DYNAMICS OF SEMI-LINEAR WAVE EQUATIONS. ERRATUM ON THE LAST VERSION OF THE COURSE

"$+n$" means line n from the top of the page.

(1) p.57, +11. In (VI.3.1),
$$\left(\lambda_n^{1/2} u_n(t_n, \cdot + x_n), \lambda_n^{3/2}\partial_t u_n(t_n, \cdot + x_n)\right),$$
should be
$$\left(\lambda_n^{1/2} u_n(t_n, \lambda_n \cdot + x_n), \lambda_n^{3/2}\partial_t u_n(t_n, \lambda_n \cdot + x_n)\right).$$

(2) p.58, +12 and +14: u_n should be f_n (4 occurrences).

(3) The inequality (VI.3.15) $\|w_n\|_{\dot{B}^1} \leq \sup_{k\in\mathbb{Z}} \|\Delta_k w_n\|_{\dot{H}^1}$, holds only up to a multiplicative constant. To be perfectly rigorous in the proof of Lemma VI.3.3, one should change the definition of the $\dot{B}^1$-norm (to $\sup_{k\in\mathbb{Z}} \|\Delta_k \cdot\|_{\dot{H}^1}$) or change the constants (i.e. replace $\varepsilon/3$ by ε/C for some large C in the definition of J_n).

(4) p.60 in (VI.4.8), w_n^{J-1} should be $w_{J-1,n}$ (twice).

(5) The end of the construction of the profile decomposition, p.60, is quite elliptic. To complete the proof, one can denote, for any sequence $(v_n)_{n\in I}$ in $\mathcal{LW}$ such that $(\vec{v}_n(0))_n$ is bounded in $\dot{\mathcal{H}}^1$,
$$\eta\big((v_n)_{n\in I}\big) = \sup_{\substack{\vec{\varphi}_0\in\dot{\mathcal{H}}^1\\ \|\vec{\varphi}_0\|_{\dot{\mathcal{H}}^1}=1}} \limsup_{n\in I} \sup_{g\in\mathcal{G}} \left(\vec{v}_n^g(0), \vec{\varphi}_0\right)_{\dot{\mathcal{H}}^1},$$
where for $g = (\lambda, T, X) \in \mathcal{G}$ and $u \in \mathcal{LW}$,
$$u^g(t,x) = \lambda^{1/2} u(\lambda t + T, \lambda x + X).$$
Proposition VI.3.2 says that if $\eta\big((v_n)_{n\in I}\big) = 0$, then
$$\lim_{n\in I} \|v_n\|_{L^\infty(\mathbb{R},L^6)} = 0.$$
Also, fixing a countable dense family of the unite sphere of $\dot{\mathcal{H}}^1$, $D = (Z_p)_{p\in\mathbb{N}}$ (it is easy to show the existence of such a family, since $\dot{\mathcal{H}}^1$ has a countable Hilbert basis), we obviously have
$$\eta\big((v_n)_{n\in I}\big) = \sup_{\vec{\varphi}_0\in D} \limsup_{n\to\infty} \sup_{g\in\mathcal{G}} \left(\vec{v}_n^g(0), \vec{\varphi}_0\right)_{\dot{\mathcal{H}}^1}.$$
Now, we want to prove (VI.4.5). We argue by contradiction, assuming that
$$\limsup_{J\to\infty} \limsup_{n\in I} \|w_{J,n}\|_{\dot{\mathcal{H}}^1} = \delta > 0. \tag{1}$$
The construction of w_n^J implies
$$\lim_{J\to\infty} \eta\big((w_{J,n})_{n\in I}\big) = 0 \tag{2}$$
By (1) and (2), there exists a sequence $(w_{J,n_J})_{J\in I'}$ where $I' \sqsubset \mathbb{N}^*$, such that
$$\sup_{\vec{\varphi}_0\in D_J} \left| \sup_{g\in\mathcal{G}} \left(\vec{w}^g_{J,n_J}(0), \vec{\varphi}_0\right)_{\dot{\mathcal{H}}^1} \right| \leq 2\eta\left((w_{J,n})_{n\in I}\right) \underset{J\in I'}{\longrightarrow} 0$$
$$\forall J \in I', \quad \|w_{J,n_J}\|_{L^\infty(\mathbb{R},L^6)} \geq \delta/2.$$
where $D_J = \{Z_0, \ldots, Z_J\}$. This contradicts Proposition VI.3.2.