RESEARCH STATEMENT

THOMAS DUYCKAERTS

Contents

My research work, detailed here, concerns partial differential equations, mainly dispersive nonlinear equations (§1), but also spectral theory (§2) and control theory (§3).

1. Dynamics of dispersive equations

A large part of my work concerns semilinear dispersive equations, and in particular nonlinear wave (NLW) and nonlinear Schrödinger (NLS) equations with focusing nonlinearities. They combine a linear equation with a dispersive behavior (the solution decays uniformly to 0 for large time) with a nonlinearity which is typically of power type. In many cases, local well-posedness is known, and most of my work concerns the asymptotic dynamics of the solutions. This dynamics depend greatly on the sign in front of the nonlinearity. In the focusing case, the effect of the nonlinearity tends to be opposite to the effect of the linear, dispersive part of the equation. Three typical types of solutions are known:

(1) global solutions that behave asymptotically like a solution of the corresponding linear solution (so-called *scattering* solutions);
(2) solution blowing-up in finite time;
(3) *soliton* (solitary wave) solutions: a well-localized solution travelling at a fixed speed c (which can be 0).

The *soliton resolution conjecture* states roughly that any solution that is not of the first two types is asymptotically a sum of decoupled solitons and a scattering solution. It was only proved for regular solutions of completely integrable equations,

Date: October 30, 2015.

using the method of inverse scattering (see e.g. [6] for Korteweg-de Vries equation). This method is not available for the equations that are considered here.

Most of my works on dispersive equations concern the classification of solutions according to their dynamics, including the proof of the soliton resolution in one particular case. I am also interested by the influence of the geometry on the behavior of the solutions, when the equation is posed on a curved manifold (see §1.5).

1.1. **Ground state threshold.** The minimal[1] solitary wave solution is often unique (modulo the invariances of the equation) and called the ground state. For energy-critical wave an Schrödinger equations, the ground state solution is the minimal energy soliton, and it is natural to expect that solutions with a lower energy do not have a soliton-like behavior. This is indeed the case: Kenig and Merle proved in [14, 15] (with a radiality assumption in the case of NLS) that solutions with energy lesser than the ground state energy must blow up in finite time in both time directions or scatter in both time directions (see also [18] for nonradial NLS in high space dimension).

In [TD22, TD21], we studied with Frank Merle the energy threshold for these two equations. We constructed two new solutions of the equation, converging to the ground state for infinite positive times and with behaviour (1) or (2) for negative times. We also classified solutions at the energy threshold: the ground state solutions and the two new solutions are the only ones that do not scatter in both time directions or blow up in both time directions.

The results above were extended to a physically relevant energy-subcritical equation, the cubic focusing nonlinear Schrödinger equation in dimension 3. This was done for radial solutions by J. Holmer and S. Roudenko [9, 10], and extended to general solutions by myself, J. Holmer and S. Roudenko [TD11]. The energy threshold was treated in [TD26], in a joint work with S. Roudenko.

In the most recent work [TD27], we show that the scattering/blow-up dichotomy can be extended, with similar methods, to some classes of solutions of nonlinear Schrödinger equations above the threshold (and indeed, with arbitrarily large energy), at least in one time direction.

1.2. **Quantitative results.** In two articles [TD23] with F. Merle, and [TD24], with F. Merle and S. Roudenko, we have given precise asymptotics of the supremum of a space-time Strichartz norm of scattering solutions, yielding a quantitative version of scattering results.

In [TD23], the classification at the ground-state threshold is used to estimate the Strichartz norm close to this threshold.

In [TD24], a precise asymptotics of the supremum of scattering space-time norms for small solutions of mass-critical NLS in low dimension is given. We also prove the uniqueness and existence of the maximizer. Note that in this context the maximizer of the relevant Strichartz inequality for the linear equation is known [7, 11].

Both works [TD23] and [TD24] rely on concentration-compactness argument, and, for [TD24], the proof of the nondegeneracy of the linear maximizer by precise explicit computations on Hermite polynomials.

1.3. **Large solutions and soliton resolution.** In a series of works with C. Kenig and F. Merle, we considered solutions of nonlinear dispersive equations that are

[1] in a sense that depends on the equation

above the ground state threshold mentioned in §1.1. Let us mention the classification of solutions slightly above the ground state energy obtained for many equations, in articles by W. Schlag, K. Nakanishi, occasionally with J. Krieger (see e.g. [27] and references therein). These results are, in a way, extension of the threshold classification (with an additional ingredient, namely a one-pass theorem) and are complementary to some of our works described below: in fact the article on the energy-critical wave equation by W. Schlag, K. Nakanishi and J. Krieger [20] uses a classification theorem proved in [TD15].

Most of our works concern the focusing energy-critical wave equation. We started to consider solutions whose energy norm is bounded by the norm of the ground state stationary solution plus a small positive number, and proved that these solutions are asymptotically the sum of a dispersive part and a rescaled stationary solution (see [TD12] for radial solutions in 3d, [TD15] for nonradial solutions). This proved in particular that the bounded, finite time blow-up solutions constructed by J. Krieger, W. Schlag and D. Tataru in [21] were the only one existing in this regime.

We then restricted to radial solutions in space dimension 3, for which we have proved the full resolution into solitons in [TD16]: any global or bounded solution is asymptotically the sum of rescaled stationary solutions and a dispersive term (see also [TD13] for a preliminary result on large solutions).

The proofs of all these classification results use a new tool, the *channels of energy* method, based on asymptotic exterior energy estimates for the linear wave equation, which was later adapted to other related equations, especially wave maps (see e.g. [13]) and wave equations with an energy-subcritical (see [29]) or supercritical (see below) nonlinearity.

The soliton resolution is not known for the energy-critical wave equation in space dimension other than 3, or without spherical symmetry assumption. We give partial results in [TD17, TD18, TD19] (joint works with C. Kenig and F. Merle).

In [TD17], we prove that any bounded solution that does not scatters converge locally in space and time, after appropriate rescaling and space translation, to a solitary wave. This is the analog of the wave maps result of Struwe [31] and Sterbenz-Tataru [30], but the proof is quite different. It relies on a compactness argument which is valid for a large class of nonlinear dispersive equations, and implies that any nonscattering solution must converge weakly, for a sequence of times, to a solution with a compact trajectory (up to the modulation of the equation). A stronger result was proved by T. Tao in the particular context of the subcritical NLS in high space dimensions [32].

The article [TD17] shows the importance of classifying solutions of nonlinear dispersive equations with a compact trajectory. It is conjectured that in many cases these solutions are simply the travelling waves. We have made progresses toward this conjecture in the case of the energy-critical wave equation, proving in [TD19] that all solutions with the compactness property are radial, and in [TD18] that the conjecture holds with an additional nondegeneracy assumption with an additional geometric nondegeneracy assumption on the solution.

1.4. **Supercritical equations.** When the power of the nonlinearity of the wave equation is greater than the energy-critical power, the equation has no solitary wave solution in the critical Sobolev space were local well-posedness holds. The analog of the soliton resolution conjecture for this type of equations is that any solution which is bounded in this critical space is a scattering solution. This was proved by

C. Kenig and F. Merle in [16] for radial solution of the defocusing wave equation and later extended to other defocusing settings (see e.g. [18, 19]). The proofs all use a monotonicity formula (e.g. Morawetz inequality) which is specific to defocusing nonlinearities. In [TD14] (joint work with C. Kenig and F. Merle), we proved the conjecture for radial solutions of the 3d *focusing* wave equation, overcoming the lack of monotonicity formula with the channels of energy method that was developed in the energy-critical case.

A refinement of this method is used in [TD28] (with T. Roy) to prove a stronger form of the conjecture, namely that any solution which is bounded along a sequence of times going to the maximal time of existence must scatter to a linear solution.

1.5. **Equations on manifolds.** In a series of works with V. Banica and R. Carles, we studied linear and nonlinear Schrödinger equations on complete, noncompact Riemannian manifold. One of the motivation is to understand the influence of the geometry of the manifold on the dynamics of the equation. Recall that finite speed of propagation does not hold for solutions of the Schrödinger equation, so that the geometry of the manifold at infinity should affect the dynamics even in very short times.

In [TD5], joint with V. Banica, we give examples of spherically symmetric manifolds such that Strichartz estimates hold for a wider range of exponents than in the Euclidean case. In particular, small data scattering holds for nonlinearities with lower power than in the Euclidean setting. The improvement of Strichartz estimates is related to the faster growth of the volume of a ball as the radius goes to infinity. An extreme case is the hyperbolic space, where this volume grows exponentially, and scattering holds in the energy space for all energy-subcritical nonlinearities (see [1, 12]). In [TD2] (with V. Banica and R. Carles), we construct a family of Riemannian manifolds interpolating between the Euclidean and hyperbolic spaces, illustrating the relation between dispersive properties and the geometry of the manifold at infinity.

In [TD3] (with V. Banica and R. Carles), we go beyond the small data regime for the focusing nonlinear Schrödinger equation, constructing blow-up solutions based on the Euclidean profile for the mass-critical equation. This confirms the heuristic that the asymptotics of global solution is influenced by the global geometry of the underlying manifold, whereas finite time blow-up solutions tends to concentrate and thus see only, at first glance, the local geometry of the manifold (that is the Euclidean one). See also [28, 8] for blow-up results on manifolds in the same spirit.

In [TD4] (with V. Banica) we are concerned with the dynamics of the focusing Schrödinger equation on the hyperbolic space. We prove (for radial solutions in low space dimensions) a scattering/blow-up dichotomy, in the spirit of the corresponding Euclidean results, below a threshold which is given by the ground state solutions of the equation. The main difficulty is to prove a monotonicity formula (generalizing the virial identity on the Euclidean space).

2. Spectral theory and resolvent estimates

2.1. **Resolvent estimates and dispersive bounds.** Local-in-time dispersive properties of the wave and Schrödinger equations are related to estimates on the resolvent for the time-independent operator, with large spectral parameters that are close to the real axis. In the works [TD6, TD9] these resolvent bounds are proved for a Laplace operator with a potential. When this potential is regular and small at infinity, the resolvent bounds follow by a perturbative argument.

In [TD6], I considered a potential with several inverse square singularities. These singularities are critical and cannot be treated as a perturbation of the Laplace operator. In [5], the case of a single inverse-square singularity was treated, using explicit computation that are not possible for more singularities. In [TD6], I prove the optimal resolvent bounds using tools from micro-local analysis, and more precisely the propagation of a Wigner measure, following a strategy first used by G. Lebeau in the study of the stability of the dissipative wave equation [23]. In [TD8], I constructed a potential with a singularity which is slightly stronger than the inverse square (by a logarithmic factor) for which all dispersive bounds fail, thus proving the criticality of the inverse- square potentials.

The propagation of a Wigner measure is also used in [TD9] to prove resolvent bounds for a Laplace operator with a smooth matrix potential. The main difficulty is caused here by the crossing of the eigenvalues of the matrix, and we prove that the resolvent bounds holds under geometric assumptions on the crossing set. Let us mention that this kind of operators appears as toy models in molecular scattering theory.

2.2. **Helmholtz resonator.** Consider a domain (the *resonator*) of $\mathbb{R}^n$, $n \geq 2$, which consists in the exterior of a compact obstacle, a strict open subset of this obstacle (the *chamber*) and a thin neck that connects the exterior and the chamber. In [TD10], we study the resonances of the Laplace operator in the resonator as the width ϵ of the neck goes to 0. It is known (see [3]) that when ϵ is small, each resonance approaches either a resonance of the exterior region, or an eigenvalue of the chamber. The imaginary part of a resonance is linked to the half-life of the corresponding vibrational mode. As $\epsilon \to 0$, an exponential upper bound of this imaginary part is known. In [TD10], we prove an optimal *lower* bound for the resonance associated to the first Dirichlet eigenvalue of the chamber, generalizing previous works of A. Martinez and L. Nédélec that consider only very particular resonators. The proof is based on various Carleman estimates, including the Carleman estimates with limiting Carleman weights appearing in [17].

3. Control and stability of evolution equations

An other domain of research that I am interested in is the control theory of partial differential equations. The proofs use microlocal analysis, functional analysis, and methods that are more specific to control theory.

In [TD7], I prove the convergence to 0, with an optimal decay rate, for a dissipative system coupling a parabolic and an hyperbolic equation. The proofs rely on microlocal techniques close to the one used in [TD6, TD9] and Carleman estimates proved by G. Lebeau and L. Robbiano [22].

The two articles [TD5, TD25] (respectively with C. K. Batty and L. Miller) consider abstract differential systems. Optimal necessary and sufficient conditions for stability [TD5] and control [TD25] of the system, in term of bounds on the resolvent of the generator, are proved. The proofs use standard functional analysis. The abstract setting include classical hyperbolic and parabolic dissipative equations. One consequence of [TD25] is that the controllability to zero in finite time of a "Schrödinger" equation $i\partial_t u + Au = 0$ (where A is a self-adjoint operator on a Hilbert space, bounded from below) implies the corresponding property for all higher order "heat" equations $\partial_t u + A^s u = 0$, $s > 1$.

In [TD29] (with X. Zhang and E. Zuazua), we explore the link between the optimality of a constant appearing in observability inequalities for linear wave and heat equations, and the construction, by V.Z. Meshkov [26], of a solution of a linear elliptic equation on $\mathbb{R}^2$ with an optimal superexponential decay. The optimal decay rate in the construction of Meshkov, as well as the observability inequality appearing in [TD29] are both related to Carleman inequalities. The same article of Meshkov was previously used in the celebrated article of J. Bourgain and C. Kenig on the Anderson model [4].

Finally, the preprint [TD1] concerns the stabilization of a dissipative wave equation through a feedback which is localized and finite-dimensional. The proof, using the *dynamic programming principle*, is an adaptation to the hyperbolic setting of the proof of a similar result on a parabolic equation in [2].

References

My publications and preprints.

[TD1] Kaïs Ammari, Thomas Duyckaerts, and Armen Shirikyan. Local feedback stabilisation to a non-stationary solution for a damped non-linear wave equation. Preprint. https://hal.archives-ouvertes.fr/hal-00759119, 2012. To be published in Math. Control Relat. Fields.

[TD2] Valeria Banica, Rémi Carles, and Thomas Duyckaerts. On scattering for NLS: from Euclidean to hyperbolic space. *Discrete Contin. Dyn. Syst.*, 24(4):1113–1127, 2009.

[TD3] Valeria Banica, Rémi Carles, and Thomas Duyckaerts. Minimal blow-up solutions to the mass-critical inhomogeneous NLS equation. *Comm. Partial Differential Equations*, 36(3):487–531, 2011.

[TD4] Valeria Banica and Thomas Duyckaerts. Large time behaviour for the focusing NLS on hyperbolic space. *Dyn. Partial Differ. Equ.*, 12(1):53–96.

[TD5] Valeria Banica and Thomas Duyckaerts. Weighted Strichartz estimates for radial Schrödinger equation on noncompact manifolds. *Dyn. Partial Differ. Equ.*, 4(4):335–359, 2007.

[TD6] Thomas Duyckaerts. Inégalités de résolvante pour l'opérateur de Schrödinger avec potentiel multipolaire critique. *Bull. Soc. Math. France*, 134(2):201–239, 2006.

[TD7] Thomas Duyckaerts. Optimal decay rates of the energy of a hyperbolic-parabolic system coupled by an interface. *Asymptot. Anal.*, 51(1):17–45, 2007.

[TD8] Thomas Duyckaerts. A singular critical potential for the Schrödinger operator. *Canad. Math. Bull.*, 50(1):35–47, 2007.

[TD9] Thomas Duyckaerts, Clotilde Fermanian Kammerer, and Thierry Jecko. Degenerated codimension 1 crossings and resolvent estimates. *Asymptot. Anal.*, 65(3-4):147–174, 2009.

[TD10] Thomas Duyckaerts, Alain Grigis, and André Martinez. Resonance widths for general Helmholtz Resonators with straight neck. Preprint arXiv:1504.05425, 2015.

[TD11] Thomas Duyckaerts, Justin Holmer, and Svetlana Roudenko. Scattering for the non-radial 3D cubic nonlinear Schrödinger equation. *Math. Res. Lett.*, 15(6):1233–1250, 2008.

[TD12] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Universality of blow-up profile for small radial type II blow-up solutions of the energy-critical wave equation. *J. Eur. Math. Soc. (JEMS)*, 13(3):533–599, 2011.

[TD13] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Profiles of bounded radial solutions of the focusing, energy-critical wave equation. *Geom. Funct. Anal.*, 22(3):639–698, 2012.

[TD14] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Scattering for radial, bounded solutions of focusing supercritical wave equations. *IMRN*, 2012.

[TD15] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Universality of the blow-up profile for small type II blow-up solutions of the energy-critical wave equation: the nonradial case. *J. Eur. Math. Soc. (JEMS)*, 14(5):1389–1454, 2012.

[TD16] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Classification of radial solutions of the focusing, energy-critical wave equation. *Cambridge Journal of Mathematics*, 1(1):75–144, 2013.

[TD17] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Profiles for bounded solutions of dispersive equations, with applications to energy-critical wave and Schrödinger equations. *Commun. Pure Appl. Anal.*, 14(4):1275–1326, 2015.

[TD18] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Solutions of the focusing, energy-critical wave equation with the compactness property. Preprint arXiv:1402.0365, 2014. To be published in Annali Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)

[TD19] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Global existence for solutions of the focusing wave equation with the compactness property. ArXiv preprint:1504.04692, 2015.

[TD20] Thomas Duyckaerts, Carlos Kenig, and Frank Merle. Concentration-compactness and universal profiles for the non-radial energy critical wave equation. Preprint arXiv:1510.01750, 2015.

[TD21] Thomas Duyckaerts and Frank Merle. Dynamics of threshold solutions for energy-critical wave equation. *Int. Math. Res. Pap. IMRP*, pages Art ID rpn002, 67, 2008.

[TD22] Thomas Duyckaerts and Frank Merle. Dynamic of threshold solutions for energy-critical NLS. *Geom. Funct. Anal.*, 18(6):1787–1840, 2009.

[TD23] Thomas Duyckaerts and Frank Merle. Scattering norm estimate near the threshold for energy-critical focusing semilinear wave equation. *Indiana Univ. Math. J.*, 58(4):1971–2001, 2009.

[TD24] Thomas Duyckaerts, Frank Merle, and Svetlana Roudenko. Maximizers for the Strichartz norm for small solutions of mass-critical NLS. *Annali della Scuola Normale Superiore di Pisa. Classe di scienze*, 10(2):427–476, 2011.

[TD25] Thomas Duyckaerts and Luc Miller. Resolvent conditions for the control of parabolic equations. *J. Funct. Anal.*, 263(11):3641–3673, 2012.

[TD26] Thomas Duyckaerts and Svetlana Roudenko. Threshold solutions for the focusing 3d cubic schrödinger equation. *Rev. Mat. Iberoam.*, 26(1):1–56, 2010.

[TD27] Thomas Duyckaerts and Svetlana Roudenko. Going beyond the threshold: scattering and blow-up in the focusing NLS equation, 2014. Preprint. http://arxiv.org/abs/1409.4821. To appear in Comm. Math. Phys.

[TD28] Thomas Duyckaerts and Tristan Roy. Blow-up of the critical norm for nonscattering radial solutions of supercritical wave equations in space dimension 3. ArXiv preprint:1506.00788.

[TD29] Thomas Duyckaerts, Xu Zhang, and Enrique Zuazua. On the optimality of the observability inequalities for parabolic and hyperbolic systems with potentials. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 25(1):1–41, 2008.

Other articles

[1] Valeria Banica, Rémi Carles, and Gigliola Staffilani. Scattering theory for radial nonlinear Schrödinger equations on hyperbolic space. *Geom. Funct. Anal.*, 18(2):367–399, 2008.

[2] Viorel Barbu, Sérgio S. Rodrigues, and Armen Shirikyan. Internal exponential stabilization to a nonstationary solution for 3D Navier-Stokes equations. *SIAM J. Control Optim.*, 49(4):1454–1478, 2011.

[3] J. Thomas Beale. Scattering frequencies of reasonators. *Comm. Pure Appl. Math.*, 26:549–563, 1973.

[4] Jean Bourgain and Carlos E. Kenig. On localization in the continuous Anderson-Bernoulli model in higher dimension. *Invent. Math.*, 161(2):389–426, 2005.

[5] Nicolas Burq, Fabrice Planchon, John G. Stalker, and A. Shadi Tahvildar-Zadeh. Strichartz estimates for the wave and Schrödinger equations with the inverse-square potential. *J. Funct. Anal.*, 203(2):519–549, 2003.

[6] W. Eckhaus and P. Schuur. The emergence of solitons of the Korteweg-de Vries equation from arbitrary initial conditions. *Math. Methods Appl. Sci.*, 5(1):97–116, 1983.

[7] Damiano Foschi. Maximizers for the Strichartz inequality. *J. Eur. Math. Soc. (JEMS)*, 9(4):739–774, 2007.

[8] Nicolas Godet. Blow up on a curve for a nonlinear Schrödinger equation on Riemannian surfaces. *Dynamics of PDE*, 10(2):99–155, 2013.

[9] Justin Holmer and Svetlana Roudenko. On blow-up solutions to the 3D cubic nonlinear Schrödinger equation. *Appl. Math. Res. Express. AMRX*, (1):Art. ID abm004, 31, 2007.

[10] Justin Holmer and Svetlana Roudenko. A sharp condition for scattering of the radial 3D cubic nonlinear Schrödinger equation. *Comm. Math. Phys.*, 282(2):435–467, 2008.
[11] Dirk Hundertmark and Vadim Zharnitsky. On sharp Strichartz inequalities in low dimensions. *Int. Math. Res. Not.*, pages Art. ID 34080, 18, 2006.
[12] Alexandru D. Ionescu and Gigliola Staffilani. Semilinear Schrödinger flows on hyperbolic spaces: scattering H^1. *Math. Ann.*, 345(1):133–158, 2009.
[13] Carlos E. Kenig, Andrew Lawrie, and Wilhelm Schlag. Relaxation of wave maps exterior to a ball to harmonic maps for all data. *Geometric and Functional Analysis*, 24(2):610–647, 2014.
[14] Carlos E. Kenig and Frank Merle. Global well-posedness, scattering and blow-up for the energy-critical, focusing, non-linear Schrödinger equation in the radial case. *Invent. Math.*, 166(3):645–675, 2006.
[15] Carlos E. Kenig and Frank Merle. Global well-posedness, scattering and blow-up for the energy-critical focusing non-linear wave equation. *Acta Math.*, 201(2):147–212, 2008.
[16] Carlos E. Kenig and Frank Merle. Nondispersive radial solutions to energy supercritical non-linear wave equations, with applications. *Amer. J. Math.*, 133(4):1029–1065, 2011.
[17] Carlos E. Kenig, Johannes Sjöstrand, and Gunther Uhlmann. The Calderón problem with partial data. *Ann. of Math. (2)*, 165(2):567–591, 2007.
[18] Rowan Killip and Monica Visan. The focusing energy-critical nonlinear Schrödinger equation in dimensions five and higher. *Amer. J. Math.*, 132(2):361–424, 2010.
[19] Rowan Killip and Monica Visan. The defocusing energy-supercritical nonlinear wave equation in three space dimensions. *Trans. Amer. Math. Soc.*, 363(7):3893–3934, 2011.
[20] Joachim Krieger, Kenji Nakanishi, and Wilhelm Schlag. Global dynamics of the nonradial energy-critical wave equation above the ground state energy. *Discrete Contin. Dyn. Syst.*, 33(6):2423–2450, 2013.
[21] Joachim Krieger, Wilhelm Schlag, and Daniel Tataru. Slow blow-up solutions for the $H^1(\mathbb{R}^3)$ critical focusing semilinear wave equation. *Duke Math. J.*, 147(1):1–53, 2009.
[22] G. Lebeau and L. Robbiano. Contrôle exact de l'équation de la chaleur. *Comm. Partial Differential Equations*, 20(1-2):335–356, 1995.
[23] Gilles Lebeau. Équation des ondes amorties. In *Algebraic and geometric methods in mathematical physics (Kaciveli, 1993)*, volume 19 of *Math. Phys. Stud.*, pages 73–109. Kluwer Acad. Publ., Dordrecht, 1996.
[24] André Martinez and Laurence Nédélec. Optimal lower bound of the resonance widths for a Helmholtz tube-shaped resonator. *J. Spectr. Theory*, 2(2):203–223, 2012.
[25] André Martinez and Laurence Nédélec. Optimal lower bound of the resonance widths for the Helmholtz Resonator. ArXiv preprint:1402.6493, 2014.
[26] V. Z. Meshkov. Weighted differential inequalities and their application for estimates of the decrease at infinity of the solutions of second-order elliptic equations. *Trudy Mat. Inst. Steklov.*, 190:139–158, 1989. Translated in Proc. Steklov Inst. Math. **1**992, no. 1, 145–166, Theory of functions (Russian) (Amberd, 1987).
[27] Kenji Nakanishi and Wilhelm Schlag. *Invariant manifolds and dispersive Hamiltonian evolution equations*. European Mathematical Society, 2011.
[28] Pierre Raphaël and Jérémie Szeftel. Existence and uniqueness of minimal blow-up solutions to an inhomogeneous mass critical NLS. *J. Amer. Math. Soc.*, 24(2):471–546, 2011.
[29] Ruipeng Shen. On the energy subcritical, non-linear wave equation with radial data for $p \in (3,5)$. Preprint arXiv:1208.2108, 2012.
[30] Jacob Sterbenz and Daniel Tataru. Regularity of wave-maps in dimension 2+1. *Comm. Math. Phys.*, 298(1):231–264, 2010.
[31] Michael Struwe. Equivariant wave maps in two space dimensions. *Comm. Pure Appl. Math.*, 56(7):815–823, 2003. Dedicated to the memory of Jürgen K. Moser.
[32] Terence Tao. A (concentration-) compact attractor for high-dimensional non-linear schrödinger equations. *Dyn. Partial Differ. Equ.*, 4(1):1–53, 2007.

Laboratoire Analyse, Géométrie et Applications (UMR 7539), Département de Mathématiques, Institut Galilée, Université Paris 13, 99 Av. J.-B. Clément 93430 Villetaneuse, France

E-mail address: duyckaer@math.univ-paris13.fr