visits Baku team members to Tbilisi , to Paris, Copenhagen, Valladolid and Muenster, in particular the INTAS conference in Tbilisi, end of March 2007. Visits to the conferences of world or european level with the corresponding subject possibilities are the conference "Banach Algebras'07" and "LAW'08").

Consumables-- Costs of materials for maintenance of printer 1000 Euros; Costs of materials for maintenance of photo copier 1000 Euros.

Equipment-- 3 computers (Pentium 4 or notebooks of low level), 800 Euros each; photo copier, 700 Euros; printer, 900 Euros.

#### **4.4.2.4 Justification Overheads**

##### Team 1 (Copenhagen)

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##### Team 2 (Muenster)

\*

##### Team 3 (Valladolid)

\*

##### Team 4 (Paris)

\*

##### Team 5 (Tbilisi)

\*

##### Team 6 (Baku)

\*

#### **4.4.2.5 Comments**

##### Team 1 (Copenhagen)

\*

##### Team 2 (Muenster)

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##### Team 3 (Valladolid)

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Team 4 (Paris)

\*

Team 5 (Tbilisi)

\*

Team 6 (Baku)

\*

4.5

#### **Project innovation potential and dissemination of results**

Inside mathematics this program is located at a place where various particular fields meet: K-theory, homological and homotopical algebra, homotopical and general theories of locally convex algebras, category theory, operator and normed algebras which are all parts of algebra and algebraic topology, but also topology and non commutative geometry, category theory and homotopy theory. Interesting and important new results are expected in the described subfields, but also it is hoped that new results and concepts will emerge from the relations between all these subfields (which has seen to be the case in the recent years).

While this kind of program has no immediate applications outside mathematics, it is an essential part to the general research effort in exact sciences and as such contributes to the general progress of knowledge.