



SÉANCE XXVIII:

Doctoribus nostris et eorum gloriis !

(The schedule below is in the Parisian time)

Lundi 23 Novembre

13.30–14.00 : Zhixiang Wu (Paris Saclay)

A local model for the trianguline variety at points with non-regular Hodge-Tate weights

The trianguline variety parameterizes trianguline Galois representations which are related to over-convergent finite slope p -adic automorphic forms. Breuil–Hellmann–Schraen developed a theory of local model for the trianguline variety at certain points with regular Hodge–Tate weights with applications on global eigenvarieties. In this talk, I will explain how to generalize the local model for certain points with non-regular Hodge–Tate weights and prove that the trianguline variety is irreducible at these points by showing that the parabolic version local model is unibranch. The result will be applied for Breuil’s locally analytic socle conjecture in the non-regular cases.

14.00–14.10 : Pause

14.10–14.40 : Johannes Girsch (Imperial College)

The Doubling Method in Algebraic Families

Local constants are an important concept in the complex representation theory of reductive p -adic groups, for example they are pivotal in the formulation of the Local Langlands correspondence. In recent years there has been progress in defining such constants for modular representations or in even more general settings. For example, Moss was able to define γ -factors for representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ with coefficients in general noetherian rings and subsequently together with Helm was able to prove a converse theorem, which was crucial for the proof of the Local Langlands correspondence in families for GL_n . The aim of this talk is to show how one can extend the Doubling Method of Piatetski–Shapiro and Rallis to families of representations of classical groups. In this setting we will introduce and prove a rationality result for the Doubling Zeta integrals. Subsequently we will show that these zeta integrals satisfy a functional equation from which one obtains γ -factors.

14.40–15.00 : Pause Café

15.00–15.30 : Amadou Bah (I.H.É.S.)

Variation of the Swan conductor of an \mathbb{F}_ℓ -sheaf on a rigid disc

Let K be a complete discrete valuation field of residue characteristic $p > 0$ and $\ell \neq p$ a prime number. To a finite dimensional \mathbb{F}_ℓ -representation M of the absolute Galois group G_K , the ramification theory of Abbes and Saito attaches a Swan conductor $\mathrm{sw}(M)$ and a characteristic cycle $\mathrm{CC}(M)$. Let D be the rigid unit disc over K and \mathcal{F} a lisse sheaf of \mathbb{F}_ℓ -modules on D . For a rational number $t \geq 0$, the normalized integral model $\mathcal{D}^{(t)}$ of the subdisc $D^{(t)}$ of radius t is defined over some finite

extension of K . The restriction of \mathcal{F} to $D^{(t)}$ defines, at the generic point $\mathfrak{p}^{(t)}$ of the special fiber of $\mathcal{D}^{(t)}$, a Galois representation M_t over a complete discrete valuation field, thus yielding a Swan conductor $\text{sw}(M_t)$ and a characteristic cycle $\text{CC}(M_t)$. The goal of the talk is to explain how we connect earlier works, of L ajtkebohmert on a discriminant function attached to a cover of D , and of Kato on the ramification of valuation rings of height 2, and prove that the function $t \mapsto \text{sw}(M_t)$ is continuous and piecewise linear with finitely many slopes which are all integers and that its right derivative is $t \mapsto -\text{ord}_{\mathfrak{p}^{(t)}}(\text{CC}(M_t)) + \dim(M_t/(M_t)^{(0)})$, where $\text{ord}_{\mathfrak{p}^{(t)}}$ is a normalized discrete valuation at $\mathfrak{p}^{(t)}$ extended to differentials and $M_t^{(0)}$ is the tame part of M_t .

15.30–15.40 : Pause

15.40–16.10 : Andrew Graham (Imperial College London)

An Euler system for conjugate self-dual automorphic representations

I will discuss the construction of an anticyclotomic Euler system for certain conjugate self-dual automorphic representations of $GL(2n)$, namely those which admit a Shalika model. The construction arises from special cycles on unitary Shimura varieties and is closely related to the branching law associated with the spherical pair $(GL(n) \times GL(n), GL(2n))$. If time permits, I will describe the expected relation with L -values and applications to the Bloch–Kato conjecture in the analytic rank one setting. This is joint work with S.W.A. Shah.

16.10–16.30 : Pause Caf 

16.30–17.00 : Sally Gilles (ENS Lyon–Imperial College)

Syntomic cohomology and period morphisms

In 2017, Colmez and Niziol proved a comparison theorem between arithmetic p -adic nearby cycles and syntomic cohomology sheaves. To prove it, they gave a local construction using (ϕ, Γ) -modules theory which allows to reduce the period isomorphism to a comparison theorem between Lie algebras. I will explain the geometric version of this local construction and how to globalize it to get a new period isomorphism. In particular, the explicit description of this new isomorphism can be used to compare previous constructions of period morphisms and prove they are equal.

Mardi 24 Novembre

13.30–14.00 : Ashwin Iyengar (King’s College London)

Irreducible components of p -adic Galois deformation rings

Let K be a p -adic field and fix an n -dimensional mod p representation ρ of the absolute Galois group of K . Gebhard Boeckle and Ann-Kristin Juschka predicted that the irreducible components of the universal lifting ring of ρ should be parametrized by the determinant map, and they proved this prediction in the 2-dimensional case. In this talk, I will discuss a proof of their conjecture in the n -dimensional case (under some mild technical assumptions) when ρ is the trivial representation.

14.00–14.10 : Pause

14.10–14.40 : Yichang Cai (LAGA)

Derived Galois deformation rings

In a recent paper of Galatius and Venkatesh, the authors studied simplicial generalizations of Galois deformation rings, and showed under certain assumptions that the cohomology of a locally symmetric space is a graded module over the homotopy groups of the corresponding simplicial deformation ring. We relax some assumptions, and in the ordinary (Fontaine–Laffaille) minimal case, we give examples where the result holds, assuming the existence and local-global compatibility of a Galois representation. As an application, Tilouine and Urban relate π_1 of the simplicial deformation ring to the p -divisible part of the corresponding dual Selmer group.

14.40–15.00 : Pause Café

15.00–15.30 : Pol van Hoften (King’s College London)

Mod p points on Shimura varieties of parahoric level

The conjecture of Langlands–Rapoport gives a conjectural description of the mod p points of Shimura varieties, with applications towards computing their (semi-simple) zeta function. The conjecture was proven by Kisin for abelian type Shimura varieties at primes of (hyperspecial) good reduction, after having constructed smooth integral models. For primes of (parahoric) bad reduction, Kisin and Pappas have constructed ‘good’ integral models and the conjecture naturally generalises to this setting. In this talk I will discuss recent work towards the conjecture for these integral models, building on earlier work of Zhou. As a byproduct of our methods we obtain irreducibility results for Ekedahl–Oort strata, Newton strata and central leaves in special fibers of Shimura varieties.

15.30–15.40 : Pause

15.40–16.10 : Juan Esteban Rodriguez (ENS-Lyon)

Dual Eichler–Shimura maps for the modular curve

Seven years ago, Andreatta–Iovita–Stevens constructed interpolations of the small slope part of the Eichler–Shimura map for the modular curve. Roughly speaking, they defined in a geometric way a map from the overconvergent modular symbols of weight k , to the overconvergent modular forms of weight $k + 2$. Then, using classicality theorems of Coleman and Ash–Stevens, they achieved a Hodge–Tate decomposition of the small slope part of overconvergent modular symbols. On the other hand, in a recent paper of Boxer–Pilloni, the authors proved that higher Coleman and Hida theories exist for the modular curve. The aim of this talk is to construct geometrically a map from the higher overconvergent modular forms of weight $-k$ to the modular symbols as above. We shall recover the Hodge–Tate decomposition of the small slope part of modular symbols, with the addition that all the maps involved are defined using the geometry of the modular curve. If time permits, we will discuss the compatibility of the previous work with duality.

16.10–16.30 : Pause Café

16.30–17.00 : Hanneke Wiersema (King’s College London)

Serre’s conjecture and two notions of minimal weight

The strong form of Serre’s conjecture states that every two-dimensional continuous, odd, irreducible mod p Galois representation arises from a modular form of a specific minimal weight, level and character. We use modular representation theory to prove the minimal weight is equal to a notion of minimal weight inspired by work of Buzzard, Diamond and Jarvis. We will also discuss this in the more general setting of mod p Galois representations of totally real fields, which is work in progress.

Mercredi 25 Novembre

13.30–14.00 : Ning Guo (Paris-Saclay)

The Grothendieck–Serre conjecture over valuation rings

We establish the Grothendieck–Serre conjecture over valuation rings: for a reductive group scheme G over a valuation ring V with fraction field K , a G -torsor over V is trivial if it is trivial over K . This result is predicted by the original Grothendieck–Serre conjecture and the resolution of singularities. The novelty of our proof lies in overcoming subtleties brought by general nondiscrete valuation rings. By using flasque resolutions and inducting with local cohomology, we prove a non-Noetherian counterpart of Colliot-Thélène–Sansuc’s case of tori. Then, taking advantage of techniques in algebraization, we obtain the passage to the Henselian rank one case. Finally, we induct on Levi subgroups and use the integrality of rational points of anisotropic groups to reduce to the semisimple anisotropic case, in which we appeal to properties of parahoric subgroups in Bruhat–Tits theory to conclude.

14.00–14.10 : Pause

14.10–14.40 : Omri Faraggi (University College London)

Cluster Pictures and the Arithmetic of Hyperelliptic Curves

Recently a new approach has been developed to study the local arithmetic of hyperelliptic curves: cluster pictures. The cluster picture of a hyperelliptic curve $C : y^2 = f(x)$ is a completely combinatorial object encoding the p -adic distances between the roots of f . From this, many invariants, such as minimal regular models, Tamagawa numbers and root numbers can be read off. In this talk, we shall give the definition of cluster pictures and describe how to read off some of these invariants in practice, as well as discussing ideas for where the future of cluster pictures lies.

14.40–15.00 : Pause Café

15.00–15.30 : Bart Michels (LAGA)

A geometric approach to extreme values of L -functions in the eigenvalue aspect

Given a CM-point on a compact arithmetic hyperbolic surface, Iwaniec–Sarnak and Milićević proved that a sequence of Maass forms takes on large values at the point. Through Waldspurger’s formula, the large values translate to extreme values of a family of L -functions in the eigenvalue aspect. In this talk, I replace the CM-point by a closed geodesic and explain how to obtain extreme values of geodesic periods, relative to their average size, and of the corresponding L -functions. I will also

discuss ongoing work on average and extreme values of maximal flat periods on quotients of split semisimple Lie groups, generalizing the results for $\mathrm{PGL}(2)$.

15.30–15.40 : Pause

15.40–16.10 : Petru Constantinescu (University College London)

Distribution of Modular Symbols

Motivated by a series of conjectures of Mazur, Rubin and Stein, the study of the arithmetic statistics of modular symbols has received a lot of attention in recent years. In this talk, I will highlight several results about the distribution of modular symbols, including their Gaussian distribution and the residual equidistribution modulo p . I will also talk about generalisations to quadratic imaginary fields and higher dimensions. ⌘

16.10–16.30 : Pause Caf 

16.30–17.00 : Alexandre Lartaux (Jussieu)

Hooley’s Delta function twisted by two characters

In this talk, we introduce Hooley’s Delta function, which measures logarithmic concentration of divisors of an entire n , and its generalisation which consists to twist this function by some Dirichlet’s characters. We investigate an upper bound of the second moment of this function twisted by two Dirichlet characters. The main arithmetical application is to count asymptotically some ideals.

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