A nonconservative hyperbolic depth-averaged model for granular-fluid mixtures: application to debris flows and landslides.

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Numerical Approximations of Hyperbolic Systems with Source Terms and Applications.
Roscoff, France
September 22, 2011

Collaborators

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Outline

- Software packages: numerical methods summary
- A few examples
- Debris flows: physics/modeling
- Debris flows: our mathematical model
- Debris flows: some numerical issues

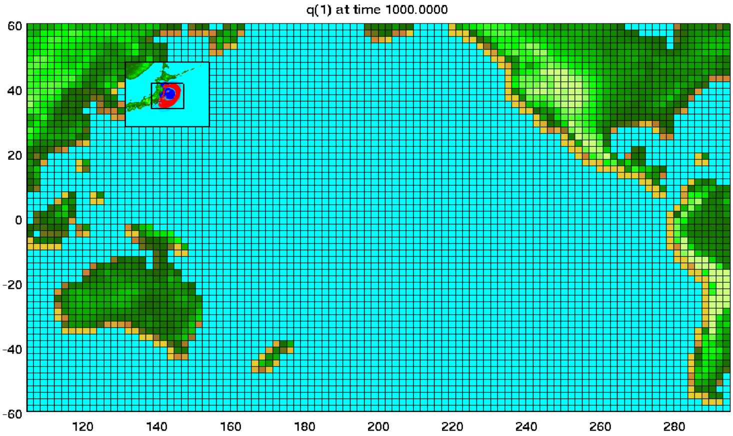
Clawpack Software

- Open-source software package for general hyperbolic problems:
- High-resolution (2nd-order) Godunov methods;
- Finite volume discretization on logically rectangular grids;
- Grid mappings for various geometries;
- Block-structured adaptive mesh refinement;
- Wave-propagation (possibly non-conservative) update from Riemann Solutions;
- Available @ `www.clawpack.org`.

The GeoClaw Software Project

- Subset of Clawpack for hyperbolic problems in geophysics:
  - free-surface flows (tsunamis, flooding, landslides etc.)
  - volcanic plume dynamics, ash clouds, pyroclastic flows etc.
  - seismic wave propagation
  - subsurface flow problems (mixed-type equations)
- Adaptive mesh refinement (AMR) schemes tailored to free-surface flows with inundation;
- Positive-depth preserving Riemann solvers;
- General inundation Riemann solver ability;
- Well-balanced Riemann solvers (to varying degree) for depth-averaged flows;
- Various user tools for geophysical data:
  - *e.g.,* dynamic processing of unaligned topography DEMs;

GeoClaw Example: Honshu-Tohoku 2011 tsunami

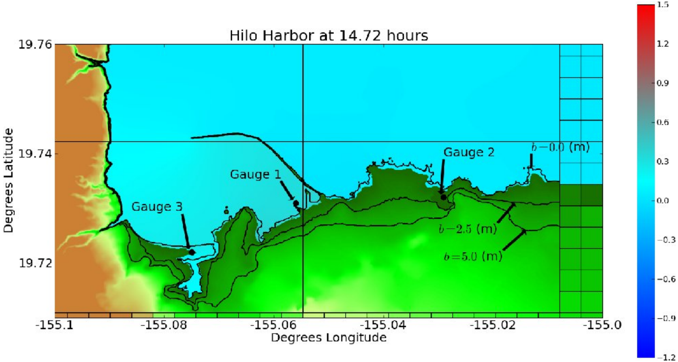


GeoClaw Example: inundation of Hilo, Hawaii

Using 5 levels of refinement with ratios 8, 4, 16, 32.

Resolution ≈ 160 km on Level 1 and ≈ 10m on Level 5.

Total refinement factor: $2^{14} = 16,384$ in each direction.



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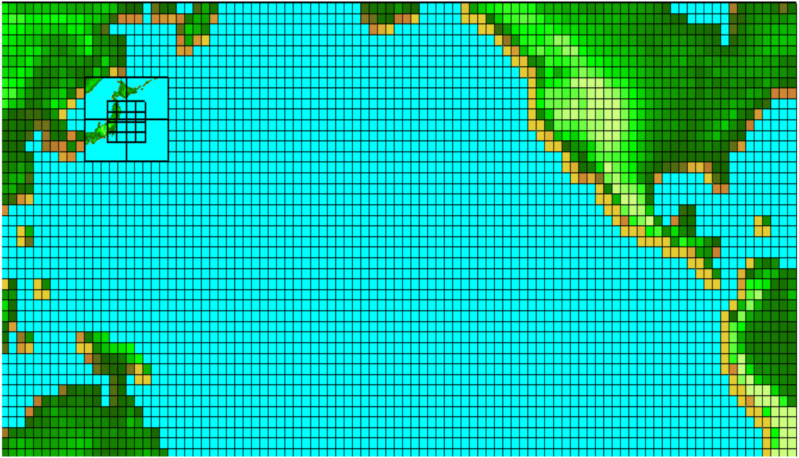
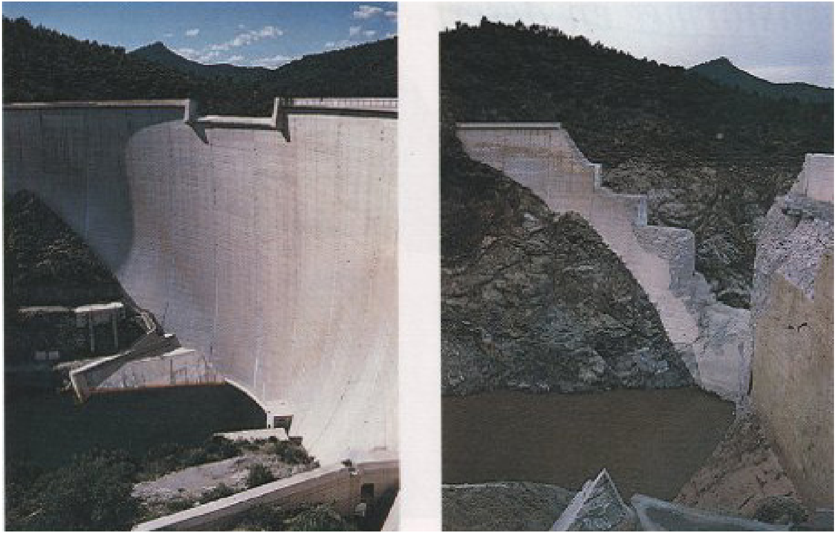


GeoClaw Example: Malpasset Dam, France 1959



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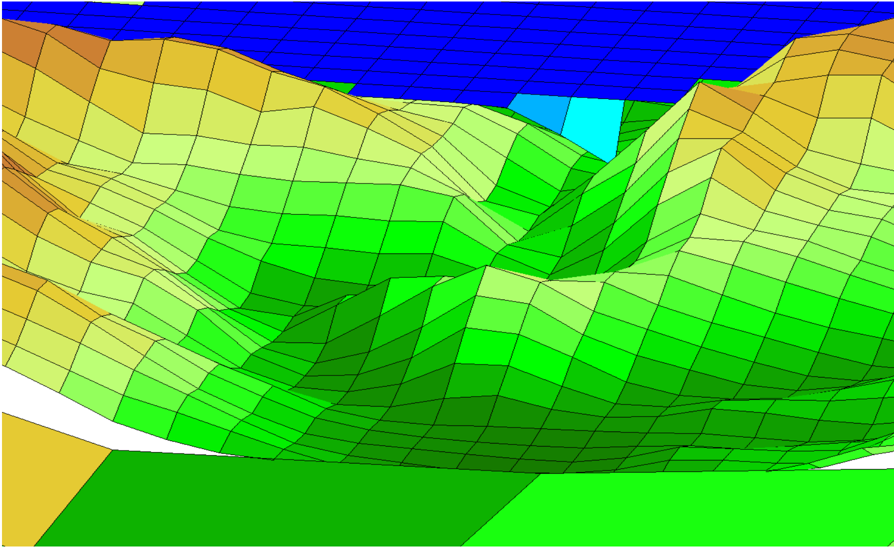

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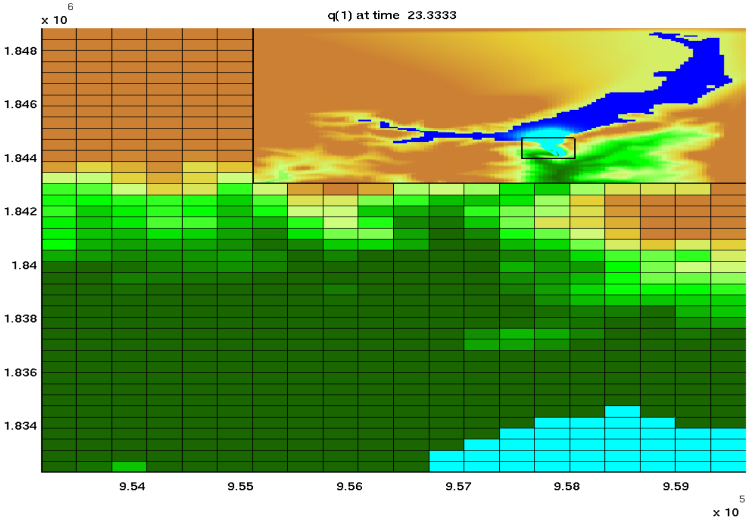

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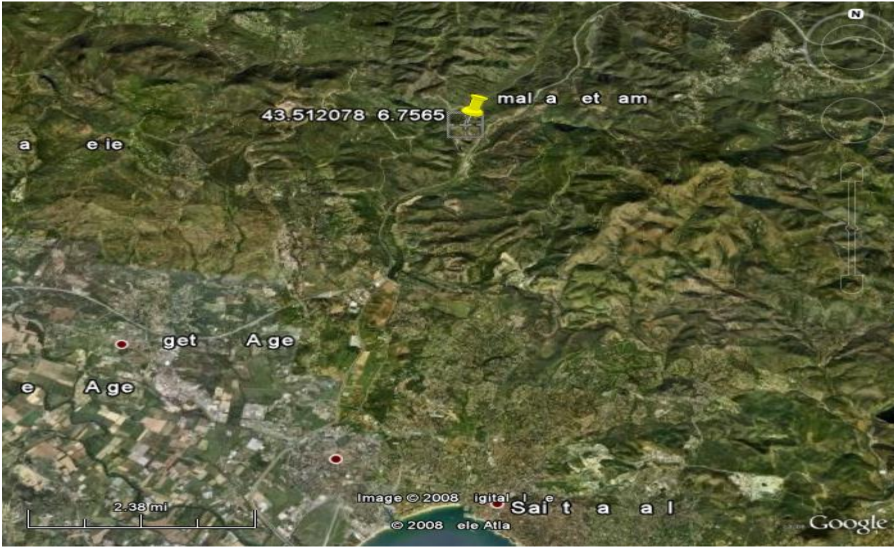


Hazardous Earth-Surface Flows

Flows characterized by a variable granular-fluid mixture:

- tsunami propagation and inundation;
- hurricane generated storm-surge inundation;
- overland/fluvial flooding, dam and levee breaches etc.;
- sediment erosion, deposition and transport;
- landslides, mudslides, lahars and debris flows;
- dry granular (or snow) avalanches.



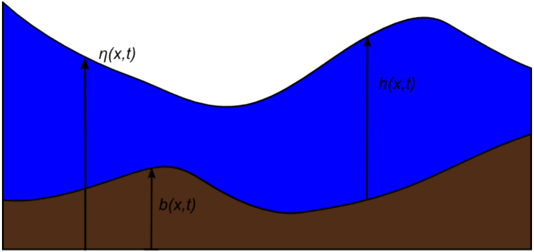
Hazardous Earth-Surface Flows

Common mathematical features and computational challenges:

- flow is shallow relative to length-scales;
- often modeled with nonconservative hyperbolic systems;
- flow moves over complex topography (singular sources);
- domain is of varying bounded extent (wet/dry problem);
- dynamics are a small perturbation to a steady state;
- feature evolving multiple spatial scales.

Hazardous Earth-Surface Flows

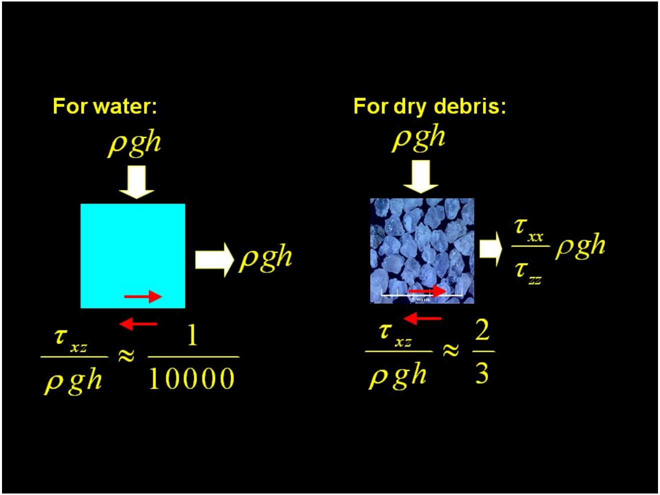
Depth-averaged models: flow between a fixed bottom $b(x, y)$ with a free surface $\eta(x, y, t)$:



- Shallow assumption and B.C.s give two-dimensional systems:

$$q_t + A(q)q_x + B(q)q_y = \psi(q),$$
$$q = (h, hu, hv, \ldots,)^T$$

Modeling fluidized-granular flow requires a stress model



Landslides and Debris Flows

Indonesian Debris Flow Movie   Ritigraben Switzerland

Landslides and Debris Flows

A true Debris flow is best described as a two or three-phase flow

- two-phase models treat the fluid and suspended fines as one phase
- shallow assumption usually holds (depth-averaging works)
- fluid-fluid stress (pressure, viscosity)
- solid-solid stress (Coulomb-Mohr friction, collisional)
- solid-fluid stress (pressure, drag)

Landslides and Debris Flows

Common categories of debris-flow models

- 3D models:
  - mixture models: single continuum rheology (usually non-Newtonian)
  - two-phase models: (usually too expensive for large-scale problems.
- Depth-averaged models:
  - Savage-Hutter models: dry-granular flows (Mohr-Coulomb friction).
  - Depth-averaged mixture models: single continuum rheology.
  - Depth-averaged multi-phase models: complex stress models for each phase
  - Quasi-two phase models: only most important contribution from fluid phase are modeled.

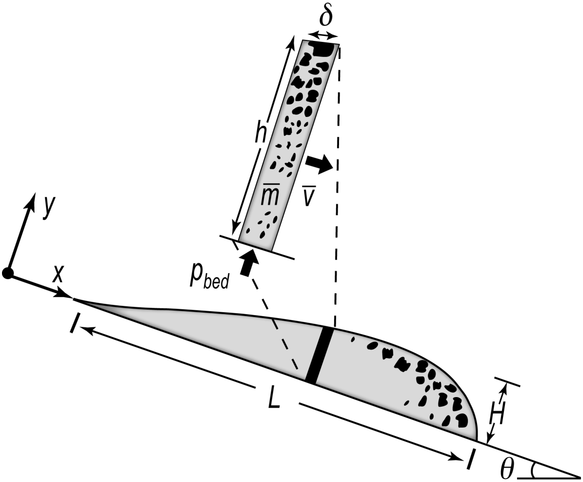
Landslides and Debris Flows

Our debris-flow model (physical fidelity $\Longleftrightarrow$ model tractability)

- quasi-two-phase:
  - mass (depth), momentum and volume fractions for solid phase are retained.
  - specific discharge is neglected. (fluid drag on solid is ignored).
  - volume-fractions are retained.
  - fluid pressure is evolved and is coupled with solid-volume fraction and momentum.
- stress model:
  - Coulomb friction for solid-solid stress: (mediated by pore-fluid pressure)
  - fluid effect on solid phase is through pressure
  - pore-fluid pressure is strongly coupled to solid-volume fraction

Landslides and Debris Flows

1D model on a fixed incline:



Landslides and Debris Flows

System for $q = (h, hu, hm, p)^T$.

$$h_t + (hu)_x = D\frac{(\rho - \rho_f)}{\rho},$$

$$(hu)_t + (hu^2 + \tfrac{1}{2}\kappa g^y h^2)_x + \frac{(1-\kappa)}{\rho} p_x = g^x h + uD\frac{(\rho - \rho_f)}{\rho} - \frac{\tau}{\rho},$$

$$(hm)_t + (hum)_x = -Dm\frac{\rho_f}{\rho},$$

$$p_t - \gamma\rho g^y u h_x + \gamma\rho g^y (hu)_x + up_x - = \zeta D - \frac{3}{h\alpha(1+\kappa)} u \tan(\psi)$$

where,

$$D = \frac{k}{h\mu}(\rho_f g^y h - p).$$

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$$q_t + f(q)_x + W(q)q_x = \psi(q, x)$$

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Landslides and Debris Flows

$$q_t + A(q)q_x = \psi(q,x)$$

Eigenstructure of $A(q)$, $q = (h, hu, hm, p)^T$:

$$r_{1,2,3,4} = \begin{bmatrix} 1 \\ u - \sqrt{g^y h\epsilon} \\ m \\ \gamma\rho g^y \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} (\kappa - 1) \\ u(\kappa - 1) \\ 0 \\ \kappa\rho g^y \end{bmatrix}, \begin{bmatrix} 1 \\ u + \sqrt{g^y h\epsilon} \\ m \\ \gamma\rho g^y \end{bmatrix},$$

$$\lambda_{1,2,3,4} = u - \sqrt{g^y h\epsilon}, u, u, u + \sqrt{g^y h\epsilon}$$

Always a full set of eigenvectors:

$$|r_{1,2,3,4}| = 2\rho g^y \sqrt{g^x h\epsilon}$$

where,

$$\epsilon = \gamma + (1-\gamma)\kappa, \quad 3/4 < \gamma < 1 \Rightarrow \epsilon > 0$$

Landslides and Debris Flows: a few numerical issues

Mobilization: $q_t + f(q)_x + W(q)q_x = \psi(q,x)$

- failure occurs when the driving forces slightly exceed the shear strength at any one point: a small perturbation to a balanced steady state: $f(q)_x + W(q)q_x \approx \psi(q,x)$
- failure: an equilibrium is perturbed...what happens next?
  1. rapid temporary instability: (shear contraction $\rightarrow$ increased pore pressure $\rightarrow$ decreased shear strength $\rightarrow$ landslide.)
  2. quick stabilization: (shear dilation $\rightarrow$ decreased pore pressure $\rightarrow$ shear strength reestablished $\rightarrow$ localized slump.)
  3. anything intermediate...*e.g.,* stick-slip.
- modeling the outcome at least requires well-balanced methods...if it can be done at all.

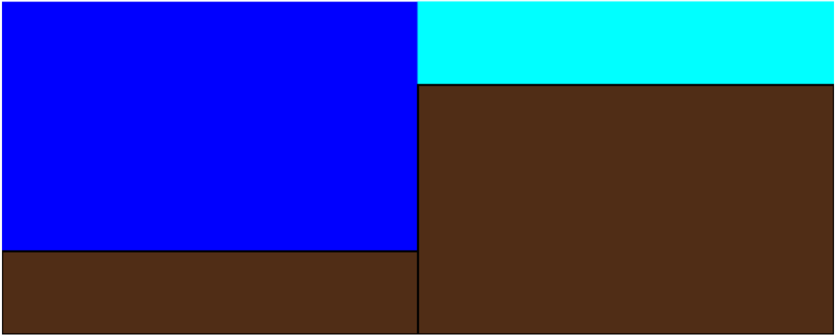
Landslides and Debris Flows: a few numerical issues

Well balancing: $q_t + f(q)_x + W(q)q_x = \psi(q,x)$

- f-wave approach: $\Delta f + W(\bar{Q})\Delta Q - \Psi = \sum \beta_p r_p$
- steady-state wave approach:
  - $q_t + f(q)_x + W(q)q_x = \psi(q,x)$
  - $q_t + A(q)q_x = \psi_1(q,x) + \psi_2(q,x)$
  - $\tilde{q}_t + \tilde{A}(\tilde{q})\tilde{q}_x = 0; \tilde{q}_t = \psi_2(q,x)$
  - $\tilde{R}(\hat{\tilde{Q}})^{-1}\Delta\tilde{Q} = \tilde{\beta}$
- steady-state solution: $\tilde{A}(\tilde{q})\tilde{q}_x = 0;$
  - $\tilde{R}(\hat{\tilde{Q}})^{-1}(\tilde{Q}_r - \tilde{Q}_l) = (0, \ldots, \tilde{\beta}_0, \ldots, 0)^T$.
  - $\tilde{Q}_r - \tilde{Q}_l = \tilde{\beta}_0 \tilde{r}_0(\hat{\tilde{Q}})$.
  - $A(\hat{\tilde{Q}})(\tilde{Q}_r - \tilde{Q}_l) = \lambda_0(\tilde{Q}_r - \tilde{Q}_l) = 0$

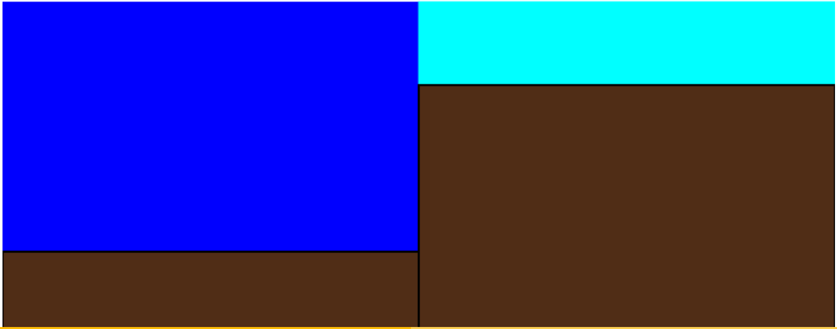
Riemann problem with source: $q_t + A(q)q_x = \psi(q, x)$

- Balanced steady-states arise: $A(q)q_x \approx \psi(q, x)$.
- Role of source term must be treated carefully in the Riemann problem.



Riemann problem with source: $q_t + A(q)q_x = \psi(q,x)$

- We solve augmented homogeneous system:
  $\mathcal{W}_t + \mathcal{A}(\mathcal{W})\mathcal{W}_x = 0$.
- steady states: stationary steady state wave only:
  $\mathcal{A}(\mathcal{W})\mathcal{W}_x = 0$



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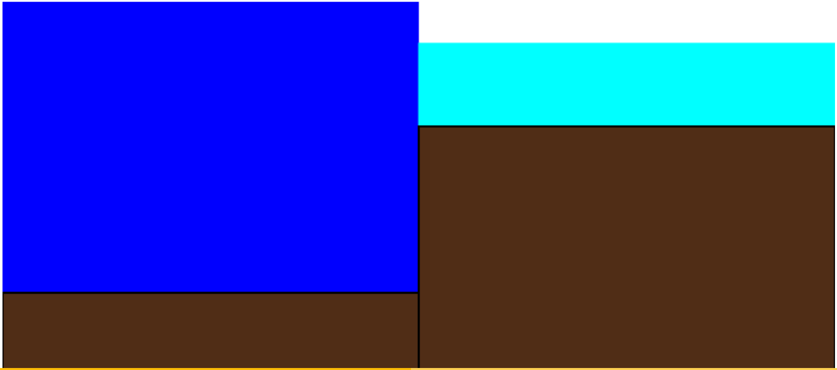

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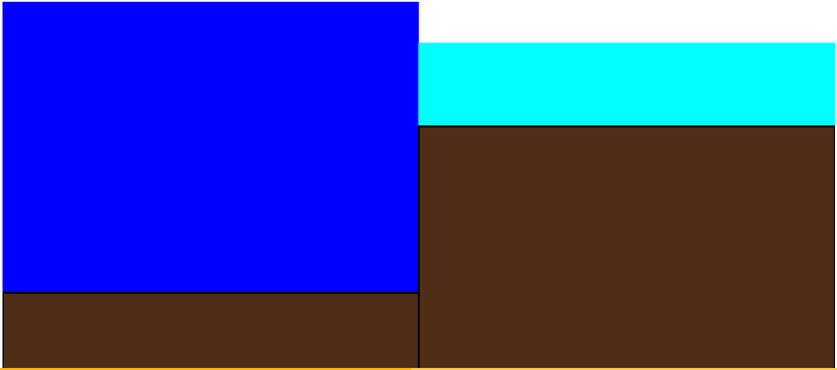
- near or non steady states: moving waves are deviation to steady state



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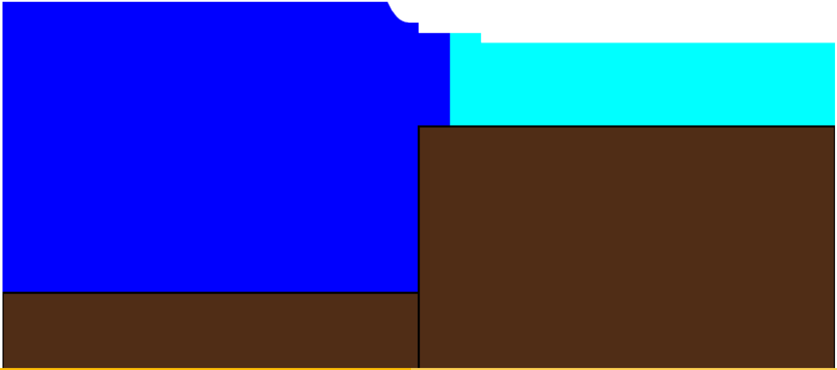

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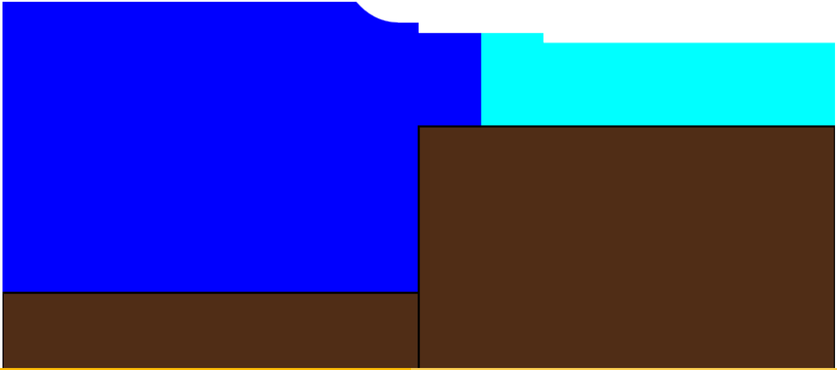

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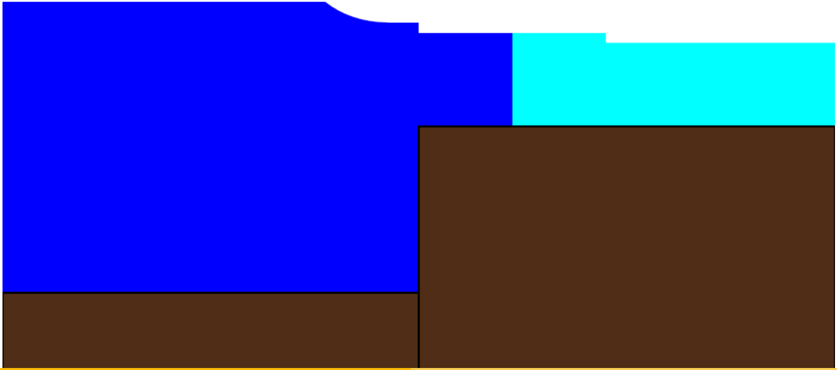

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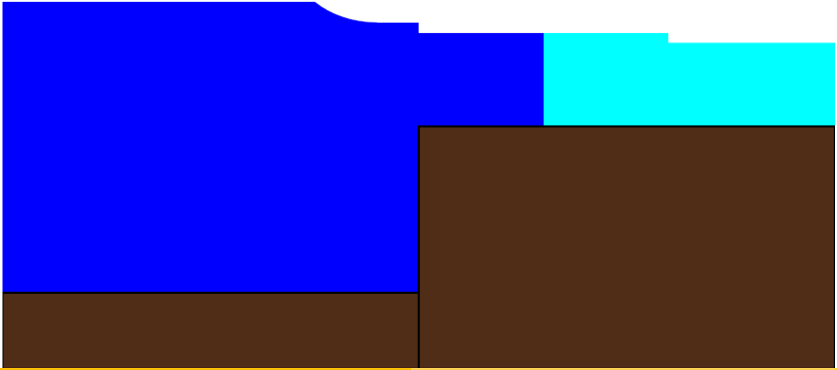

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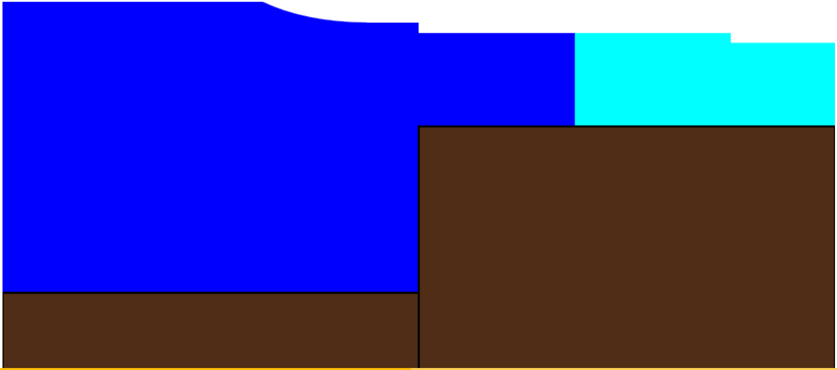

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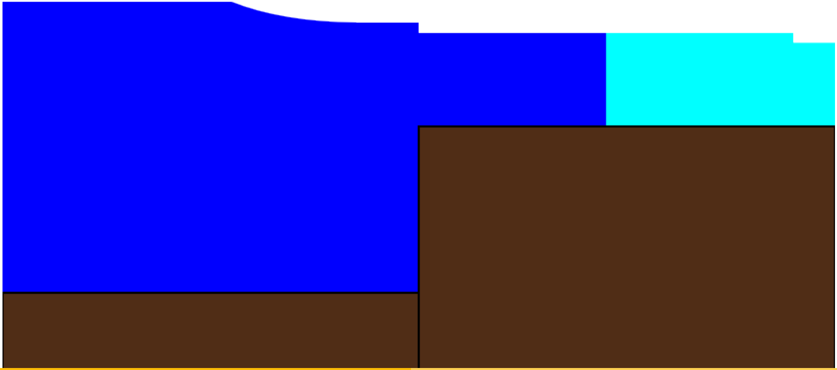

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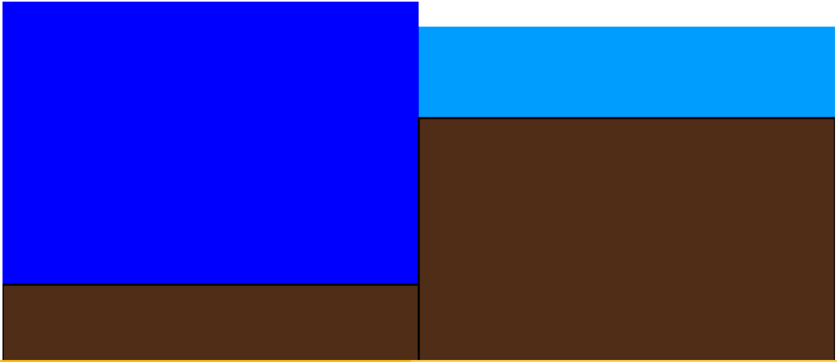
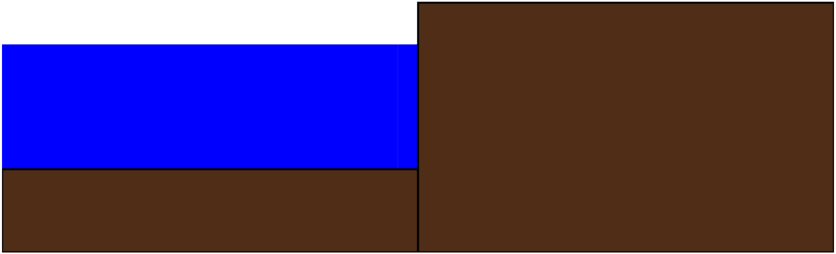


The Riemann problem at the shoreline.

motionless steady state at the shoreline: no waves.



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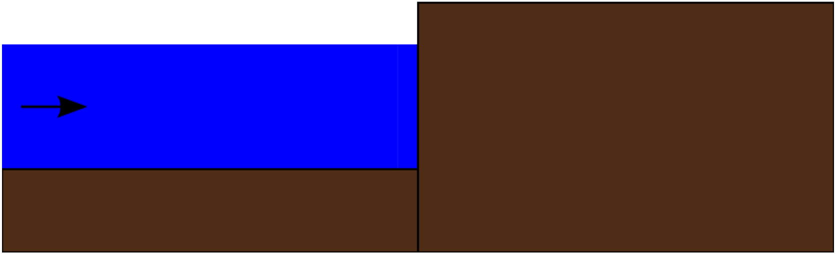
flow at the shoreline: velocity insufficient for inundation.



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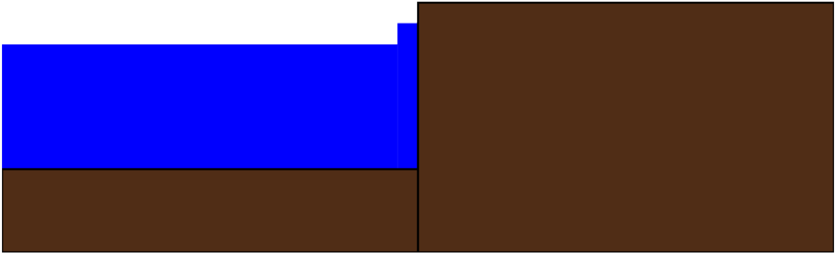

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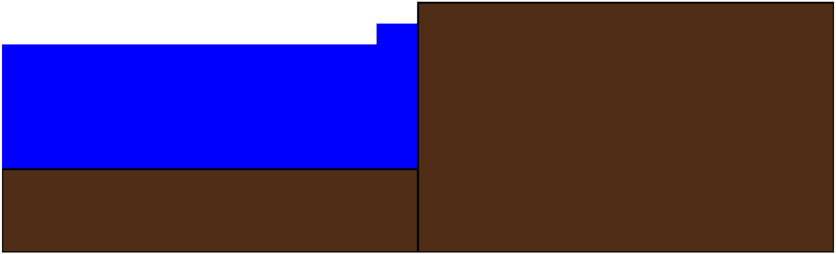

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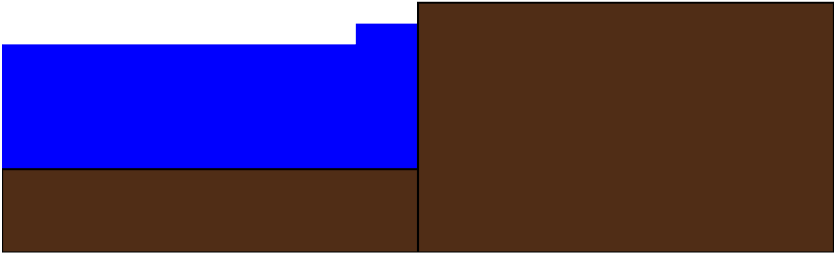

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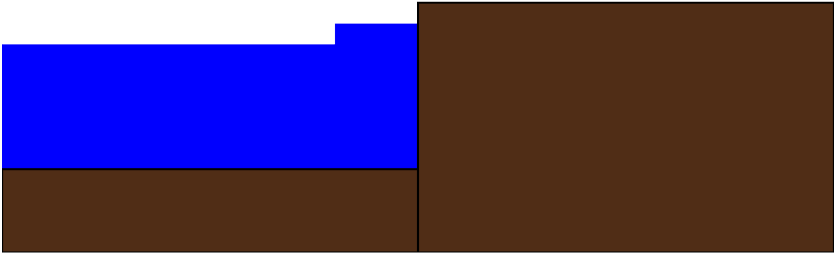

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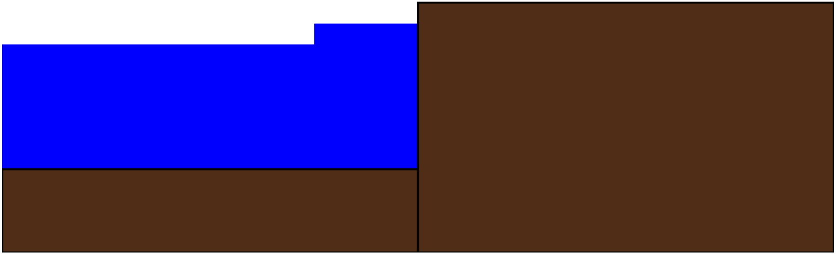

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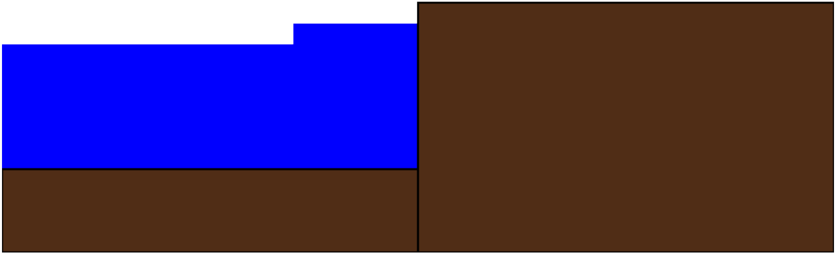

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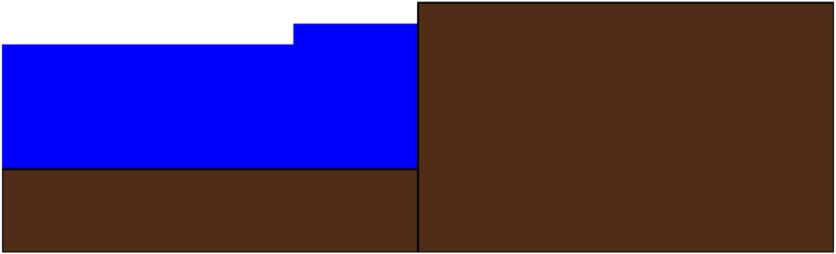

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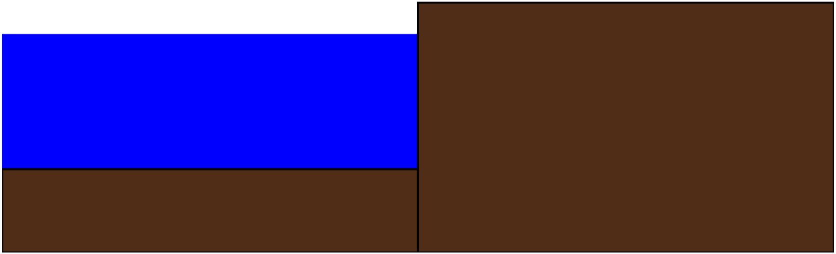

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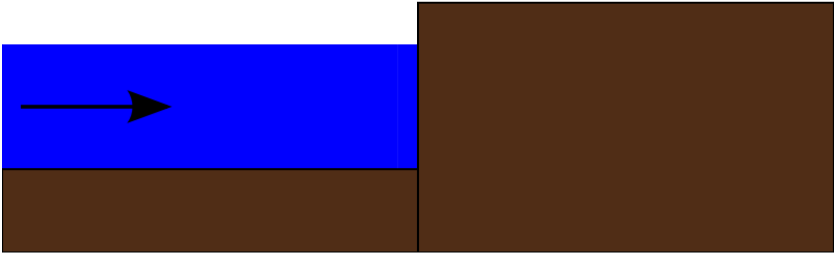

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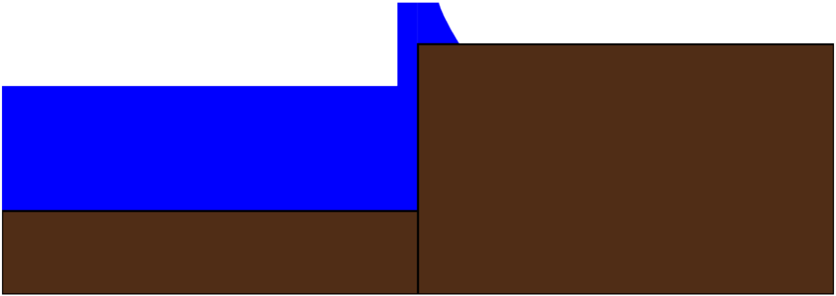

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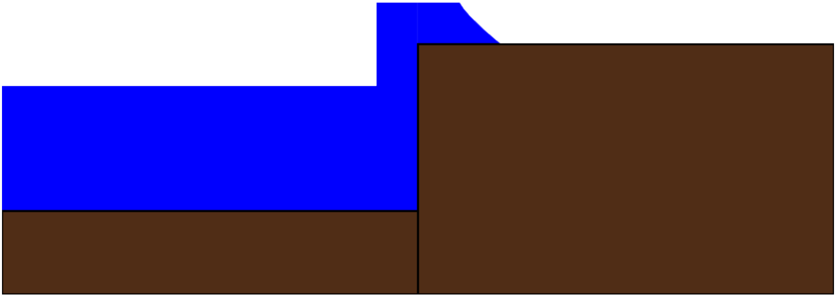

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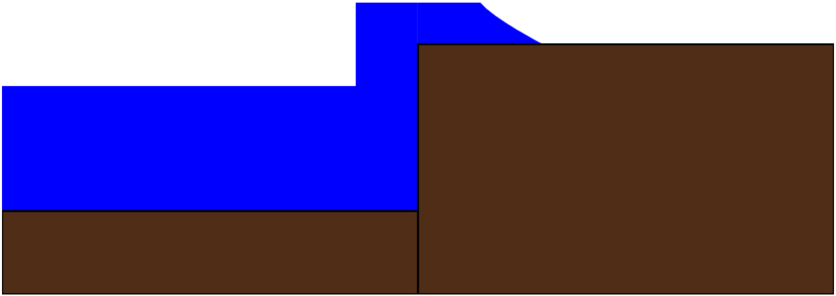

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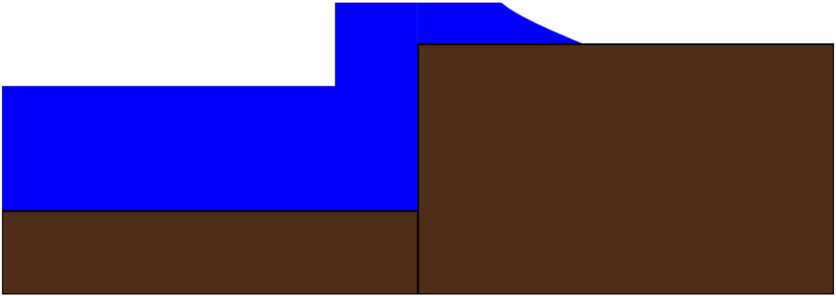

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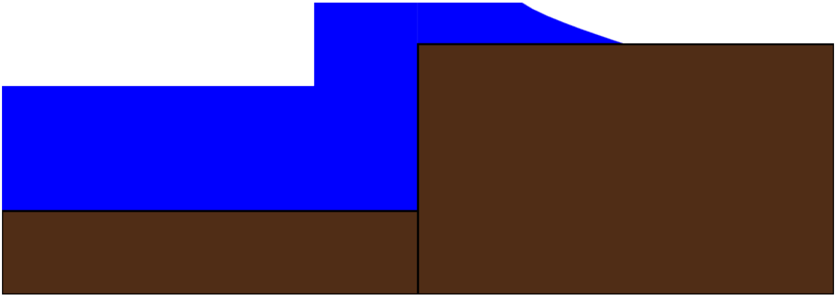

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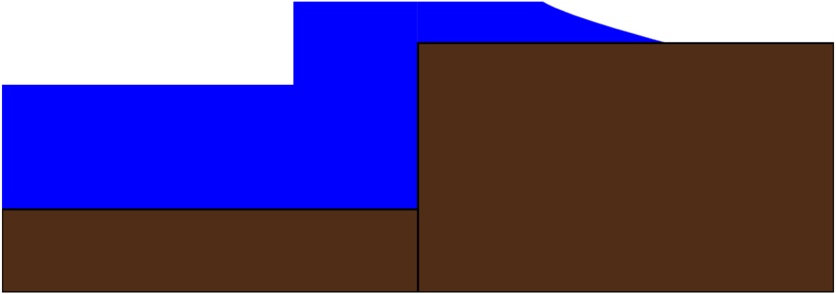

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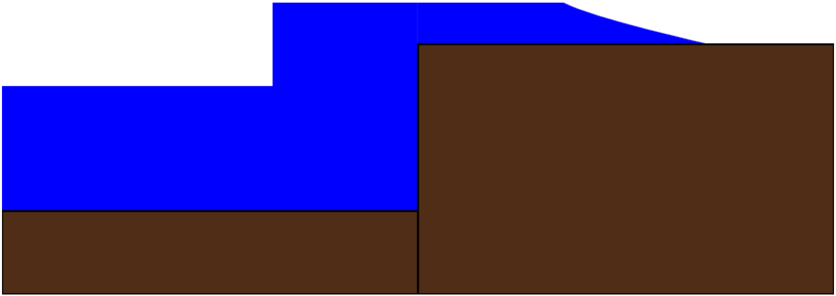

The Riemann problem at the shoreline.

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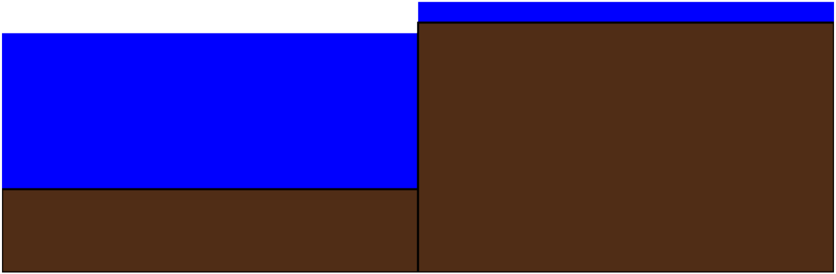
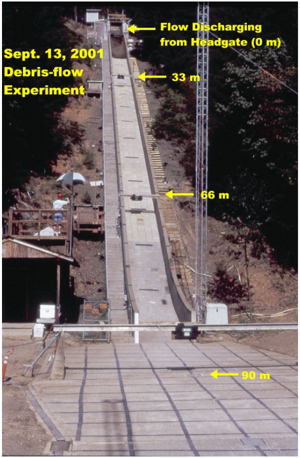
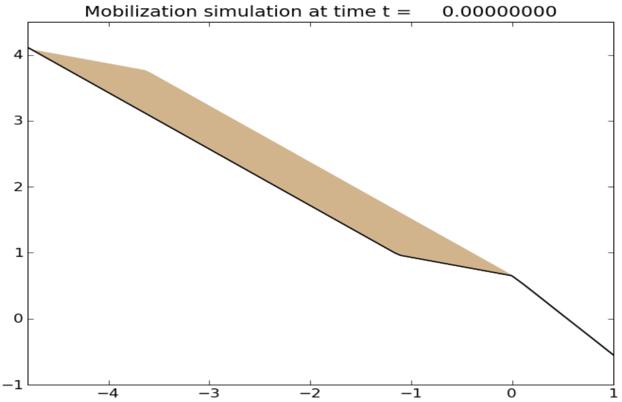


USGS experimental debris-flow flume



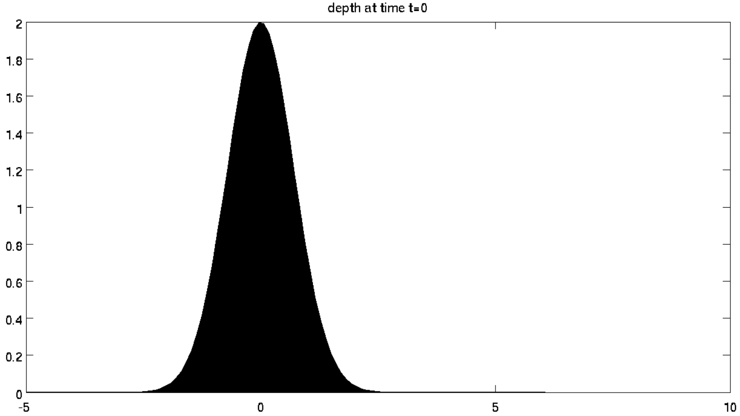
Flow Dynamics Movie
Mobilization Movie
Flow Dynamics and Segregation

Testing mobilization

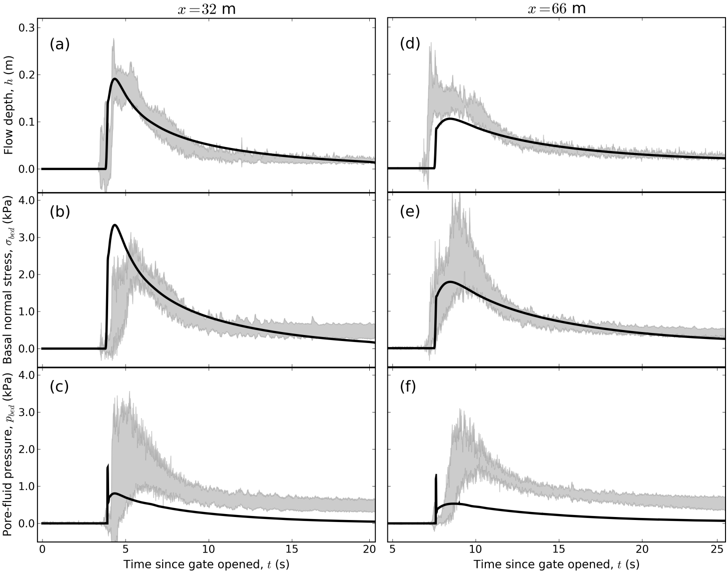


Testing flow dynamics

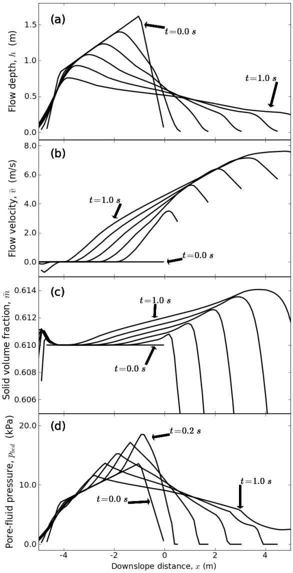
Model test:

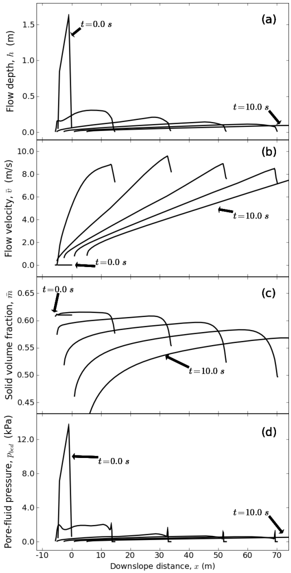


Testing flow dynamics



Testing flow dynamics





Future Directions and Conclusions

- Future Directions
  - incorporation of particle size segregation
  - entrainment
  - arbitrary topography
  - vertical acceleration corrections?
- Thank you for your attention.