Séminaire de Théorie des Nombres "Paris–Londres"

Séance XXXIII: To the memory and mathematical legacy of Yuri Manin

Imperial College London

Lundi 12 Juin (room 140, Imperial College)

9:30–10:00: Welcome! The coffee is served!

10:00–11:00 : Kestutis Cesnavicius (CNRS–Université Paris Saclay)
The Manin constant and the modular degree
The Manin constant c of an elliptic curve E over $\mathbb{Q}$ is the nonzero integer that scales the differential ω determined by the normalized newform f associated to E into the pullback of a Néron differential under a minimal parametrization $\phi : X_0(N) \to E$. Manin conjectured that $c = 1$ for optimal parametrizations. We show that in general $c \mid \deg(\phi)$, under a minor assumption at 2 and 3, which improves the status of the Manin conjecture for many E. Our core result that gives this divisibility is the containment $\omega \in H^0(X_0(N), \Omega)$, which we establish by combining automorphic methods with techniques from arithmetic geometry (here the modular curve $X_0(N)$ is considered over $\mathbb{Z}$ and Ω is its relative dualizing sheaf over $\mathbb{Z}$). To overcome obstacles at 2 and 3, we analyze nondihedral supercuspidal representations of $GL_2(\mathbb{Q}_2)$ and exhibit new cases in which $X_0(N)$ has rational singularities over $\mathbb{Z}$. This is joint work with Abhishek Saha and Michalis Neururer.

11:30–12:30 : Adam Morgan (University of Glasgow)
On the Hasse principle for Kummer varieties
Conditional on finiteness of relevant Shafarevich–Tate groups, Harpaz and Skorobogatov established the Hasse principle for Kummer varieties associated to 2-coverings of a principally polarised abelian variety A, under certain large image assumptions on the Galois action on $A[2]$. However, their method stops short of treating the case where the image is the full symplectic group, due to the possible failure of the Shafarevich–Tate group to have square order in this case. I will explain work in progress which overcomes this obstruction by combining second descent ideas in the spirit of Harpaz and Smith with new results on the 2-parity conjecture.

12:30–14:30: Déjéneur

14:30–15:30 : Anna Cadoret (Institut Mathématique de Jussieu)
On toric points of p-adic local systems arising from geometry
For a smooth variety over a number field and a p-adic local systems arising from geometry on it, classical conjectures on algebraic cycles predict that the toric points should fit with the CM points of the associated variation of Hodge structure; in particular, they should have similar properties in terms of sparsity. I will discuss results in this direction. This is a joint work with Jakob Stix.

16:00–17:00 : Soheyla Feyzbakhsh (Imperial College London)
Moduli spaces of stable vector bundles on curves on $K3$ surfaces
Consider the moduli space M of stable rank r vector bundles on a curve C with canonical determinant, and let h be the maximum number of linearly independent global sections of these bundles. If C embeds in a $K3$ surface X, I will show the sublocus M' of M consisting of bundles with h global

sections is a smooth projective hyperkahler manifold. As a result, I will prove Mukai's conjecture saying that the $K3$ surface X containing C can be obtained uniquely out of the curve C.

Mardi 13 Juin (ROOM 140, IMPERIAL COLLEGE)

10:00–11:00 : Olivier Wittenberg (CNRS, Université Sorbonne Paris Nord)
Supersolvable descent for rational points
The by now classical formalism of descent under a torus introduced by Colliot-Thélène and Sansuc in the 1980's admits an analogue in which the torus is replaced with a supersolvable finite group. I will explain this formalism and discuss applications to rational points of homogeneous spaces of linear groups over number fields and to the inverse Galois problem with prescribed norms. This is joint work with Yonatan Harpaz.

11:30–12:30 : Alec Shute (ISTA)
The Hasse principle for polynomials represented by norm forms
A central question in arithmetic geometry asks under what circumstances the Hasse principle holds for the affine equation given by a polynomial equal to a norm form. In this talk, I present results which establish the Hasse principle for a wide new family of polynomials and number fields, which includes polynomials of arbitrarily large degree. The proof makes use of the beta sieve developed by Rosser and Iwaniec, and also has applications to the study of rational points in fibrations.

12:30–14:30: DÉJÉNEUR

14:30–15:30 : Daniel Loughran (University of Bath)
The arithmetic of cubic surfaces
I will explain some of my protracted love affair with cubic surfaces which began when I first picked up Manin's book on cubic forms.

16:00–17:00 : Efthymios Sofos (University of Glasgow)
The second moment method for rational points
In a joint work with Alexei Skorobogatov we used a second-moment approach to prove asymptotics for the average of the von Mangoldt function over the values of a typical integer polynomial. As a consequence, we proved Schinzel's Hypothesis in 100% of the cases. In addition, we proved that a positive proportion of Châtelet equations have a rational point. I will explain subsequent joint work with Tim Browning and Joni Teräväinen [arXiv:2212.10373] that builds on the second-moment method and establishes asymptotics for averages of a general arithmetic function over the values of typical polynomials. The new tools come from the theory of averages of arithmetic functions in short intervals. One of the applications is that the Hasse principle holds for 100% of Châtelet equations. This agrees with the conjecture of Colliot-Thélène stating that the Brauer–Manin obstruction is the only obstruction to the Hasse principle for rationally connected varieties.

Mercredi 14 Juin (ROOM 140, IMPERIAL COLLEGE)

10:00–11:00 : Yuri Zarhin (Pennsylvania State University)
Central simple representations and superelliptic jacobians

Let $f(x)$ be a polynomial of degree at least 5 with complex coefficients and without repeated roots. Suppose that all the coefficients of $f(x)$ lie in a subfield K such that:

(1) K contains a primitive p-th root of unity;
(2) $f(x)$ is irreducible over K;
(3) the Galois group $\mathrm{Gal}(f)$ acts doubly transitively on the set of roots of $f(x)$;
(4) the index of every maximal subgroup of $\mathrm{Gal}(f)$ does not divide $\deg(f) - 1$.

Then the endomorphism ring of the Jacobian of the superelliptic curve $y^p = f(x)$ is isomorphic to the p-th cyclotomic ring for all primes $p > deg(f)$. We outline the proof, which is based on ideas from representation theory.

11:30–12:30 : Loic Merel (Institut Mathématique de Jussieu)

On the conjecture of Harris and Venkatesh for modular forms of weight one

(with E. Lecouturier) This is part of the vast world of (a variant of) Venkatesh's conjectures about derived Hecke algebras. The situation of modular forms of weight one considered by Harris and Venkatesh might be the simplest significant case. Consider the one-to-one correspondence between newforms f of weight one and two-dimensional, complex, odd, irreducible representation ρ of the absolute Galois group. Consider such f and ρ with coefficients in a subring A of $\mathbb{C}$. Let q be a prime number unramified for ρ (equivalently, not dividing the level of f). Consider the multiplicative group $\mathbb{F}_q^*$ of the finite field $\mathbb{F}_q$. Harris and Venkatesh attach, up to sign, two elements in the tensor product of $\mathbb{F}_q^*$ and A:

- From f, without knowing ρ, a pseudo-eigenvalue of the derived Hecke operator at q,
- From ρ, without knowing f, the localisation at q of a global cohomology class well defined up to a scalar in A.

The conjecture asserts that the second collection is equal to the first, up to an element of A. Original explanations will be given, and the conjecture will be expanded into new directions, based on the theory of modular symbols, as developed by Manin.

Le séminaire de Théorie de Nombres Paris–Londres est organisé par Kevin Buzzard, Fred Diamond, Vladimir Dokchitser, Steve Lester, Yiannis Petridis, Alexei Skorobogatov (Londres) et Marc Hindry, Stefano Morra, Matthew Morrow (Paris). Le séminaire est soutenu par l'Institut de Mathématiques de Jussieu–Paris Rive Gauche, le département de Mathématiques d'Orsay, le Laboratoire Analyse Géométrie Applications, le CNRS-Imperial "Abraham de Moivre" International Research Laboratory, Cecilia Tanner Research Funding Schemes, l'Heilbronn Institute for Mathematical Research