A New Parareal Algorithm for Problems with Discontinuous Sources

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TECHNISCHE UNIVERSITÄT DARMSTADT

GRADUATE SCHOOL computational engineering

Outline of the Talk

1. Introduction
   - Motivation
   - The eddy current problem
2. Systems with highly-oscillatory excitations
   - Modified Parareal with reduced coarse dynamics
   - Convergence results for nonsmooth sources
   - Numerical example: induction machine
3. Acceleration of convergence to the steady state
   - Time-periodic eddy current problem
   - Parareal for time-periodic problems
   - Results for the induction machine
4. Conclusions and outlook

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   - Motivation
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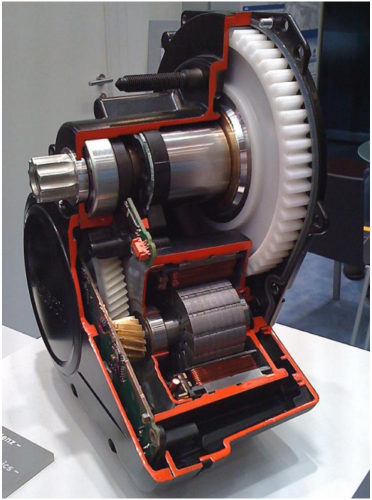
2. Systems with highly-oscillatory excitations

3. Acceleration of convergence to the steady state

4. Conclusions and outlook

Motivation

- E-bike with a synchronous machine
- Robust geometry optimization
- Expensive time domain simulations



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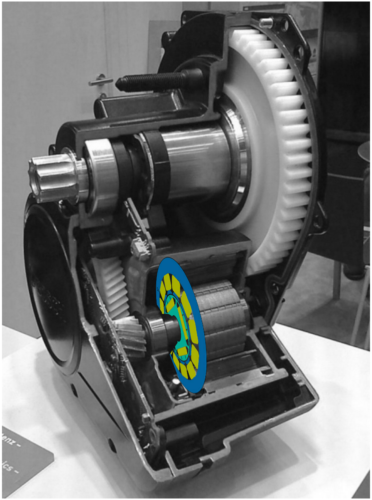

- E-bike with a synchronous machine
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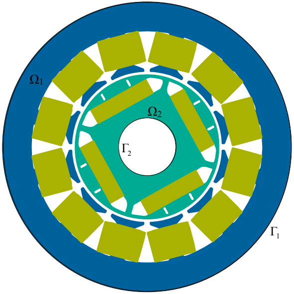
The eddy current problem

- Eddy current problem on domains Ω_1 and Ω_2

$$\sigma \frac{\partial \vec{A}}{\partial t}(\vec{x}, t) = -\nabla \times \big(\nu \nabla \times \vec{A}(\vec{x}, t)\big) + \vec{J}_{\mathrm{s}}(\vec{x}, t)$$

with magnetic vector potential $\vec{A}(\vec{x}, 0) = \vec{A}_0(\vec{x})$, current density in coils and magnets $\vec{J}_{\mathrm{s}}$, conductivity $\sigma(\vec{x})$ and reluctivity $\nu(\vec{x}, \vec{A})$.



- Spatial discretization yields initial value problem

$$\begin{aligned} \mathbf{M}\mathrm{d}_t\mathbf{u}(t) &= \mathbf{f}\big(t, \mathbf{u}(t)\big), \quad t \in (0, T], \\ \mathbf{u}(0) &= \mathbf{u}_0, \end{aligned}$$

with unknown $\mathbf{u}(t)$, mass matrix $\mathbf{M}$ and right-hand-side $\mathbf{f}(\ldots)$.

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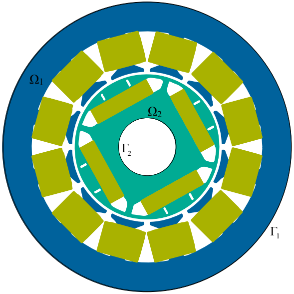

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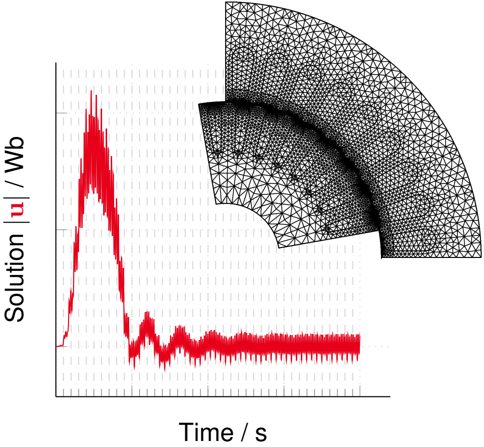
Challenges



- Machines operate most of their life time in steady state
- Long simulation time until steady state is reached
- Effects on several time scales, e.g. due to pulsed excitations
- Many time steps yield time-consuming computation

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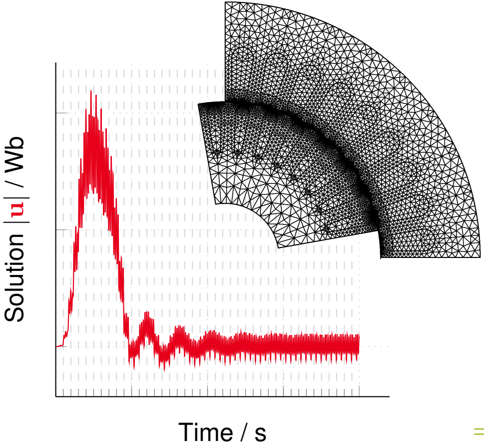

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$\Longrightarrow$ parallel-in-time method

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Parareal for highly-oscillatory discontinuous excitation

Parareal

- PWM (pulse width modulation): excitation contains high-order frequency components
- Propagators: fine $\mathcal{F}$ and coarse $\mathcal{G}$
- Solver $\mathcal{F}$ resolves high-frequency pulses
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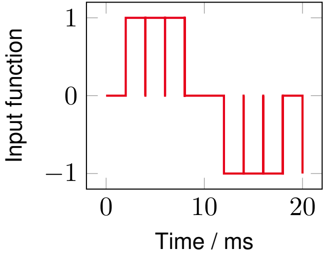

PWM signal with a switching frequency of 500 Hz.

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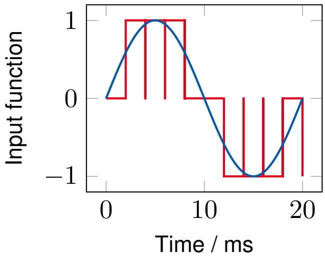
Idea

- Solve coarse problem for slowly-varying smooth input
- Low-frequency component: sinusoidal waveform $\sin\left(\frac{2\pi}{T}t\right)$



PWM signal with a switching frequency of 500 Hz and a sine wave of 50 Hz.

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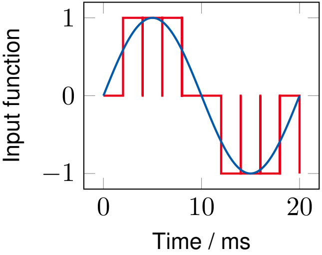
Question

- What about convergence?



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Modified Parareal with reduced coarse dynamics

- Splitting of the nonsmooth excitation for $t \in (0, T]$

$$\mathbf{M}\mathrm{d}_t\mathbf{u}(t) = \mathbf{f}(t, \mathbf{u}(t))$$

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- Reduced coarse propagator $\bar{\mathcal{G}}$

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- Modified Parareal update formula

$$\mathbf{U}_0^{(k+1)} = \mathbf{u}_0,$$
$$\mathbf{U}_n^{(k+1)} = \mathcal{F}\big(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}\big) + \bar{\mathcal{G}}\big(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k+1)}\big) - \bar{\mathcal{G}}\big(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}\big)$$

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- **Proof**: based on perturbation results for ODEs with discontinuities

Convergence of the modified approach

Theorem (Gander, K.-R., Schöps, Niyonzima, '18)

- For $\mathcal{I} := [0, T]$ let $\Delta T = T/N$ denote window length and $\mathcal{F}\big(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}\big)$ be the exact solution to the original problem at T_n, with the RHS $\mathbf{f} = \bar{\mathbf{f}} + \tilde{\mathbf{f}}$. For $p \geq 1$ we denote $C_p = \|\tilde{\mathbf{f}}\|_{\boldsymbol{L}^p(\mathcal{I}, \mathbb{R}^n)}$ and let $q \geq 1$ be given by $1/p + 1/q = 1$.

- Assume $\bar{\mathcal{G}}\big(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}\big)$ is an approximation to the reduced problem with the smooth RHS $\bar{\mathbf{f}}$. The error is bounded by $C_3 \Delta T^{l+1}$, and let $\bar{\mathcal{G}}$ satisfy the Lipschitz condition:

$$\|\bar{\mathcal{G}}\big(t + \Delta T, t, U\big) - \bar{\mathcal{G}}\big(t + \Delta T, t, Y\big)\| \leq (1 + C_2 \Delta T)\|U - Y\|.$$

Then at iteration k we have: $||\mathbf{u}(T_n) - \mathbf{U}_n^k|| \leq$

$$\bar{C}_1^k \left[\bar{C}_4 C_p \Delta T^{(l+1)k + 1/q} + \bar{C}_3 \left(\Delta T^{l+1}\right)^{k+1}\right] \frac{(1 + C_2 \Delta T)^{n-k-1}}{(k+1)!} \prod_{j=0}^{k} (n - j).$$

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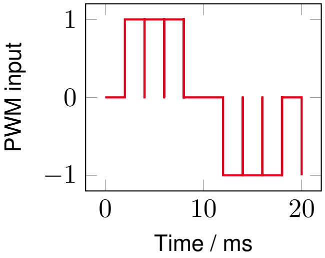
Numerical verification

RL-circuit model:

$$\frac{1}{R}\phi'(t) + \frac{1}{L}\phi(t) = f(t), \quad t \in (0, T],$$
$$\phi(0) = 0,$$

with $R = 0.01\ \Omega$, $L = 0.001$ H, $T = 0.02$ s;
$f-$ supplied PWM current source of 20 kHz.



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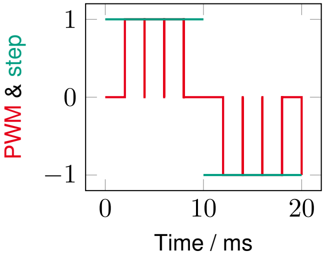

Choice of the coarse reduced input:

$$\bar{f}_{\mathbf{step}}(t) = \begin{cases} 1, & t \in [0, T/2), \\ -1, & t \in [T/2, T) \end{cases}$$

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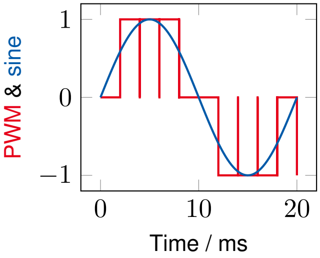

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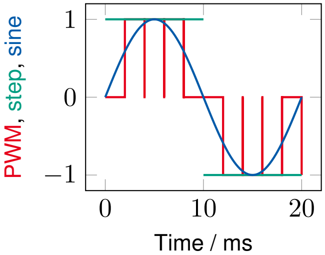

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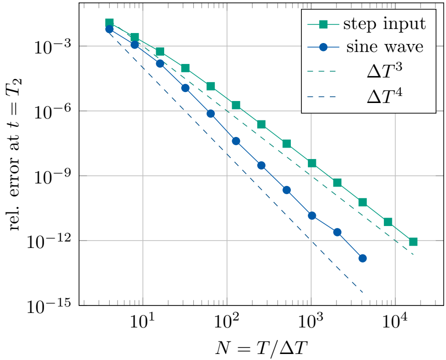
$$\Longrightarrow\ \tilde{f}(t) := f(t) - \bar{f}(t) \in \boldsymbol{L}^{\infty}(0,T) \iff 1/q = 1.$$

Numerical verification

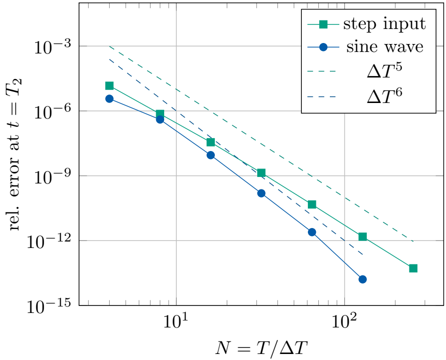
Step input: $\Delta T^{(l+1)k+1/q} = \Delta T^3$; **Sine** wave: $\left(\Delta T^{l+1}\right)^{k+1} = \Delta T^4$



Convergence of the Parareal iteration $k = 1$ using the implicit Euler method $(l = 1)$ and the reduced coarse step- and sine-input.

Numerical verification

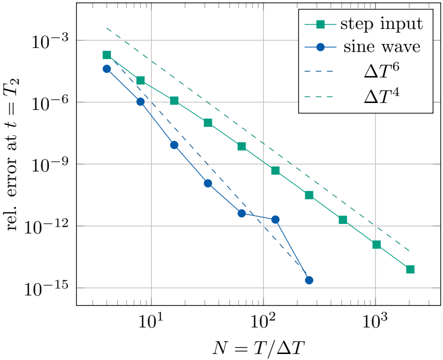
Step input: $\Delta T^{(l+1)k+1/q} = \Delta T^5$; **Sine** wave: $\left(\Delta T^{l+1}\right)^{k+1} = \Delta T^6$



Convergence of the Parareal iteration $k = 2$ using the implicit Euler method $(l = 1)$ and the reduced coarse step- and sine-input.

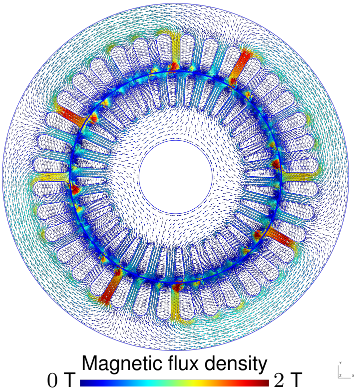
Numerical verification

Step input: $\Delta T^{(l+1)k+1/q} = \Delta T^4$; **Sine** wave: $\left(\Delta T^{l+1}\right)^{k+1} = \Delta T^6$

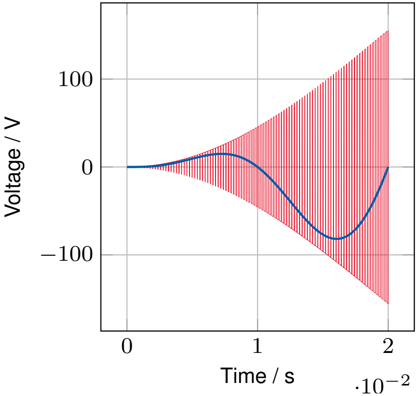


Convergence of the Parareal iteration $k = 1$ using the Crank-Nicolson scheme $(l = 2)$ and the reduced coarse step- and sine-input.

Application to an induction machine

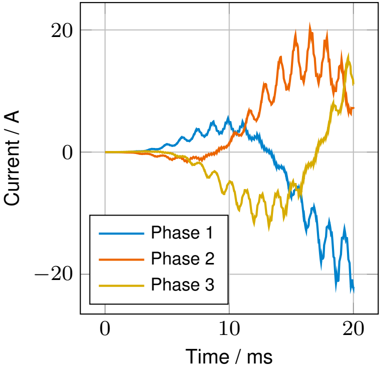


Four-pole squirrel cage 'im_3kw' model and its magnetic field at $t = 20$ ms if excited by a sinusoidal voltage source (author: Gyselinck).

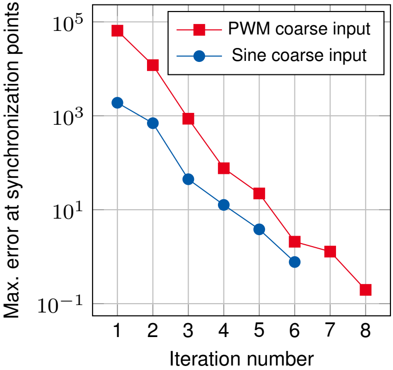


PWM voltage source of 5 kHz with a ramp-up and phase 1 of the corresponding sinusoidal waveform of 50 Hz.

Numerical results



Stator currents for the three-phase PWM voltage source of 20 kHz on $[0,\ 20]$ ms. Software: implicit Euler within GetDP.



Convergence of the standard Parareal and the modified Parareal algorithms to reach the prescribed tolerance $1.5 \cdot 10^{-5}$.

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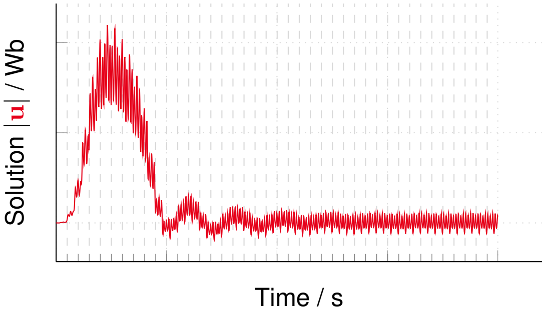
- Time-periodic eddy current problem
- Parareal for time-periodic problems
- Results for the induction machine

4 Conclusions and outlook

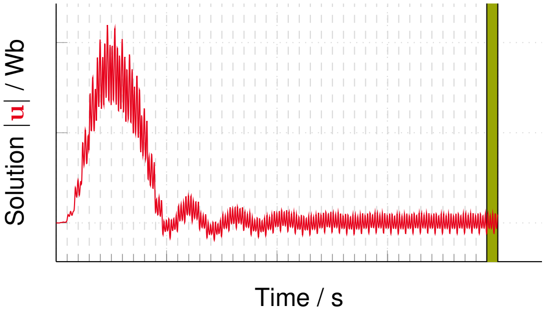
Time-periodic eddy current problem

- **Goal:** obtain the steady-state solution



Time-periodic eddy current problem

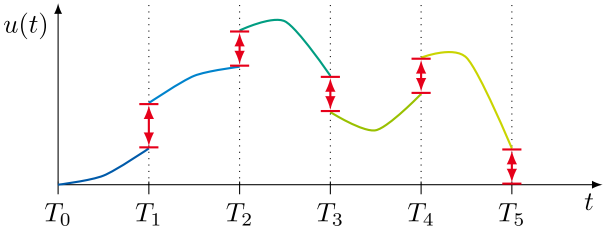
- **Goal:** obtain the steady-state solution



- Solve periodic boundary value problem in time:

$$\mathbf{M}\mathrm{d}_t\mathbf{u}(t) = \mathbf{f}(t, \mathbf{u}(t)),\ t \in (0, T) \qquad \text{with} \quad \mathbf{u}(0) = \mathbf{u}(T).$$

Parareal for time-periodic problems



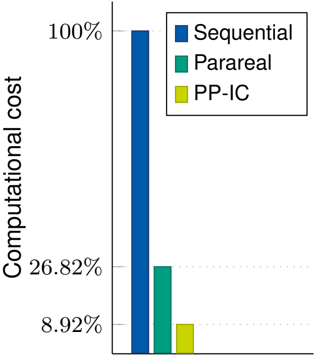
PP-IC: periodic parareal algorithm with initial value coarse problem:

$$\mathbf{U}_0^{(k+1)} = \mathbf{U}_N^{(k)},$$

$$\mathbf{U}_n^{(k+1)} = \mathcal{F}(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}) + \mathcal{G}(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k+1)}) - \mathcal{G}(T_n, T_{n-1}, \mathbf{U}_{n-1}^{(k)}).$$

M. J. Gander et al., *Analysis of Two Parareal Algorithms for Time-Periodic Problems*, SIAM Journal on Scientific Computing 35 (5), 2013.

Results for induction machine with PWM voltage source



Computational efforts to obtain the periodic (steady-state) solution:

- Sequential: 9 periods until the steady state $\Longrightarrow$ $2\,176\,179$ system solves
- Parareal: calculation on $[0, 9T]$, needs effectively $583\,707$ linear solutions
- PP-IC: applied on one period $[0, T]$, requires $194\,038$ linear solutions

Period $T = 0.02$ s, available CPUs $N = 20$

Fine propagator $\mathcal{F}$: three-phase PWM excitation of 20 kHz, $\delta T = 10^{-6}$ s

Coarse solver $\bar{\mathcal{G}}$: three-phase sinusolidal source of 50 Hz, $\Delta T = 10^{-3}$ s

Outline of the Talk

1 Introduction

2 Systems with highly-oscillatory excitations

3 Acceleration of convergence to the steady state

4 Conclusions and outlook

Conclusions and outlook

Conclusions

- Introduced a new Parareal algorithm with reduced coarse dynamics
- Developed convergence theory for problems with (highly-oscillatory) discontinuous excitation
- Applied the modified Parareal method to the time-periodic eddy current problem for an induction machine model

Conclusions and outlook

Conclusions

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Outlook

- Prove convergence of the modified PP-IC algorithm
- Further development of parallel-in-time methods for the periodic eddy current problem with PWM excitation
- Combine the time-parallel techniques with spatial domain decomposition for simulation of electric machines

Thank you!

ACKNOWLEDGEMENT

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