

$\mu(I)$ rheology for granular flows with applications for avalanches and column collapse



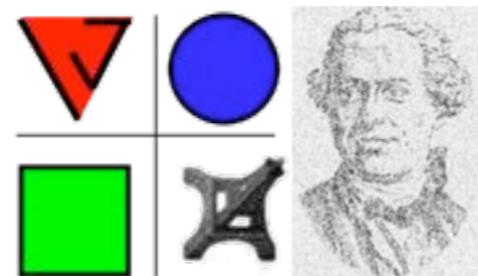
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NumHyp Roscoff 20/09/11





outline

- what is a granular fluid? some images
- the $\mu(l)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(l)$ friction law
- the **Saint Venant Savage Hutter Hyperbolic model**
- implementing the $\mu(l)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse (limits of Saint Venant Savage Hutter Hyperbolic model)



FIG. 1.2 - Les milieux granulaires forment une famille extrêmement variée.

- What is a granular media?
- size > 100µm
- grains of sand, small rocks, glass beads, animal feed pellet, medicines, cereals, wheat, sugar, rice...
- 50 % of the traded products

PHYSIQUE

**LES MILIEUX GRANULAIRES
ENTRE FLUIDE ET SOLIDE**

Bruno Andreotti, Yoël Forterre et Olivier Pouliquen

Sable, riz, sucre, neige, ciment... Bien qu'omniprésents dans notre vie quotidienne, les milieux granulaires continuent de défrayer l'industrie, de fasciner le chercheur et d'intriguer l'amateur. Pourquoi le sable est-il aussi solide pour former un tas ou soutenir le poids d'un immeuble, et coule-t-il aussi comme un liquide lorsqu'il tombe d'un sac ? Comment le vent sculpte-t-il les rôles de sable sur la planète et les dunes dans le désert ? Longtemps l'apanage des ingénieurs et des géologues, l'étude des milieux granulaires constitue aujourd'hui un sujet de recherche actif à la frontière de nombreuses disciplines — physique, mécanique, chimie, électronique, géologie, géophysique, sciences de l'ingénierie... Cet ouvrage s'adresse à dresser l'état des connaissances sur les milieux granulaires et à présenter les avancées récentes du domaine, tant de cours de Master et d'école d'ingénieur, à l'adresse aux étudiants des trois cycles universitaires, aux chercheurs et aux ingénieurs, qui trouveront là une présentation des propriétés fondamentales des milieux granulaires : interactions entre grains, comportement solide, liquide et gazeux, couplage avec un fluide, comportement au transport de sédiments et à la formation des vagues, phénomènes de transition entre les différents états... Des phénomènes simples et formels, permettant de pénétrer des domaines aussi variés que l'élasticité, la plasticité, la physique statistique, la mécanique des fluides ou la géomorphologie. De nombreux encadrés permettent d'approfondir certains phénomènes et illustrent les propriétés singulières des milieux granulaires au travers de leurs manifestations les plus spectaculaires (chant des dunes, sables mouvants, avalanches de neige...).

Bruno Andreotti, est professeur à l'université Paris VII et effectue ses recherches à l'ESPCI. Ses recherches portent sur l'hydrodynamique, le mouillage et la géomorphogénèse.
Yoël Forterre, est chercheur CNRS au laboratoire IUSTI à Marseille. Il travaille sur le comportement des fluides complexes, des milieux granulaires et sur la biomécanique des plantes.
Olivier Pouliquen, est chercheur CNRS au laboratoire IUSTI à Marseille. Ses travaux portent sur les matériaux granulaires, les suspensions et fluides complexes.

Série Physique et collection dirigée par Michèle LEDUC
SAVOIRS ACTUELS

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En vente, écrit par des docteurs, offerts aux enseignants dispensant dans le cadre de la formation à la recherche. Ils s'adressent donc aux étudiants avancés, aux chercheurs désireux de perfectionner leurs connaissances ainsi qu'à tout lecteur passionné par la science contemporaine.

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**LES MILIEUX GRANULAIRES
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SAVOIRS **PHYSIQUE** **ACTUELS**

LES MILIEUX GRANULAIRES

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spoil tip (boney pile, gob pile, bing or pit heap), «terril» in french

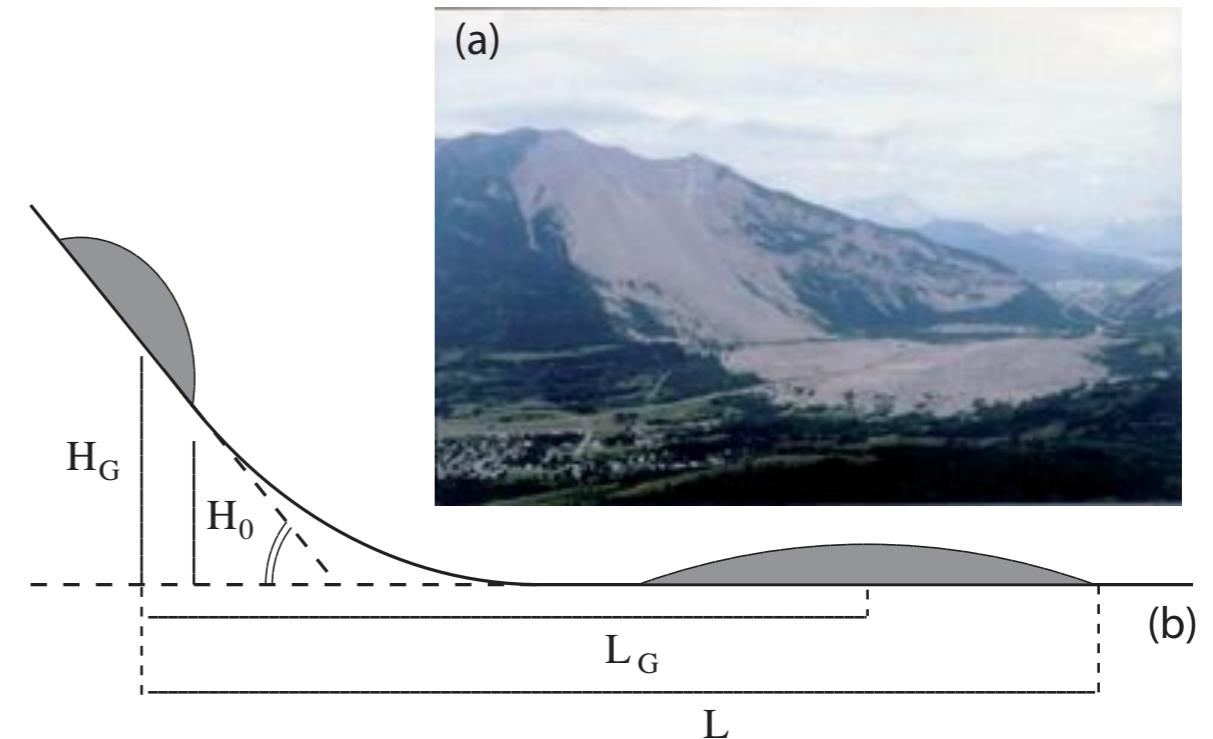




Fig. 20. Frank slide.

Environmental Modelling & Software xx (2006) 1e18
www.elsevier.com/locate/envsoft

The effect of the earth pressure coefficients on the runout of granular material
Marina Pirulli ^{a,*}, Marie-Odile Bristeau ^b, Anne Mangeney ^c, Claudio Scavia



Staron



<http://www.pbase.com/image/63044602>

2006 Gary Hebert

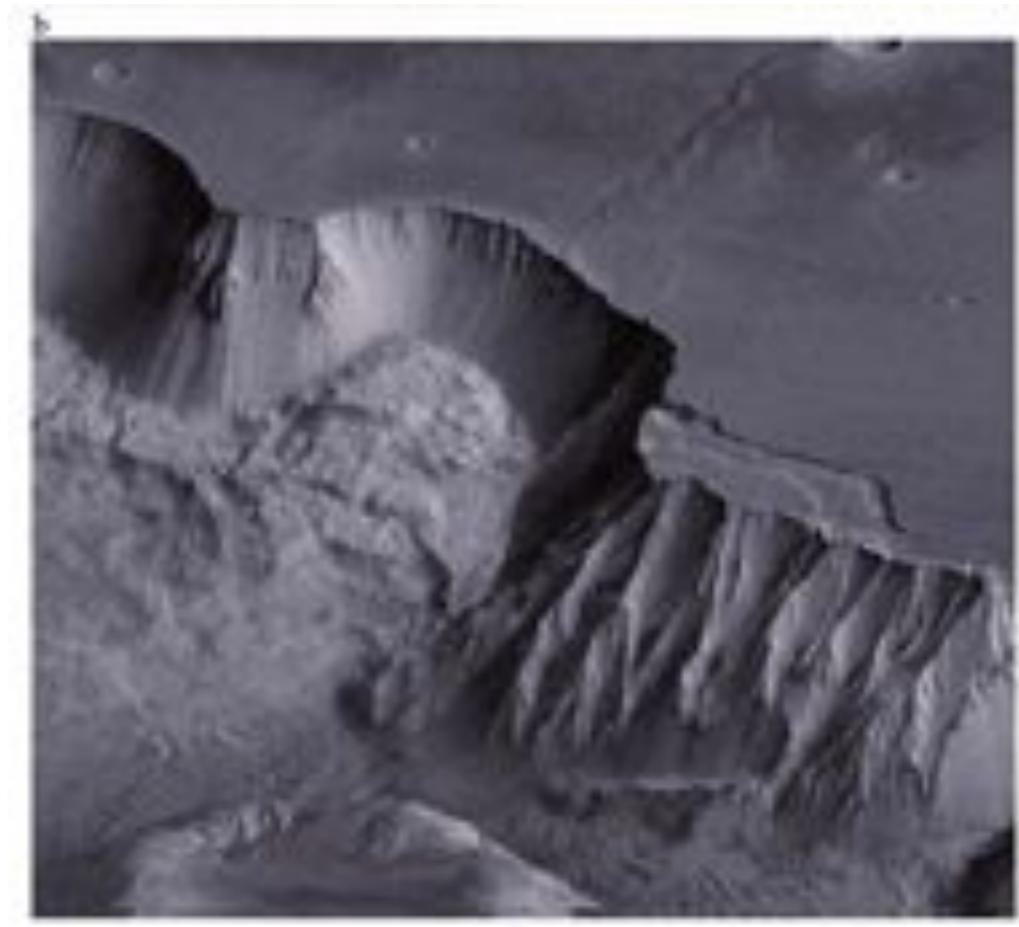


Lofoten Norway

8

photo PYL





<http://books.google.fr/books?id=HY6Z5od4-E4C&pg=PA49&dq=granular>

+flow&hl=fr&ei=lamtTaa_NYyVOoToldcL&sa=X&oi=book_result&ct=result&resnum=10&ved=0CFkQ6AEwCTgK#v=onepage&q&f=true

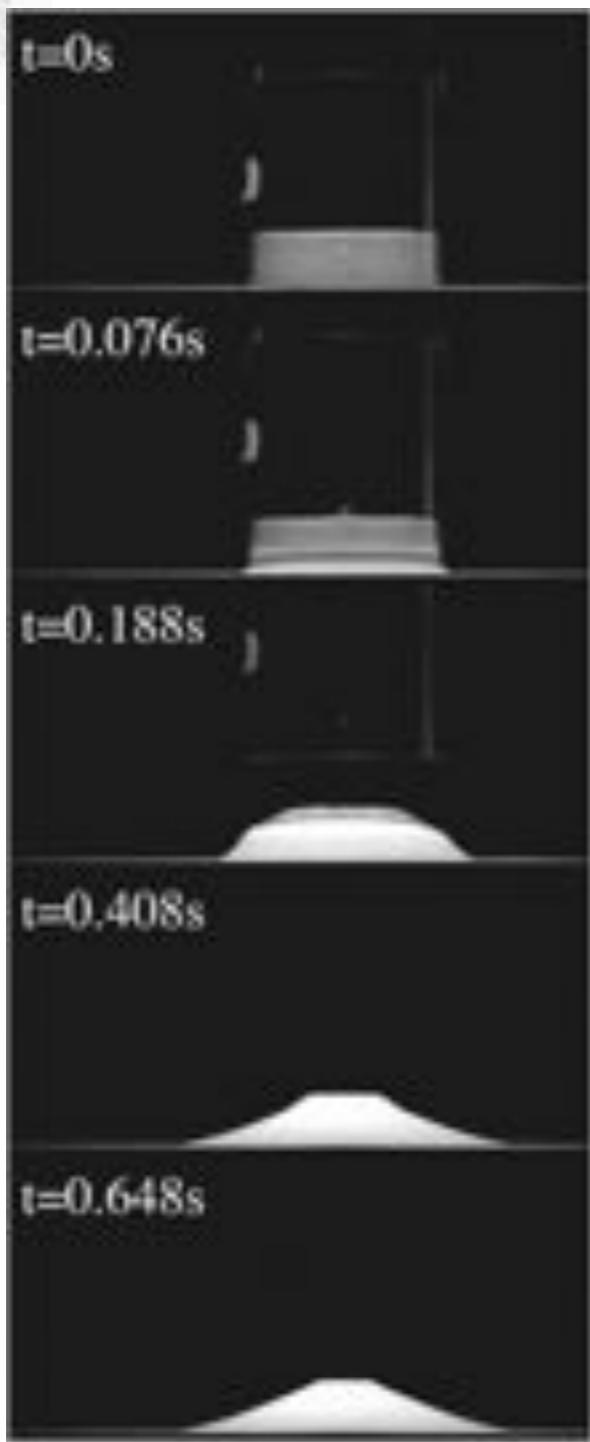


http://www.cieletespace.fr/image-du-jour/5126_la-saison-des-avalanches-sur-mars

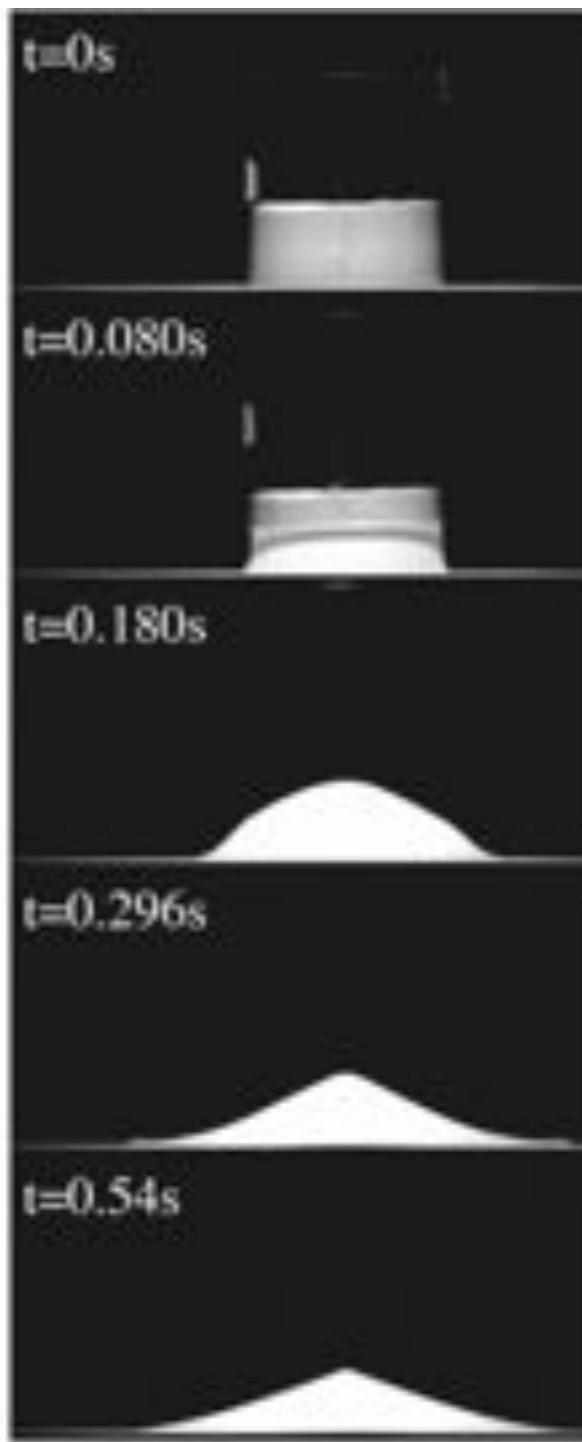
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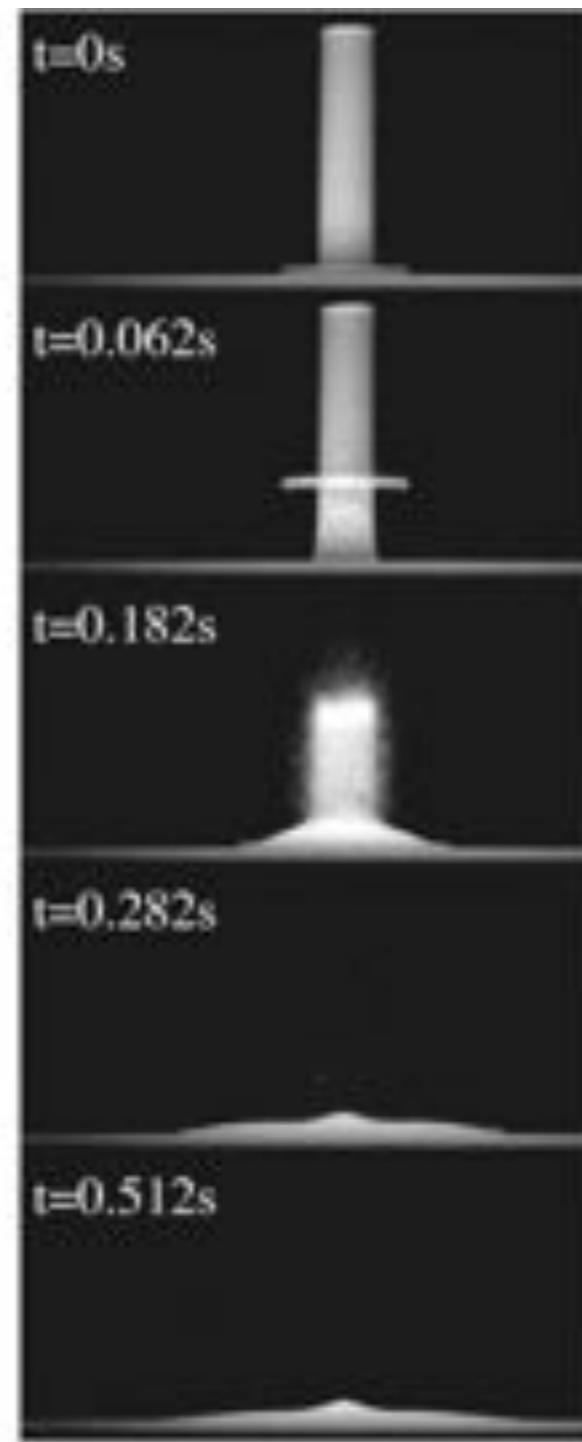
Granular Column Collapse



(a)



(b)



(c)



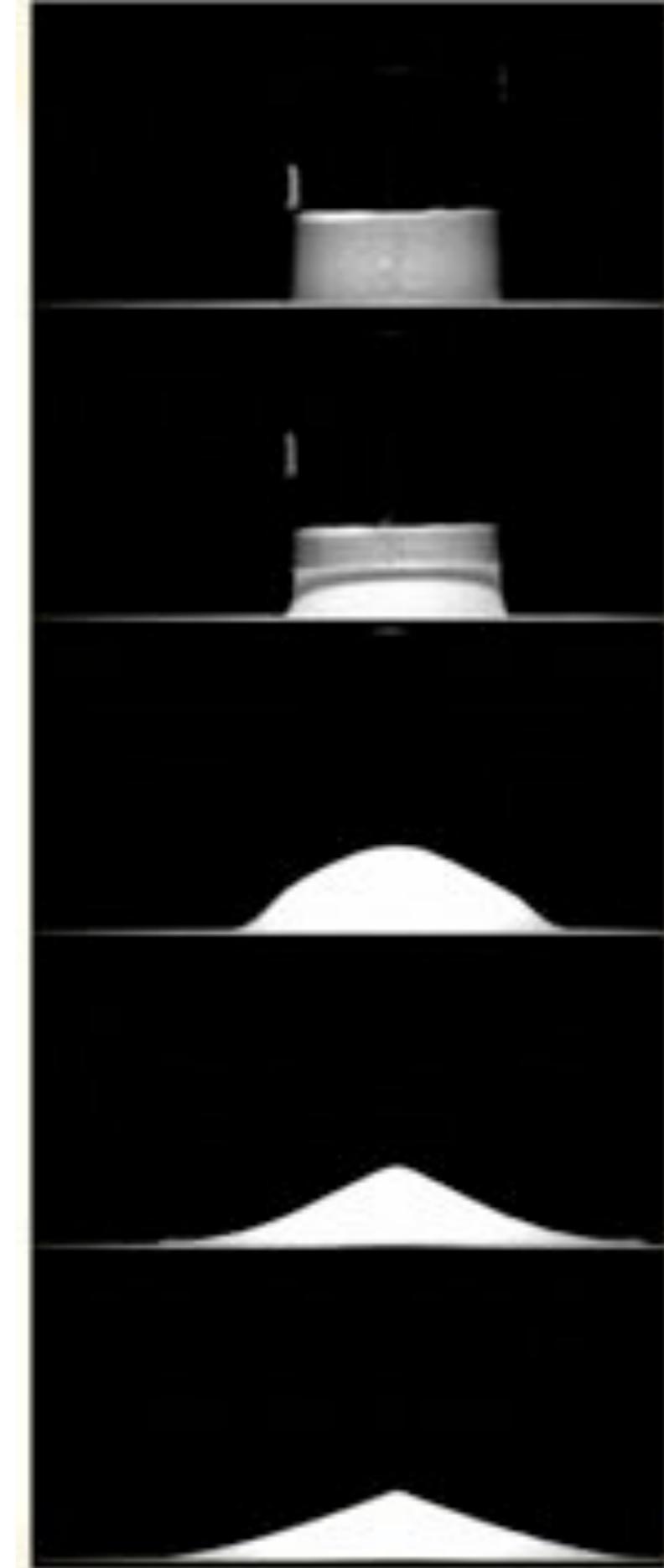
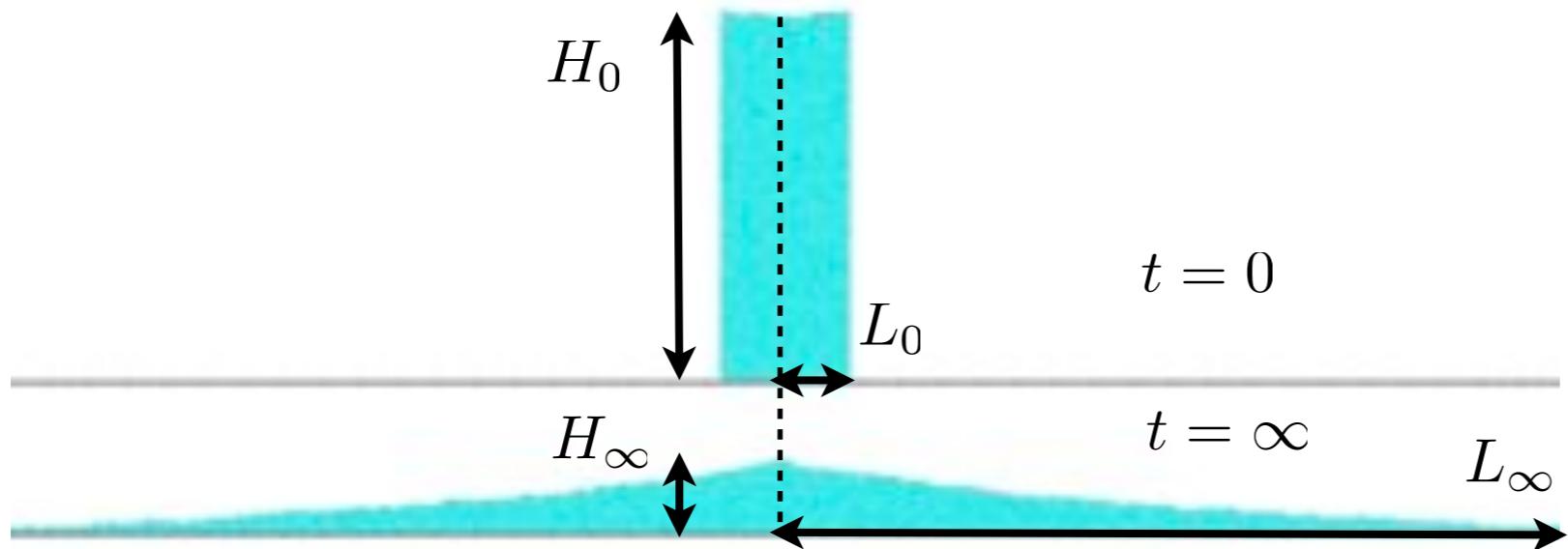
The sand pit problem: quickly remove the bucket of sand

<http://www.mylot.com/w/photokeywords/pail.asp>



Granular Column Collapse

aspect ratio $a = H_0/R_0 = H_0/L_0$



The sand pit problem: quickly remove the bucket of sand



Granular Column Collapse



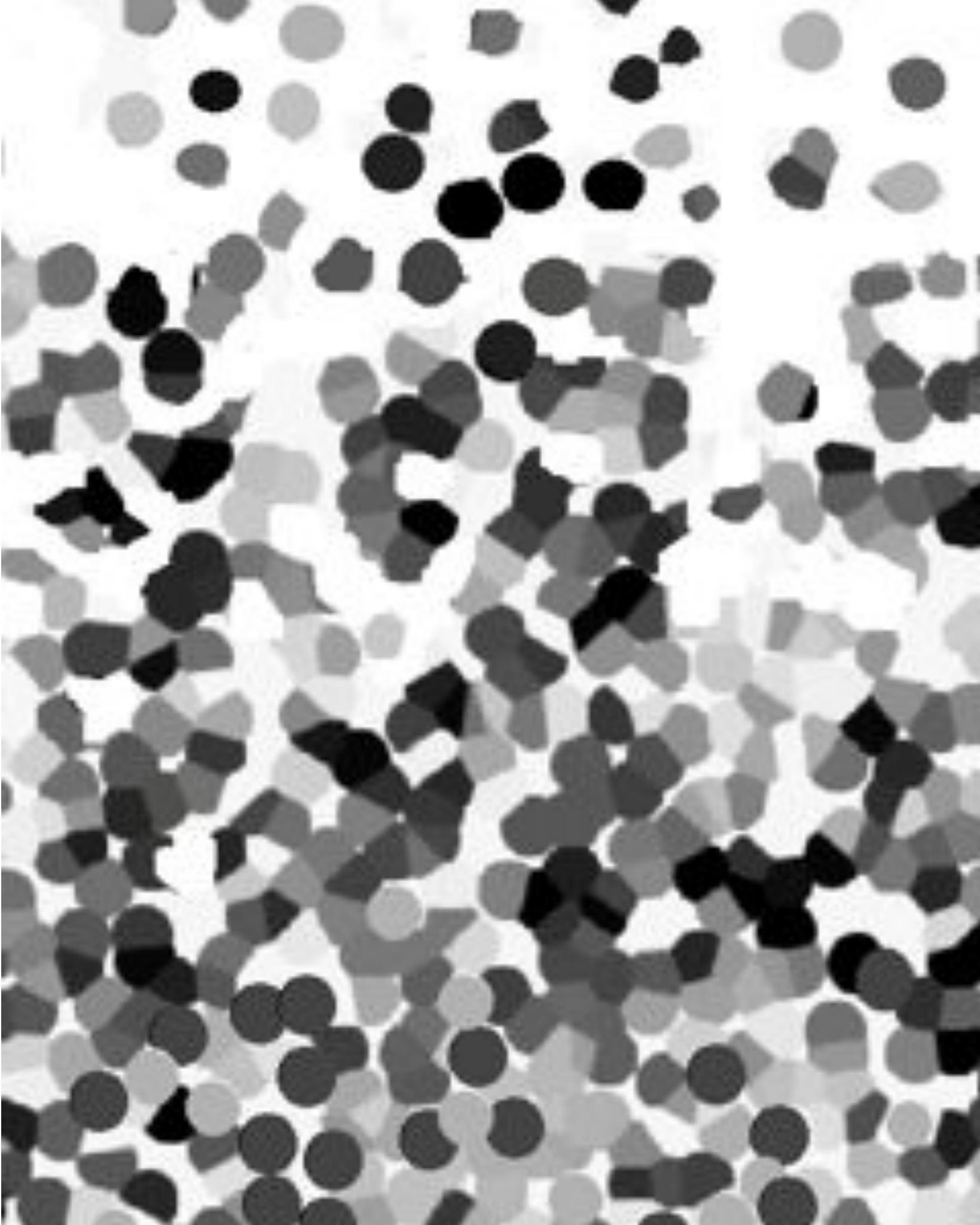
A possible experimental set up is a container filled by sand (left), the aspect ratio (height/length) is a . At initial time, the gate is opened quickly. After the avalanche, the grains stop, the final configuration is at rest (right). We compare results from Discrete Contact Method Simulations (simulation of the displacement of each grain) to a continuum Navier Stokes simulation with the $\mu(I)$ rheology *Gerris*.

The sand pit problem: quickly remove the bucket of sand



outline

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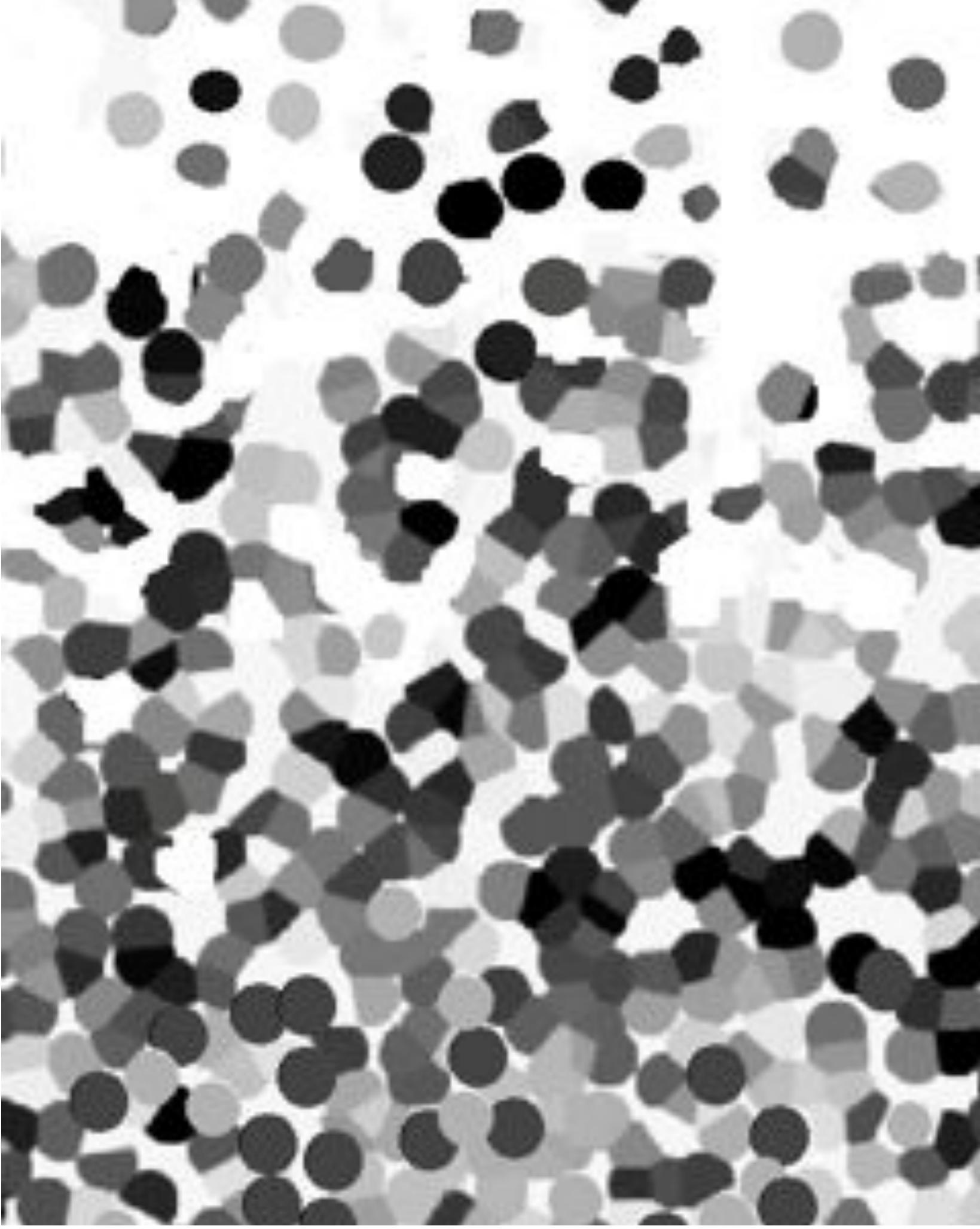
grains

impacts:
suspension

granular media
contacts

like a solid:

from the grains to the fluid



grains

ϕ_{min}

0.5 (2D) 0.55 (3D)

$\phi_{min} < \phi < \phi_{Max}$

ϕ_{max}

0.8 (2D) 0.65 (3D)

from the grains to the fluid



~~grains~~

continuum media
hypothesis

from the grains to the fluid



- experiments on model material (glass bead, sand), rheology, images
- numerical experiments of contact dynamics (disks, polygons, spheres)
- Simple configuration: shear/ inclined plane

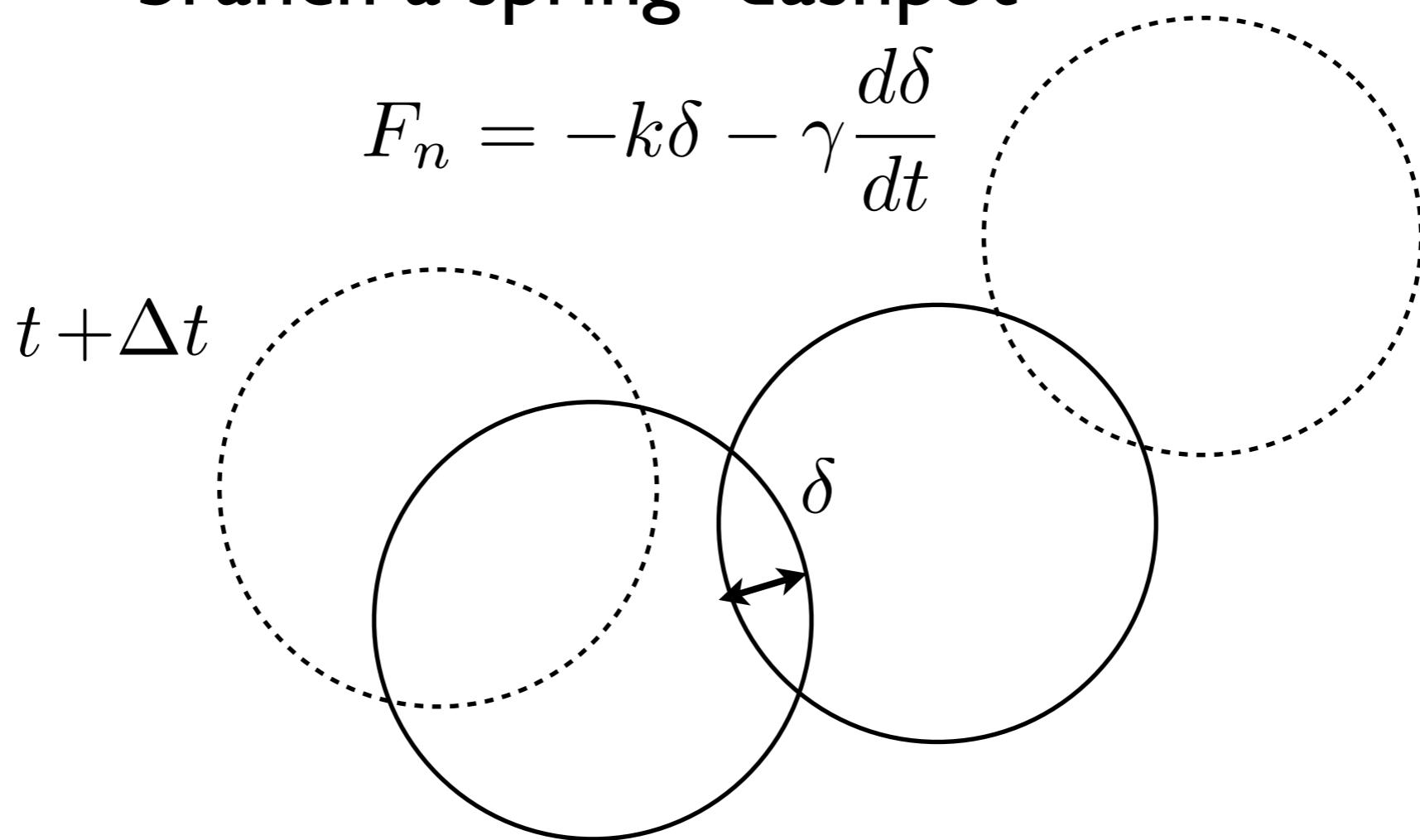


Molecular Dynamics:

$$m \frac{d}{dt} \vec{U} = \vec{F} + \vec{F}_n + \vec{F}_t \quad \text{Newton's equations}$$

branch a spring -dashpot

$$F_n = -k\delta - \gamma \frac{d\delta}{dt}$$



tangential Coulombic Friction $F_t < \mu F_n$



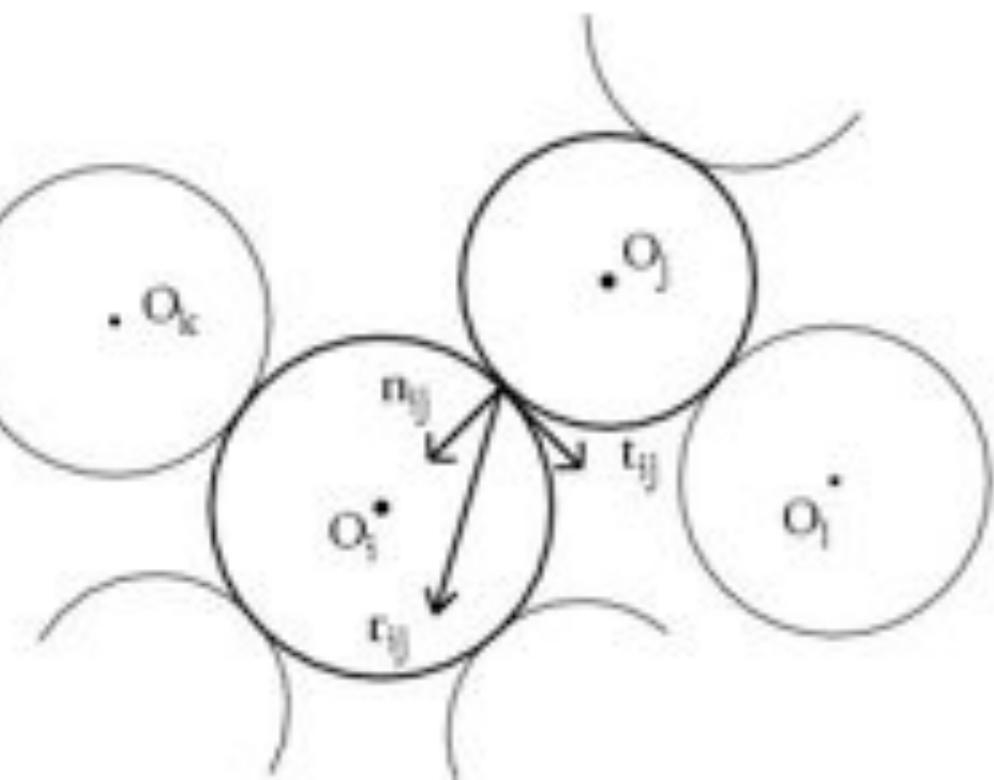
Contact Dynamics de (Moreau 1988)

rigid grains

coefficient of friction μ

$$m \frac{d}{dt} \vec{U} = \vec{F}$$

Newton's equations





Contact Dynamics de (Moreau 1988)

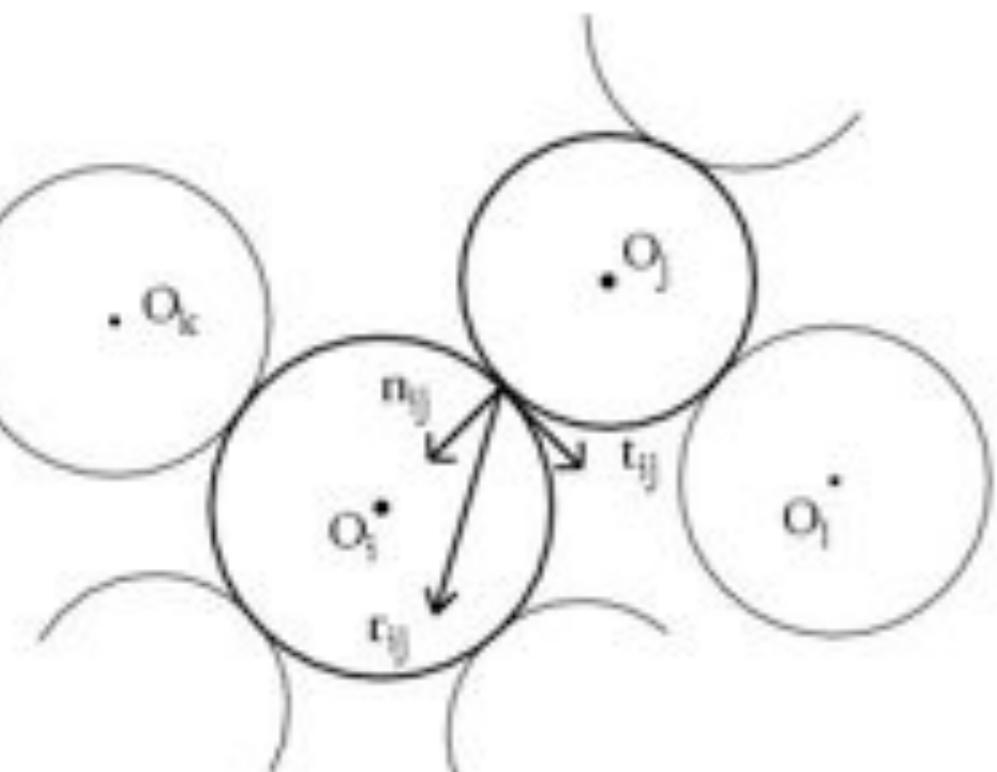
rigid grains

coefficient of friction μ

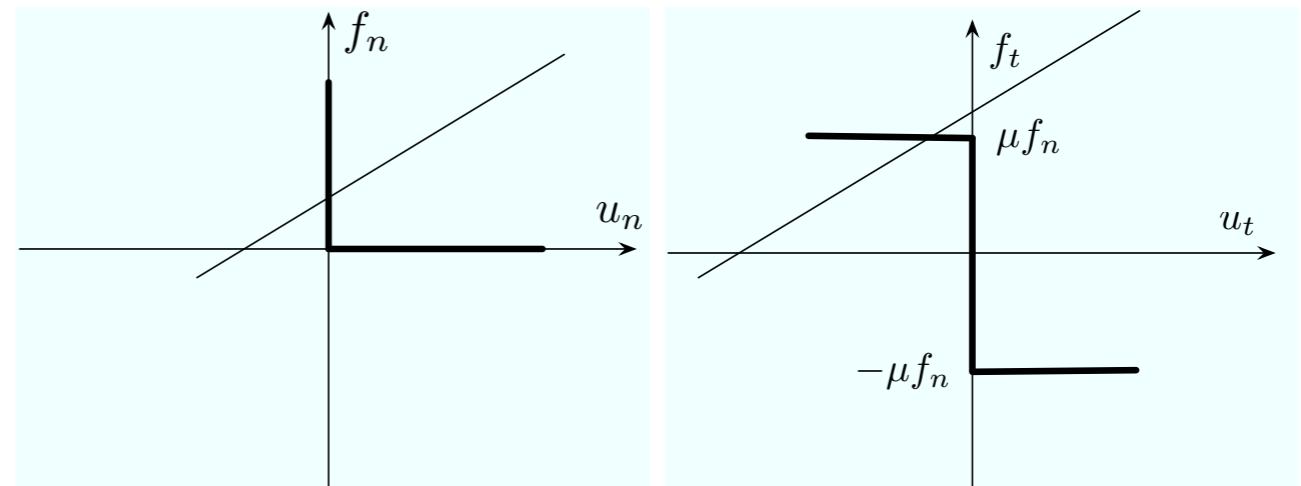
$$m(\vec{U}^+ - \vec{U}^-) = \vec{F} \delta t \quad \text{Newton's equations}$$

take the form of an equality between the change of momenta
and the average impulse during δt .

written for each grain at the contact

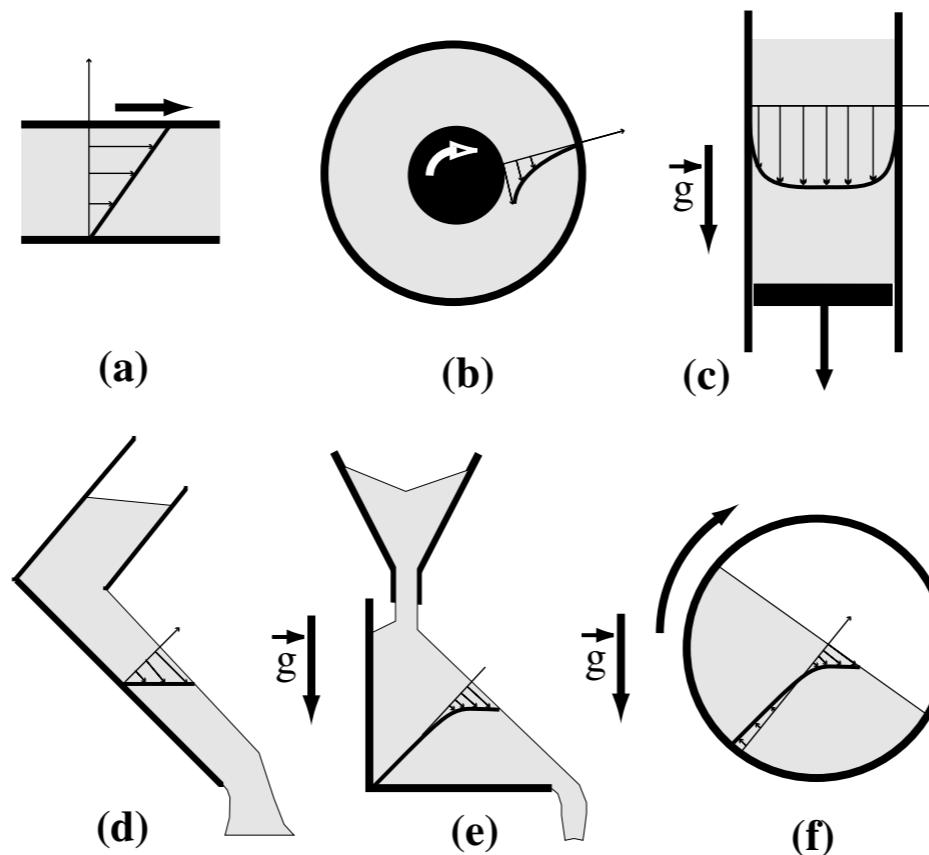


u_n, u_t





- Looking for a continuum description
- Lot of recent experiments
- Simulations with Contact Dynamics



GDR MiDi EPJ E 04

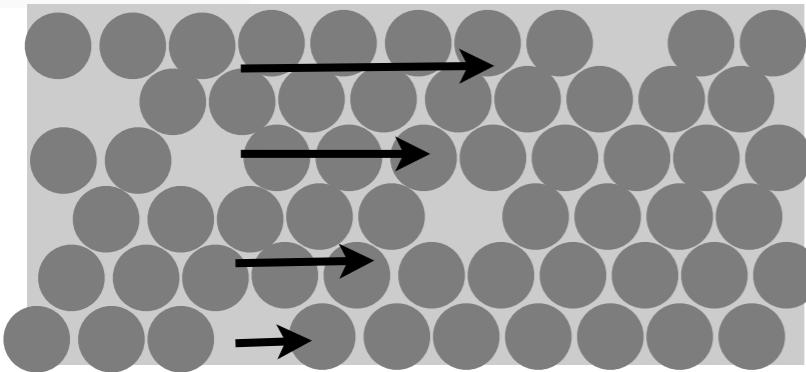
Fig. 1. The six configurations of granular flows: (a) plane shear, (b) annular shear, (c) vertical-chute flows, (d) inclined plane, (e) heap flow, (f) rotating drum.



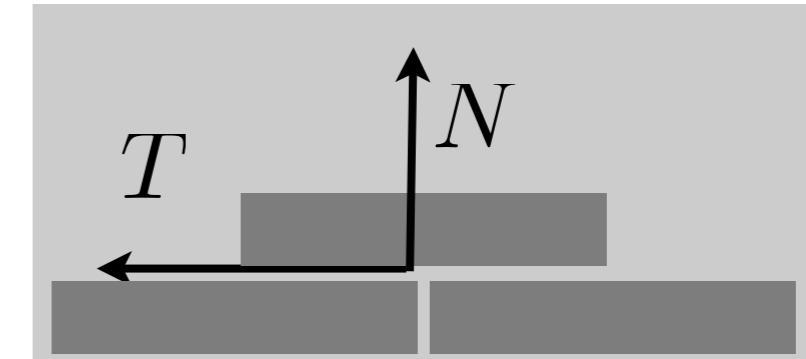
- Looking for a continuum description
 - Lot of recent experiments
 - Simulations with Contact Dynamics
-
- Defining a «viscosity»
 - Implement it in the Navier Stokes solver *Gerris*
 - Test on exact «Bagnold» avalanche solution
 - Test on granular collapse



$$V(z)$$

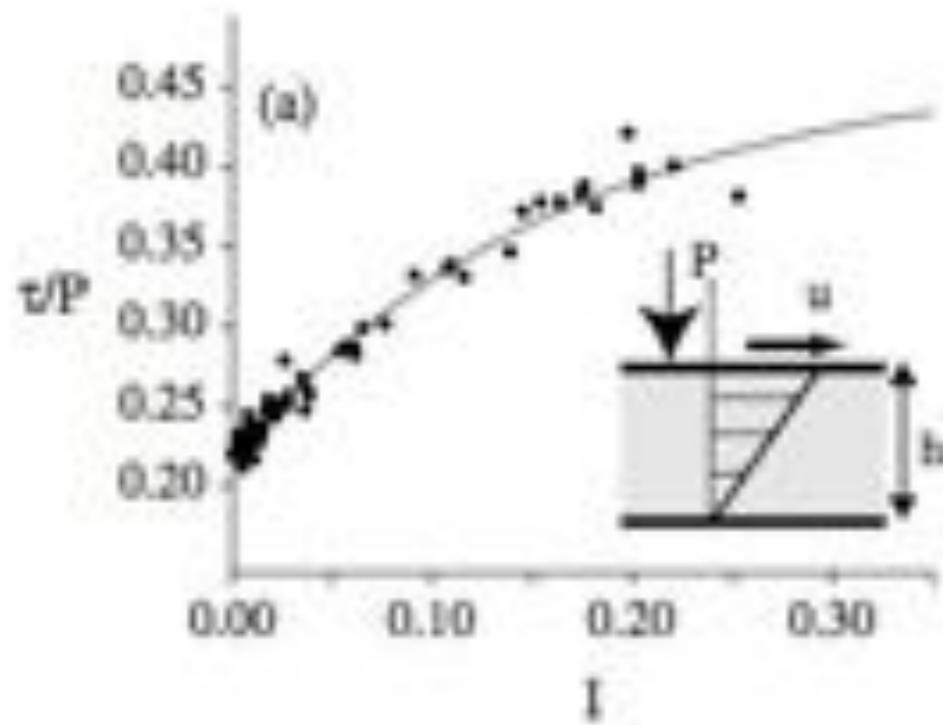


$$V(z)$$



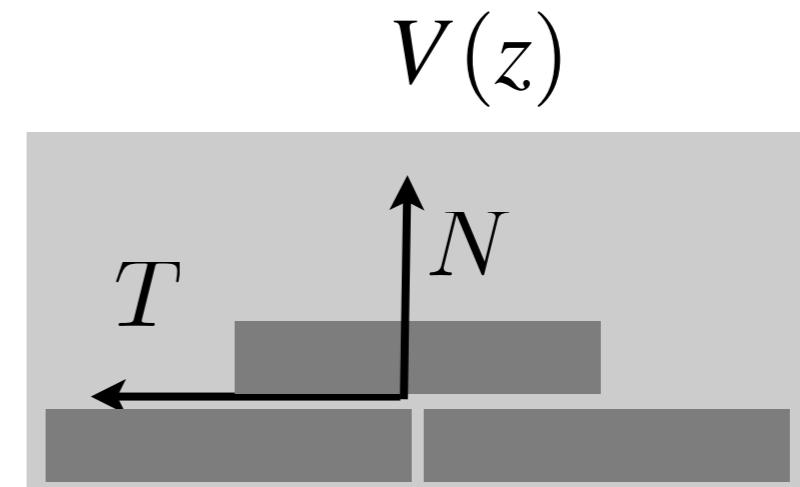
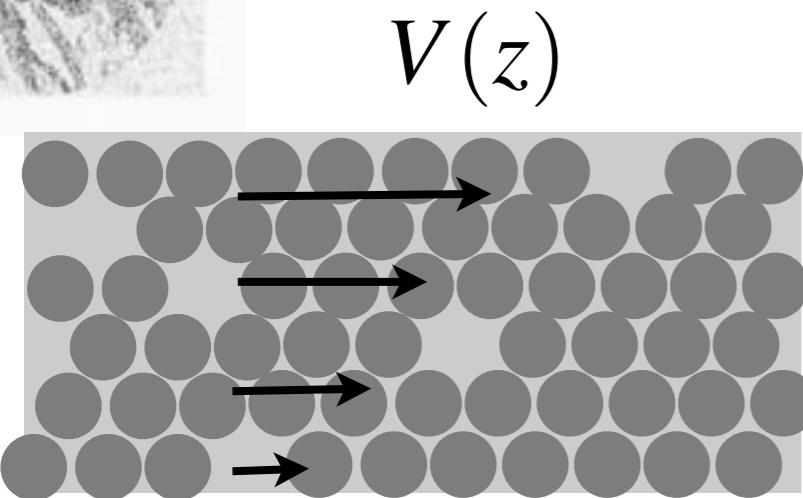
$$T = \mu N$$

constitutive law?

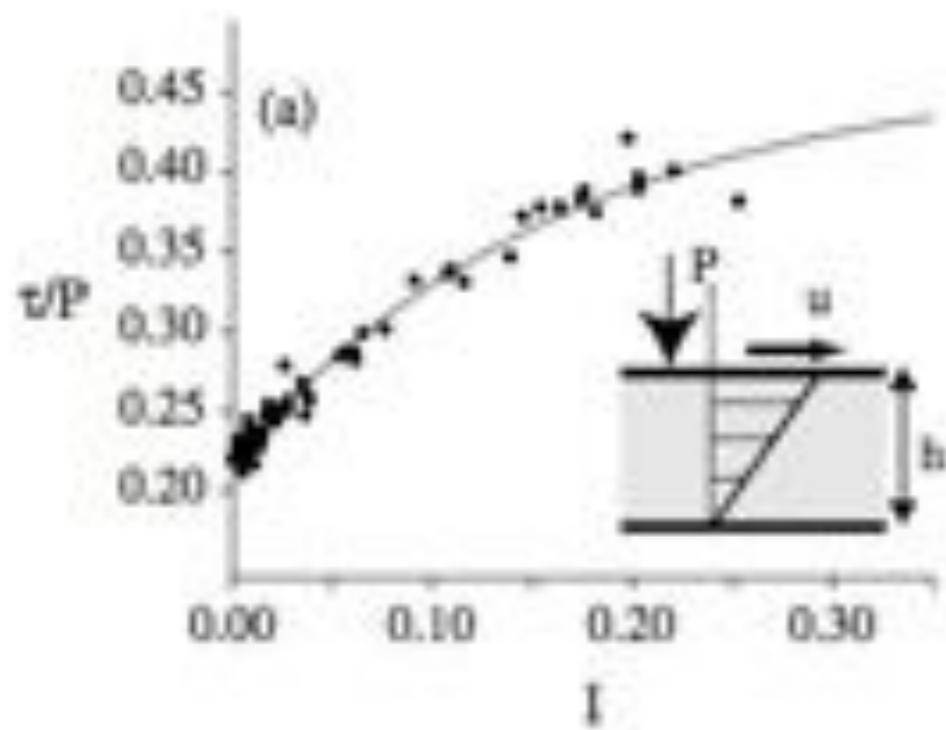


Coulomb dry friction
Coulomb friction law

$$\tau = \mu P$$



$$T = \mu N$$

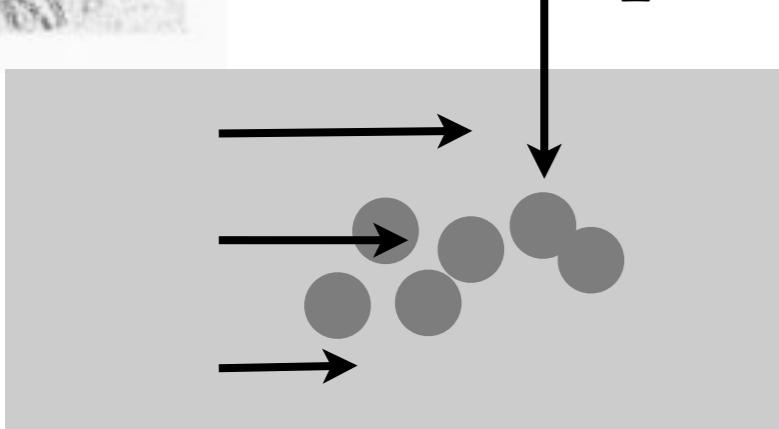


$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

Coulomb friction law

$$\tau = \mu(I)P$$

non dimensional number: «Froude»
local «Inertial Number» (Da Cruz 04-05)



$$md^2y/dt^2 = Pd^2$$

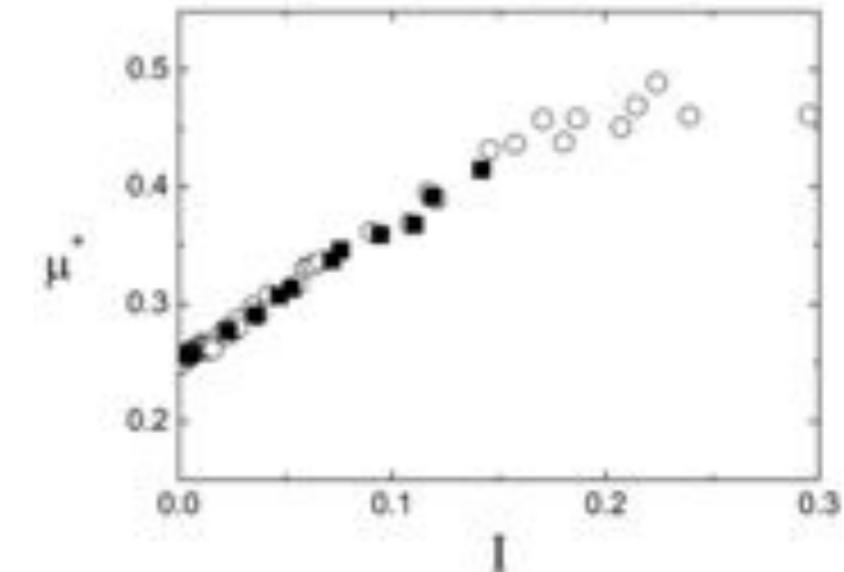
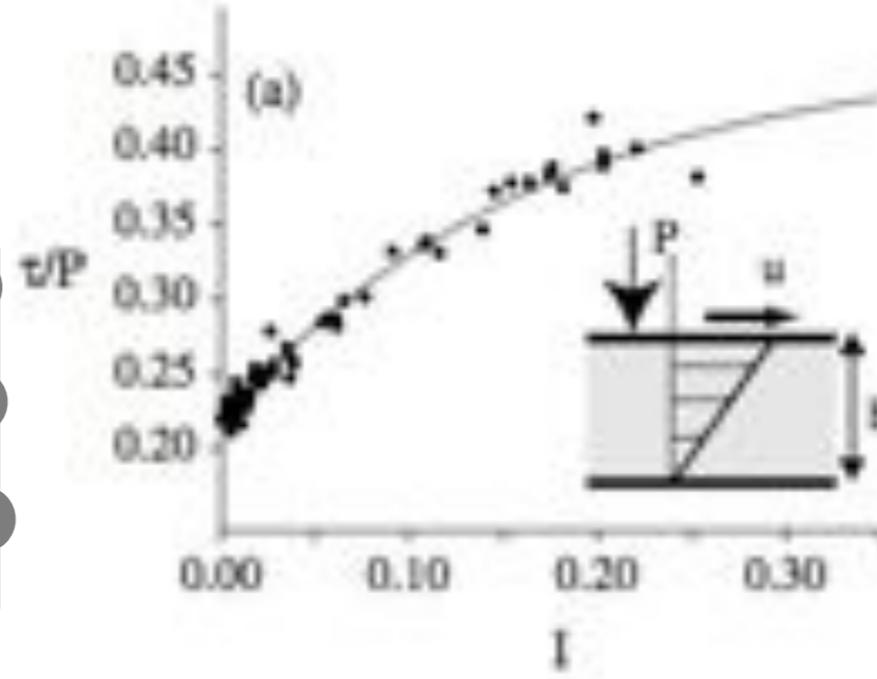
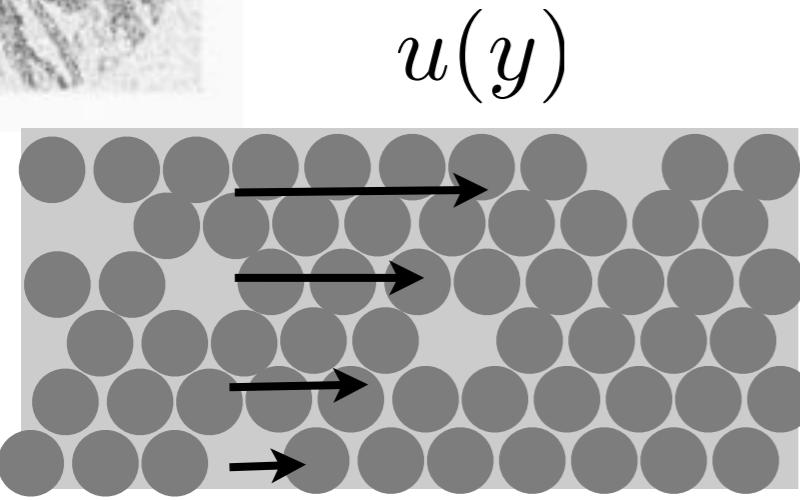
$$\updownarrow t^2 = \rho d^2/(P)$$

$$\frac{dx}{dt} = d\frac{\partial u}{\partial y} \quad t = 1/\frac{\partial u}{\partial y}$$

falling time

displacement time

$$I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$



Da Cruz PRE 05

Coulomb friction law

$$\tau = \mu(I)P$$

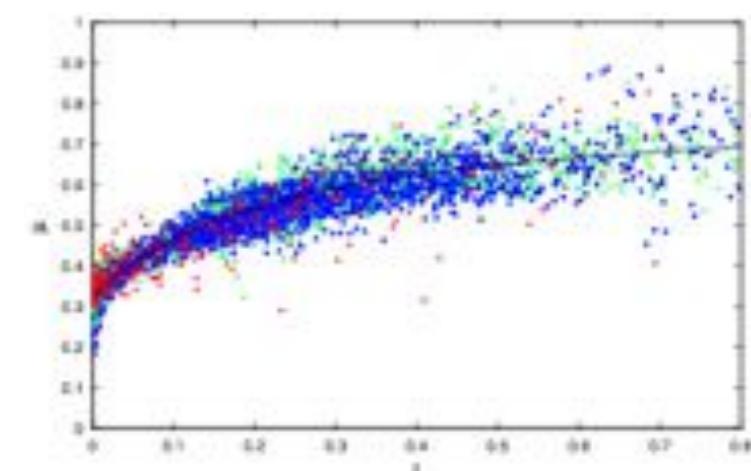
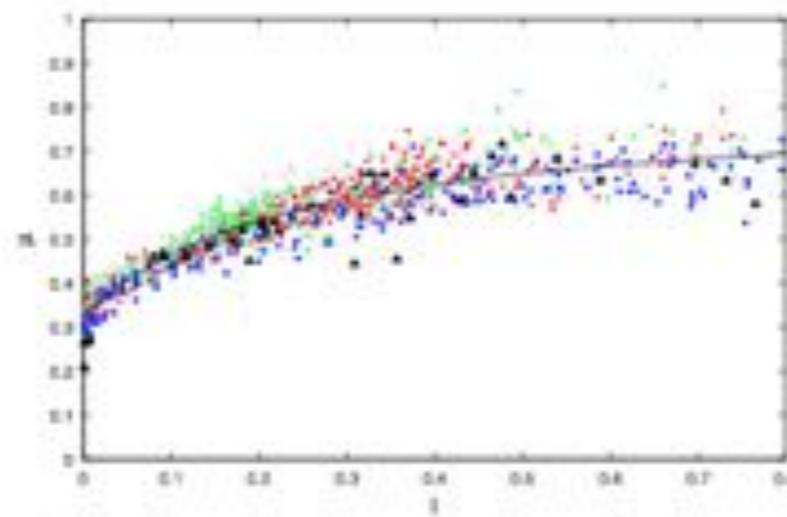
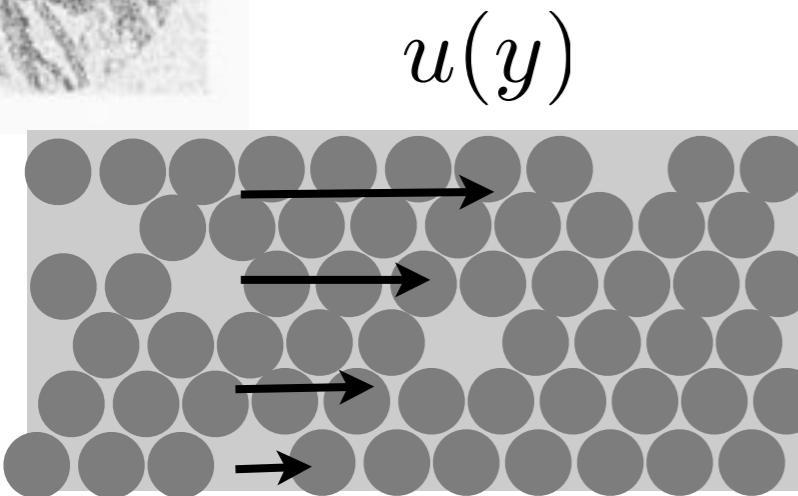
falling time
displacement time

$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

Pouliquen 99
Pouliquen Forterre JSM 06
Da Cruz 04-05
GDR Midi 04
Josserand Lagrée Lhuillier 04



by grain dynamics



Lacaze Kerswell 09

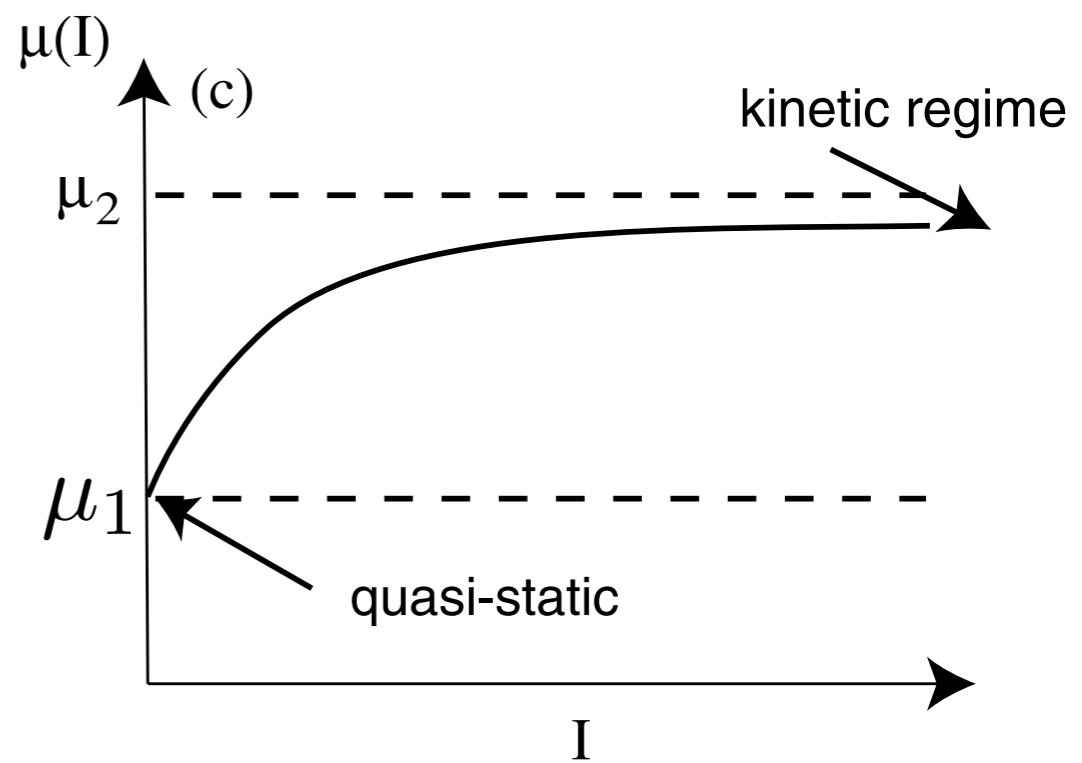
Coulomb friction law

$$\tau = \mu(I)P$$

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

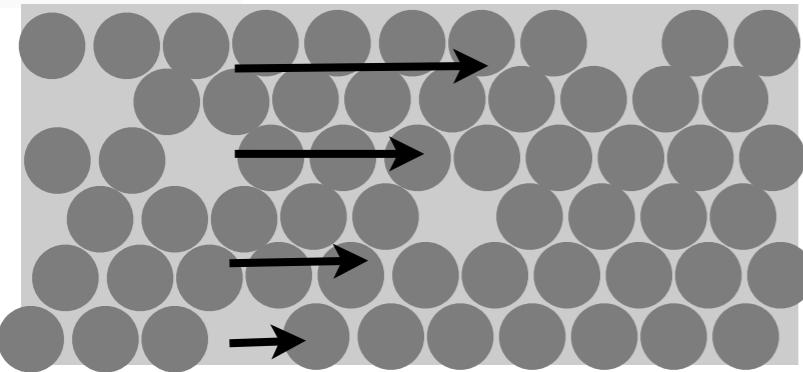
$$\mu_1 \simeq 0.32 \quad (\mu_2 - \mu_1) \simeq 0.23 \quad I_0 \simeq 0.3$$

$$I = \frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$





$u(y)$



Coulomb friction law

$$\tau = \mu(I)P$$

$$I = \frac{d\frac{\partial u}{\partial y}}{\sqrt{P/\rho}}$$

«Drucker-Prager»
plastic flow

equivalent viscosity

$$\eta \frac{\partial u}{\partial y} = \mu(I)p \rightarrow \eta = \frac{\mu(I)p}{\frac{\partial u}{\partial y}}$$



implementation in Navier Stokes?

Jop Forterre Pouliquen 2005

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$D_2 = \sqrt{D_{ij}D_{ij}} \quad D_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$$

construction of a viscosity based on the D_2 invariant and redefinition of I

$$I = d\sqrt{2}D_2/\sqrt(|p|/\rho).$$

$$\eta = \left(\frac{\mu(I)}{\sqrt{2}D_2} p \right)$$

«Drucker-Prager»

$$\nabla \cdot \mathbf{u} = 0, \quad \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho g,$$

Boundary Conditions: no slip and $P=0$ at the interface



outline

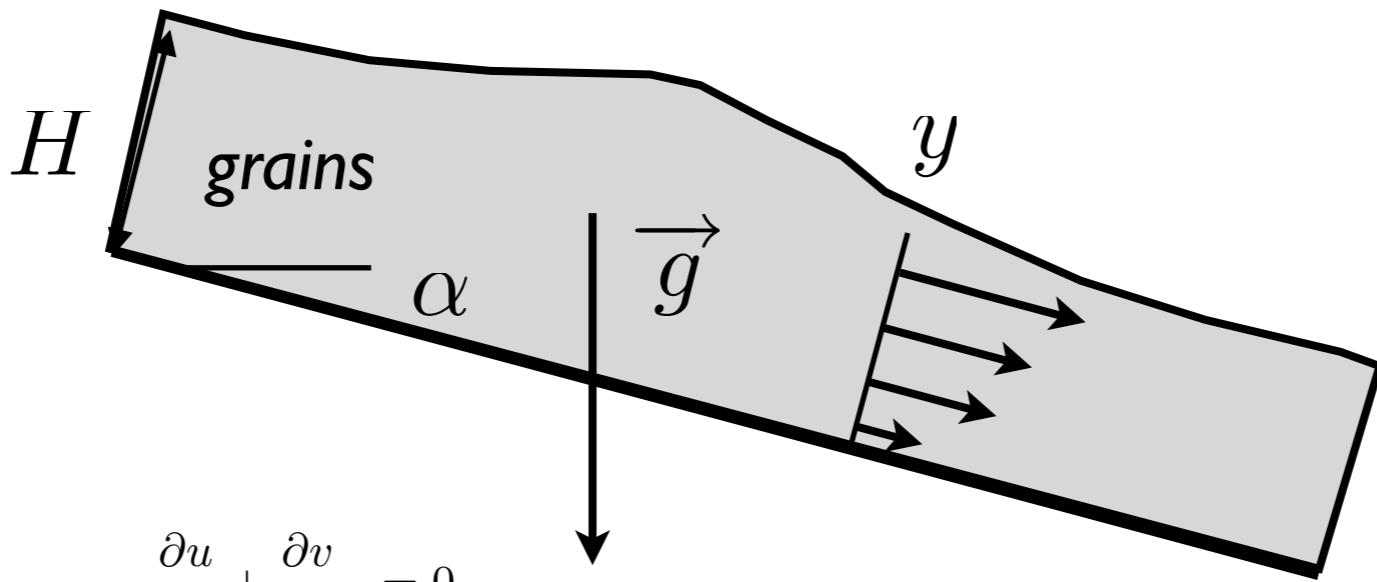
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ST VENANT



● Couche Mince Saint Venant Shallow Water Savage Hutter



$$\varepsilon = \dot{H}/L$$

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ \rho \left(\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} \right) = -g\rho \sin \alpha - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \\ \rho \left(\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} \right) = -g\rho \cos \alpha - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \end{array} \right.$$

$$p = -\rho g \cos \alpha (\eta(x, t) - y)$$

$$\tau_{xy} = \rho g H \sin \alpha \bar{\tau}_{xy}$$

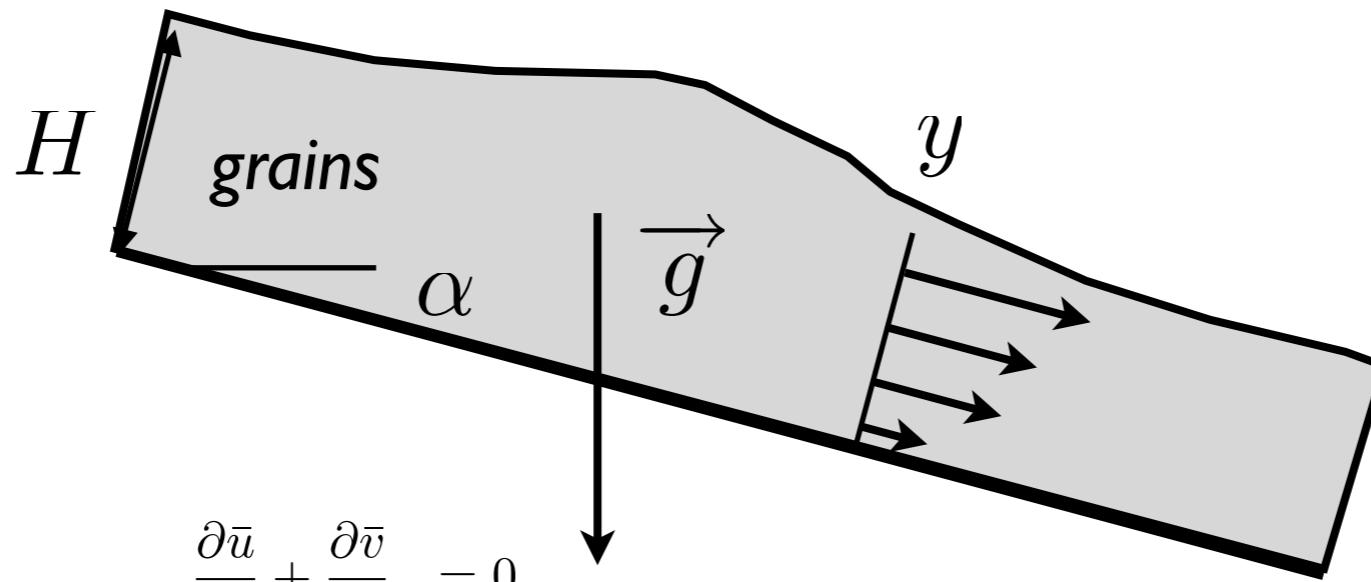
$$\rho U_0^2/L \longleftrightarrow -\frac{\partial p}{\partial x} = -\rho g \frac{\partial \eta}{\partial x}$$

$$\rho U_0^2/(H/\varepsilon) = \rho g \varepsilon.$$

$$U_0 = \sqrt{gH}.$$



Saint-Venant Savage Hutter



$$\varepsilon = \dot{H}/L$$

$$\left\{ \begin{array}{l} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \\ \varepsilon \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{u}^2}{\partial \bar{x}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{y}} \right) = -\sin \alpha - \varepsilon \frac{\partial \bar{p}}{\partial \bar{x}} + \sin \alpha \varepsilon \frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} \\ \varepsilon^2 \left(\frac{\partial \bar{v}}{\partial \bar{t}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{x}} + \frac{\partial \bar{v}^2}{\partial \bar{y}} \right) = -\cos \alpha - \frac{\partial \bar{p}}{\partial \bar{y}} + \sin \alpha \varepsilon \frac{\partial \bar{\tau}_{yy}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{yx}}{\partial \bar{y}} \end{array} \right.$$

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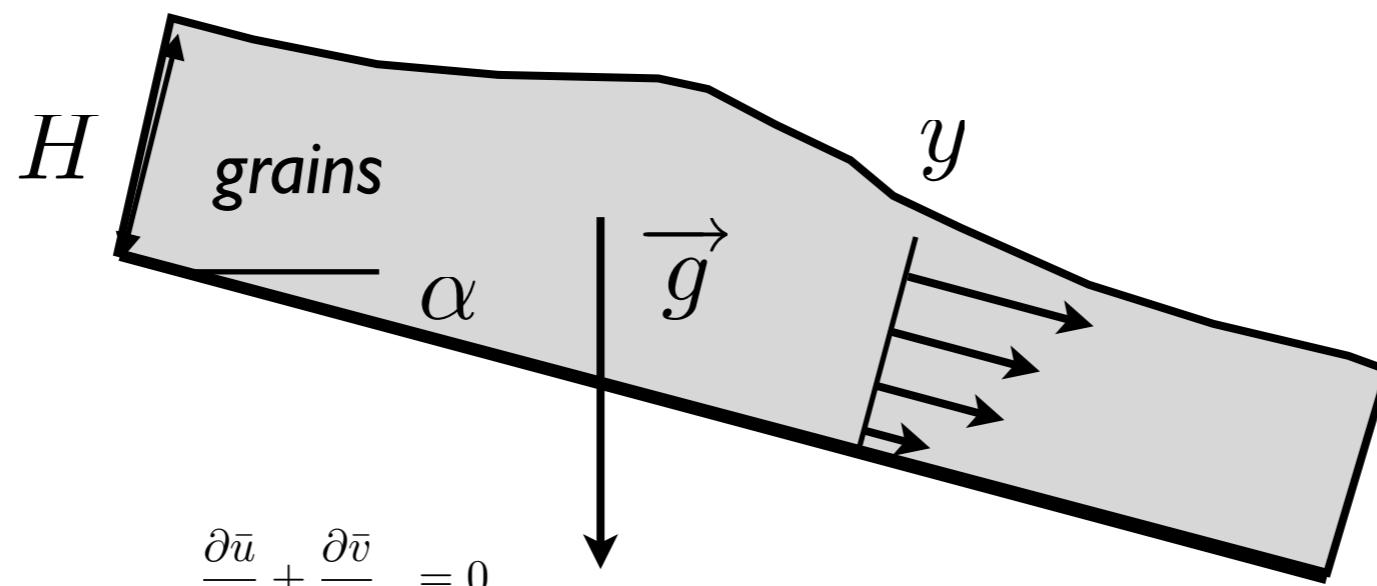
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Saint-Venant Savage Hutter



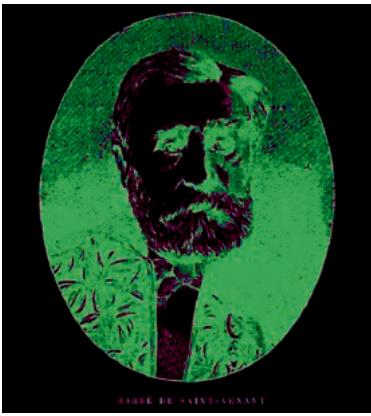
$$\left\{ \begin{array}{l} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \\ \varepsilon \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\partial \bar{u}^2}{\partial \bar{x}} + \frac{\partial \bar{u}\bar{v}}{\partial \bar{y}} \right) = -\sin \alpha - \varepsilon \frac{\partial \bar{p}}{\partial \bar{x}} + \sin \alpha \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}} \\ 0 = -\cos \alpha - \frac{\partial \bar{p}}{\partial \bar{y}} \end{array} \right.$$

$$\int \Bigg| dy$$

$\mu(l)$ at the wall

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \int_f^\eta u(x, y, t) dy = 0$$

$$\frac{\partial}{\partial t} \int_f^\eta u(x, y, t) dy + \frac{\partial}{\partial x} \left(\int_f^\eta u(x, y, t)^2 dy + \cos \alpha \frac{g}{2} (h^2) \right) = -gh \sin(\alpha) - \cos \alpha g h \frac{d}{dx} f - \mu(I(0)) gh \cos(\alpha).$$



Saint-Venant Savage Hutter

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2) \right) = -gh\mu(I) \frac{Q}{|Q|}$$

Integral over the layer of grains



Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2)\right) = -gh\mu(I)\frac{Q}{|Q|}$$

Gerris is a free finite volume code by Stéphane Popinet
one part of the code is a Shallow Water solver

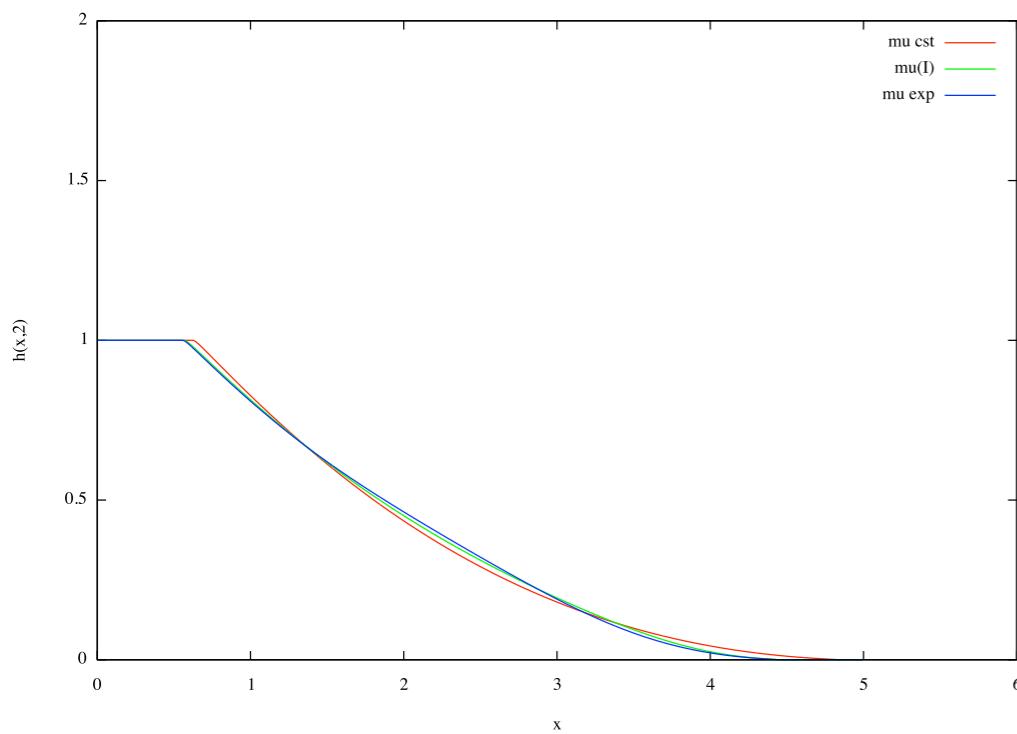
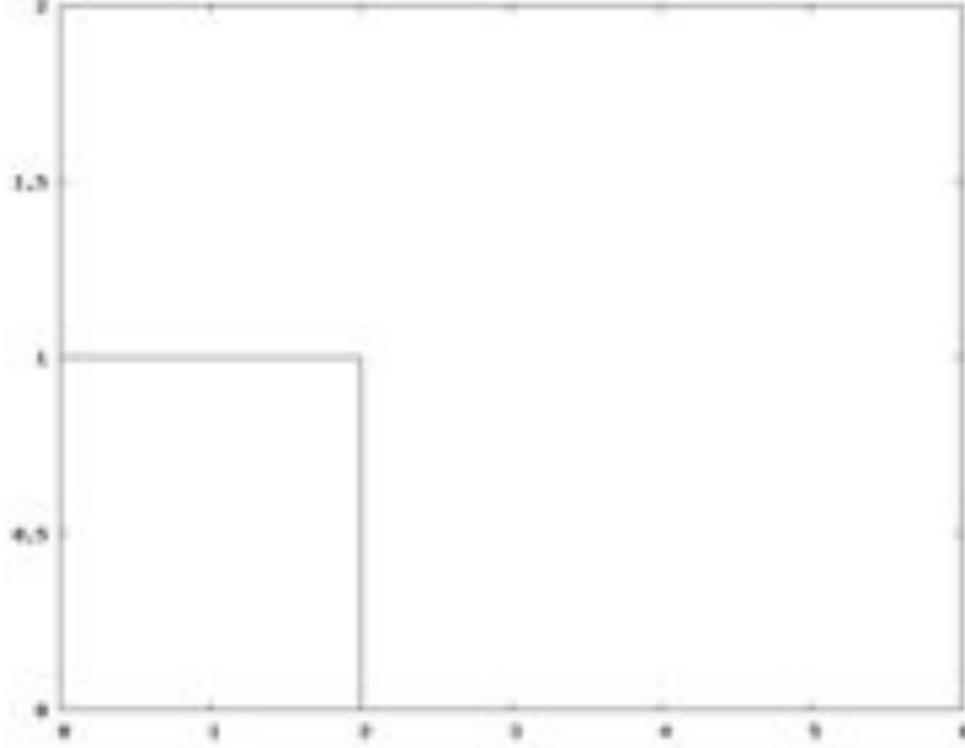
$$\frac{Q^* - Q^n}{\Delta t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{h} + \frac{g}{2}(h^2)\right) = 0 \quad \frac{Q^{n+1} - Q^*}{\Delta t} = -gh^*\mu(I^*)\frac{Q^{n+1}}{|Q^*|}$$

Audusse et al.



Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2) \right) = -gh\mu(I) \frac{Q}{|Q|}$$



$$\mu(I) = \mu_s$$

$$\mu(I) = \mu_s + \frac{\Delta\mu}{\frac{I_0}{I} + 1}$$

$$\mu(I) = \mu_s + \Delta\mu e^{-\beta/I}$$

$$\mu = 0.45$$

$$\mu = (0.4 + 0.26/(0.4/\ln + 1))$$

$$\mu = (0.4 + 0.26 * \exp(-0.136/\ln))$$



Saint-Venant Savage Hutter *Gerris*

$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2)\right) = -gh\mu(I)\frac{Q}{|Q|}$$

valid by hypothesis for small aspect ratio

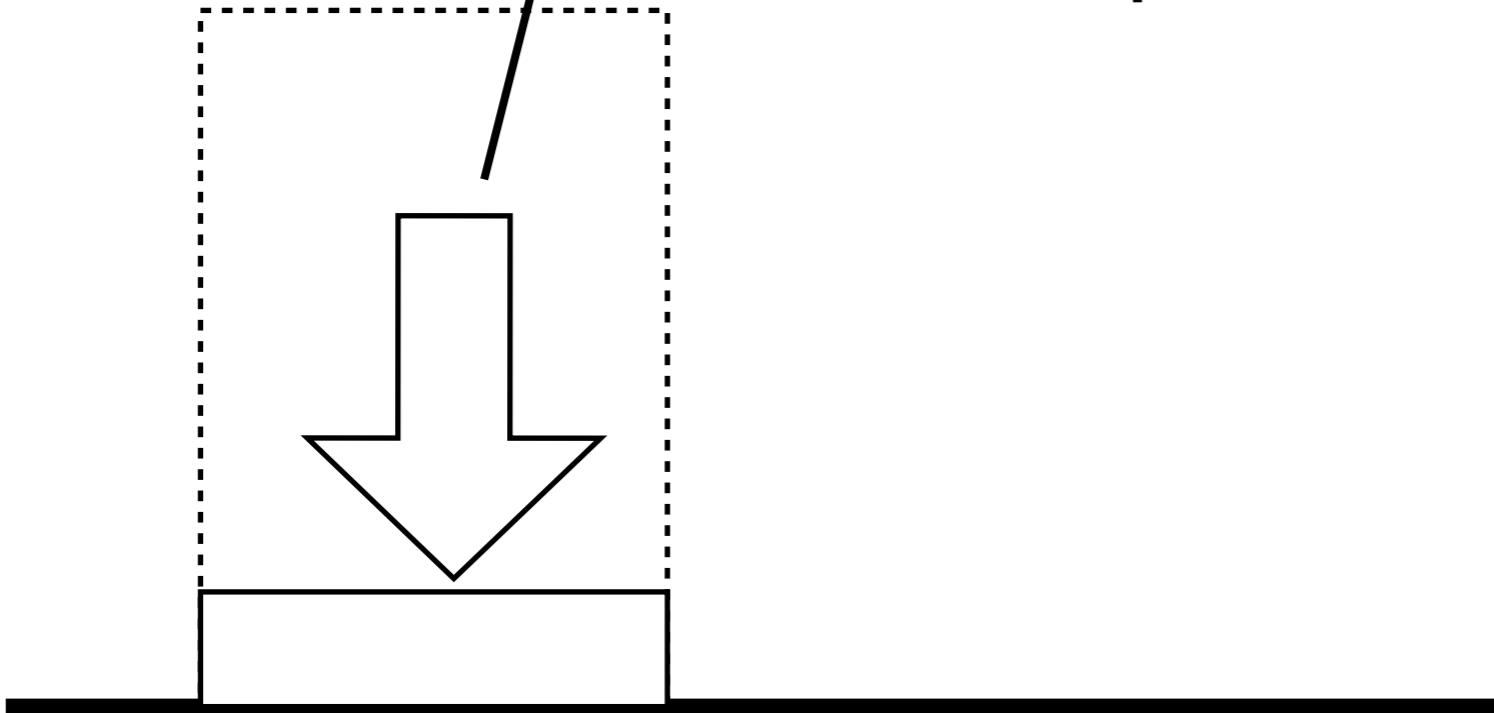




Saint-Venant Savage Hutter *Gerris*

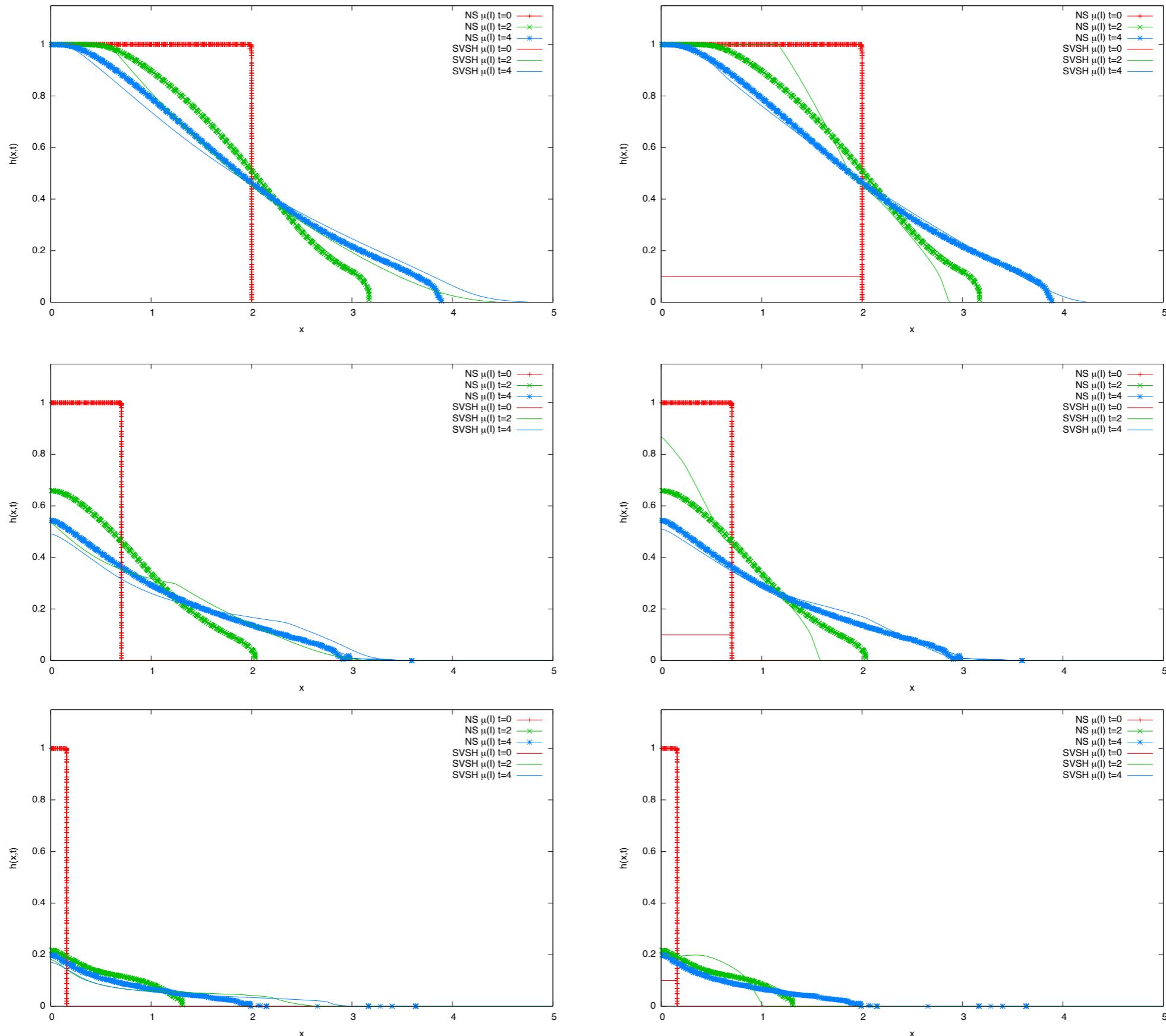
$$\frac{\partial h}{\partial t} + \frac{\partial(Q)}{\partial x} = 0 \quad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5Q^2}{4h} + \frac{g}{2}(h^2)\right) = -gh\mu(I)\frac{Q}{|Q|}$$

add a source term corresponding
to pluviation





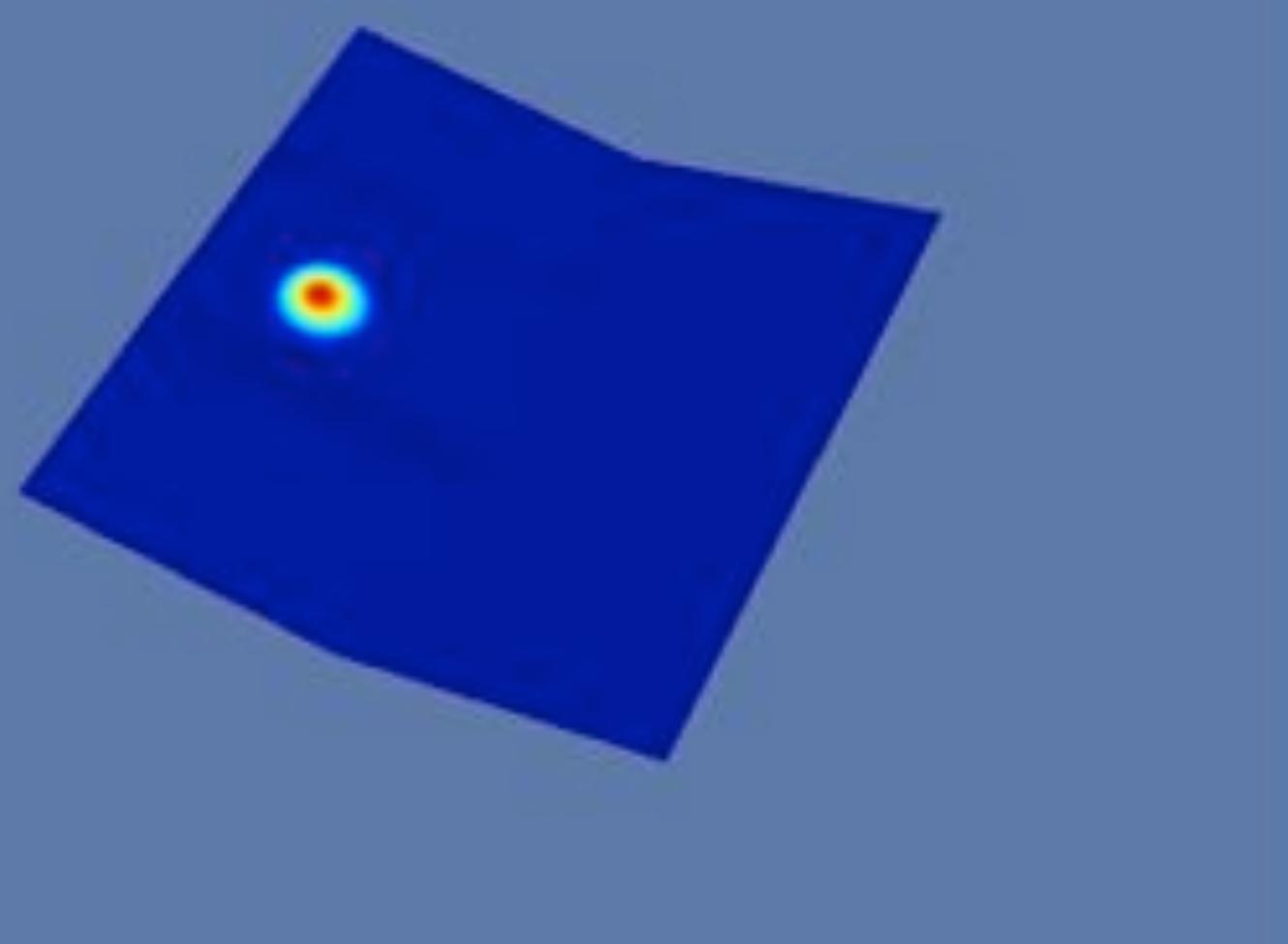
Saint-Venant Savage Hutter *Gerris*



L. Satron, J. Hinch (DAMTP), A. Mangeney (IPGP), E. Larieu (IMFT)



Saint-Venant Savage Hutter *Gerris*

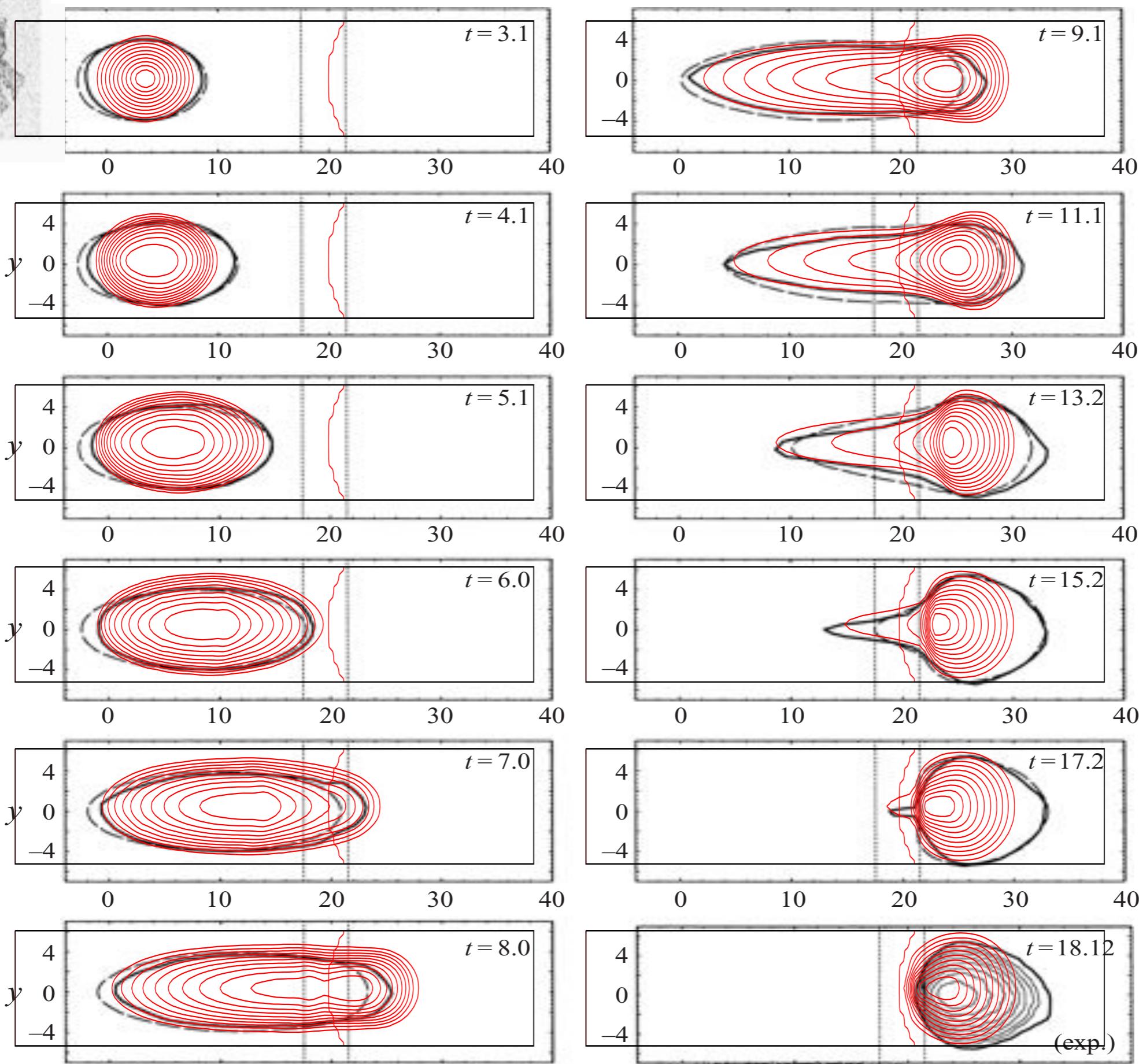




Saint-Venant Savage Hutter *Gerris*

WIELAND, J. M. N. T. GRAY AND K. HUTTER 1999

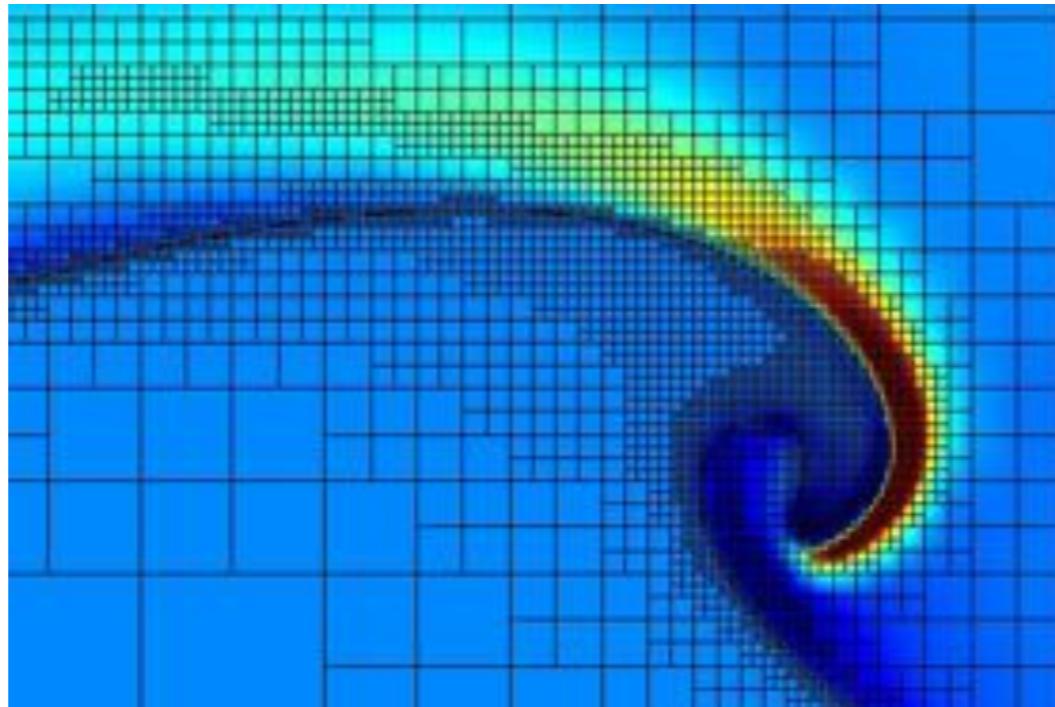
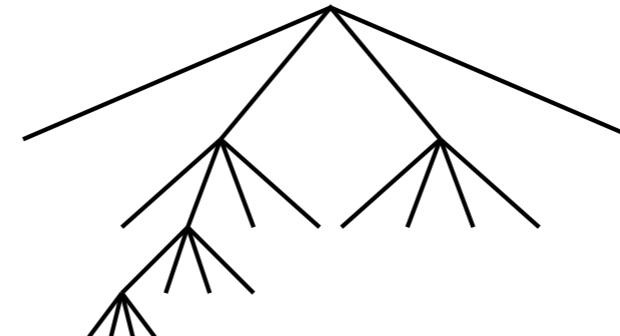
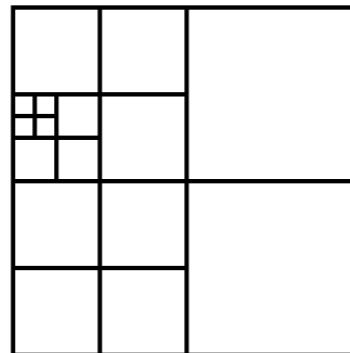
Marina Pirulli, Marie-Odile Bristeau Anne Mangeney, Claudio Scavia 2006





outline

- what is a granular fluid? some images
- the $\mu(l)$ friction law obtained from experiments and discrete simulation
- the viscosity associated to the $\mu(l)$ friction law
- the Saint Venant Savage Hutter Hyperbolic model
- implementing the $\mu(l)$ friction law in Navier Stokes
- Examples of flows: focusing on the granular column collapse (limits of Saint Venant Savage Hutter Hyperbolic model)



- *Gerris* is a finite volume code by Stéphane Popinet NIWA
one part of the code is a Navier Stokes solver
- automatic mesh adaptation
- Volume Of Fluid method for two phase flows
- free on sourceforge



rheology; defining a viscosity

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$\eta \frac{\partial u}{\partial y} = \mu(I) P$$

local equilibrium

$$\eta = \frac{\mu \left(\frac{d \frac{\partial u}{\partial y}}{\sqrt{P/\rho}} \right) P}{\frac{\partial u}{\partial y}}$$

construction of a viscosity

P.Jop, Y. Forterre, O. Pouliquen, (2006) "A rheology for dense granular flows", Nature 441, pp. 727-730



implementation in *Gerris* flow solver?

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

$$D_2 = \sqrt{D_{ij} D_{ij}} \quad D_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$$

construction of a viscosity based on the D_2 invariant and redefinition of I

$$\eta = \min(\eta_{max}, \max\left(\frac{\mu(I)}{\sqrt{2}D_2} p, 0\right)) \quad I = d\sqrt{2}D_2/\sqrt(|p|/\rho).$$

- the «min» limits viscosity to a large value
- always flow, even slow

Boundary Conditions: no slip and $P=0$ at the interface



implementation in *Gerris* flow solver?

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

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$$\nabla \cdot \mathbf{u} = 0, \quad \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot (2\eta \mathbf{D}) + \rho g,$$

Boundary Conditions: no slip and $P=0$ at the interface



implementation in *Gerris* flow solver?

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

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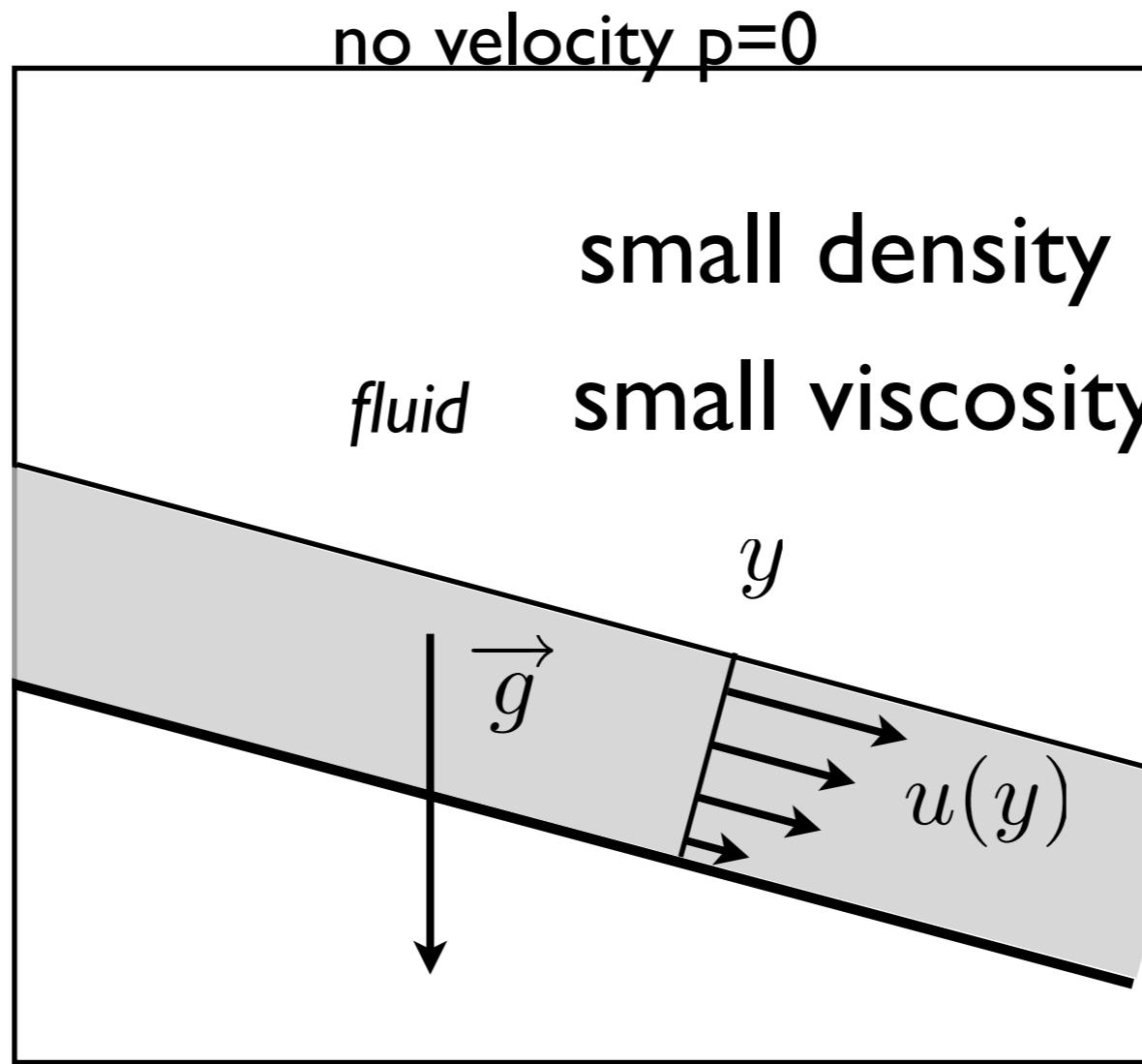
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$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0, \quad \rho = c\rho_1 + (1-c)\rho_2, \quad \eta = c\eta_1 + (1-c)\eta_2$$

The granular fluid is covered by a passive light fluid (it allows for a zero pressure boundary condition at the surface, bypassing an up to now difficulty which was to impose this condition on a unknown moving boundary).

Boundary Conditions: no slip and $P=0$ at the top



$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = 0, \quad \rho = c\rho_1 + (1 - c)\rho_2, \quad \eta = c\eta_1 + (1 - c)\eta_2$$

The granular fluid is covered by a passive light fluid (it allows for a zero pressure boundary condition at the surface, bypassing an up to now difficulty which was to impose this condition on a unknown moving boundary).

Boundary Conditions: no slip and $P=0$ at the top



Projection Method



$$\begin{aligned}\rho_{n+\frac{1}{2}} \left(\frac{\mathbf{u}_* - \mathbf{u}_n}{\Delta t} + \mathbf{u}_{n+\frac{1}{2}} \cdot \nabla \mathbf{u}_{n+\frac{1}{2}} \right) &= \nabla \cdot (\eta_{n+\frac{1}{2}} \mathbf{D}_*) - \nabla p_{n-\frac{1}{2}}, \\ \mathbf{u}_{n+1} &= \mathbf{u}_* - \frac{\Delta t}{\rho_{n+\frac{1}{2}}} (\nabla p_{n+\frac{1}{2}} - \nabla p_{n-\frac{1}{2}}), \\ \nabla \cdot \mathbf{u}_{n+1} &= 0.\end{aligned}$$

multigrid solver for Laplacien of pressure

$$\nabla \cdot \left(\frac{\Delta t}{\rho_{n+\frac{1}{2}}} \nabla p_{n+\frac{1}{2}} \right) = \nabla \cdot \left(\mathbf{u}_* + \frac{\Delta t}{\rho_{n+\frac{1}{2}}} \nabla p_{n-\frac{1}{2}} \right)$$

implicit for \mathbf{u}^*

$$\frac{\rho_{n+\frac{1}{2}}}{\Delta t} \mathbf{u}_* - \frac{1}{2} \nabla \cdot (\eta_{n+\frac{1}{2}} \nabla \mathbf{u}_*) = \rho_{n+\frac{1}{2}} \left[\frac{\mathbf{u}_n}{\Delta t} - \mathbf{u}_{n+\frac{1}{2}} \cdot \nabla \mathbf{u}_{n+\frac{1}{2}} \right] - \nabla p_{n-\frac{1}{2}} + \frac{1}{2} \nabla \mathbf{u}_n^T \nabla \eta_{n+\frac{1}{2}}.$$

VOF reconstruction

$$\frac{c_{n+\frac{1}{2}} - c_{n-\frac{1}{2}}}{\Delta t} + \nabla \cdot (c_n \mathbf{u}_n) = 0$$

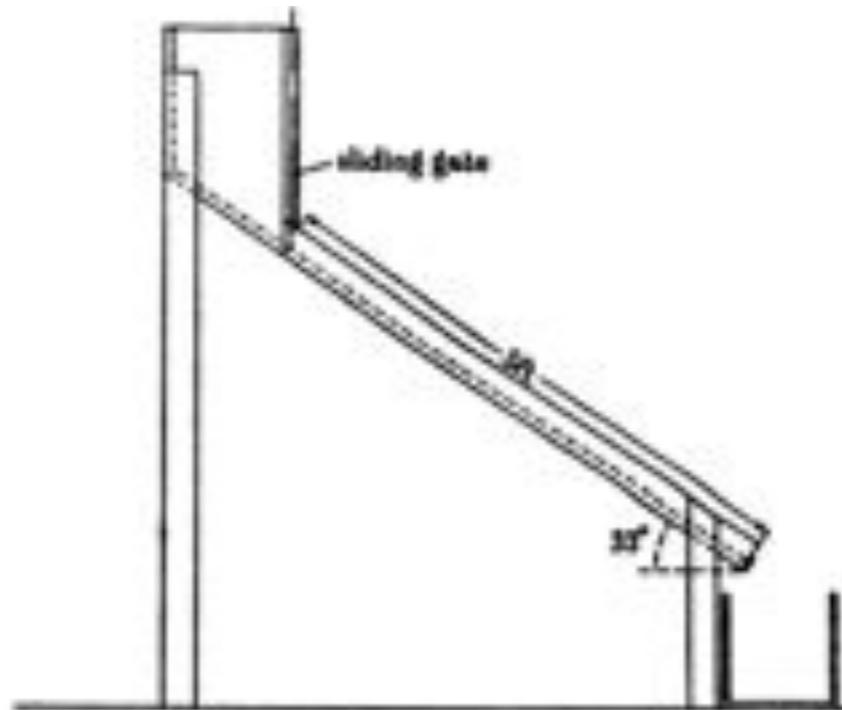


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Test of the code: «Bagnold» avalanche



kind of Nußelt solution

$$T \propto \sigma(\lambda D)^2 (dU/dy)^2$$

$$U = \frac{2}{3} \times 0.165 (g \sin \beta)^{1/2} \frac{y^{3/2}}{D} .$$

TABLE 1.

| flow height Y (cm) | measured speed (cm/sec) | speed, from (9) (cm/sec) | ratio |
|-----------------------|----------------------------|-----------------------------|-------|
| 0.5 | 17.2 | 26.4 | 1.53 |
| 0.65 | 27.5 | 38.8 | 1.41 |
| 0.75 | 30.0 | 48.0 | 1.6 |
| 0.9 | 39.0 | 63.0 | 1.61 |

Bagnold 1954



Test of the code: «Bagnold» avalanche

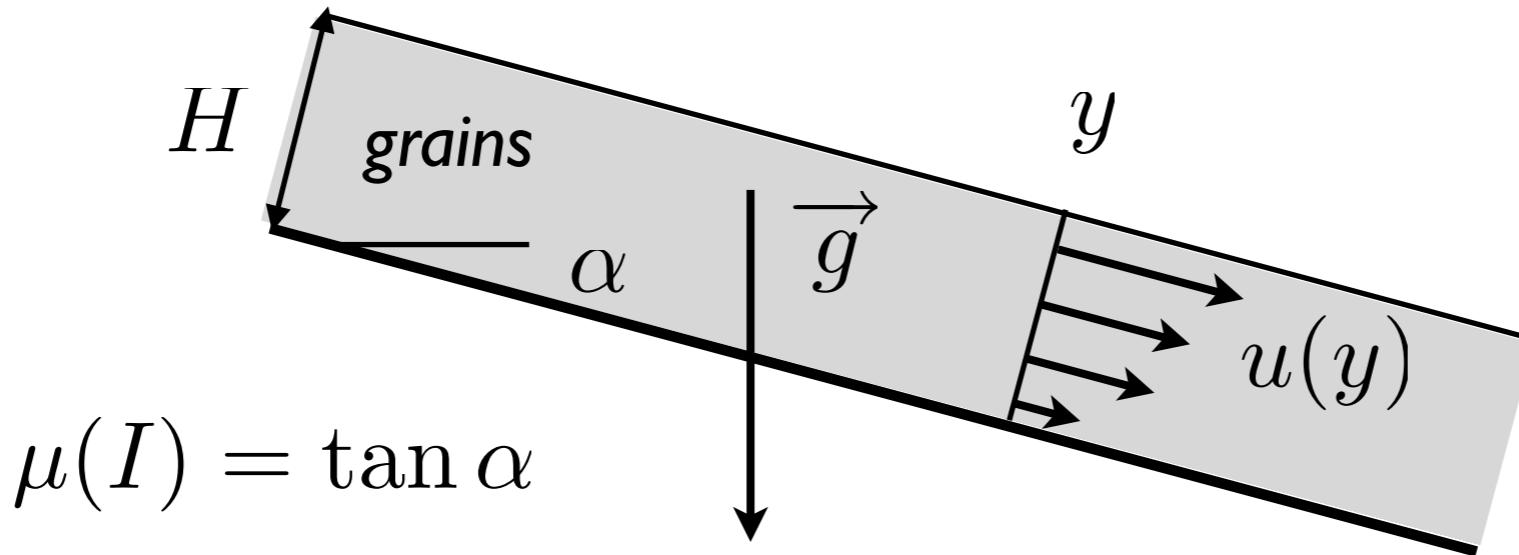


kind of Nußelt solution

Contact Dynamic
simulation Lydie Staron



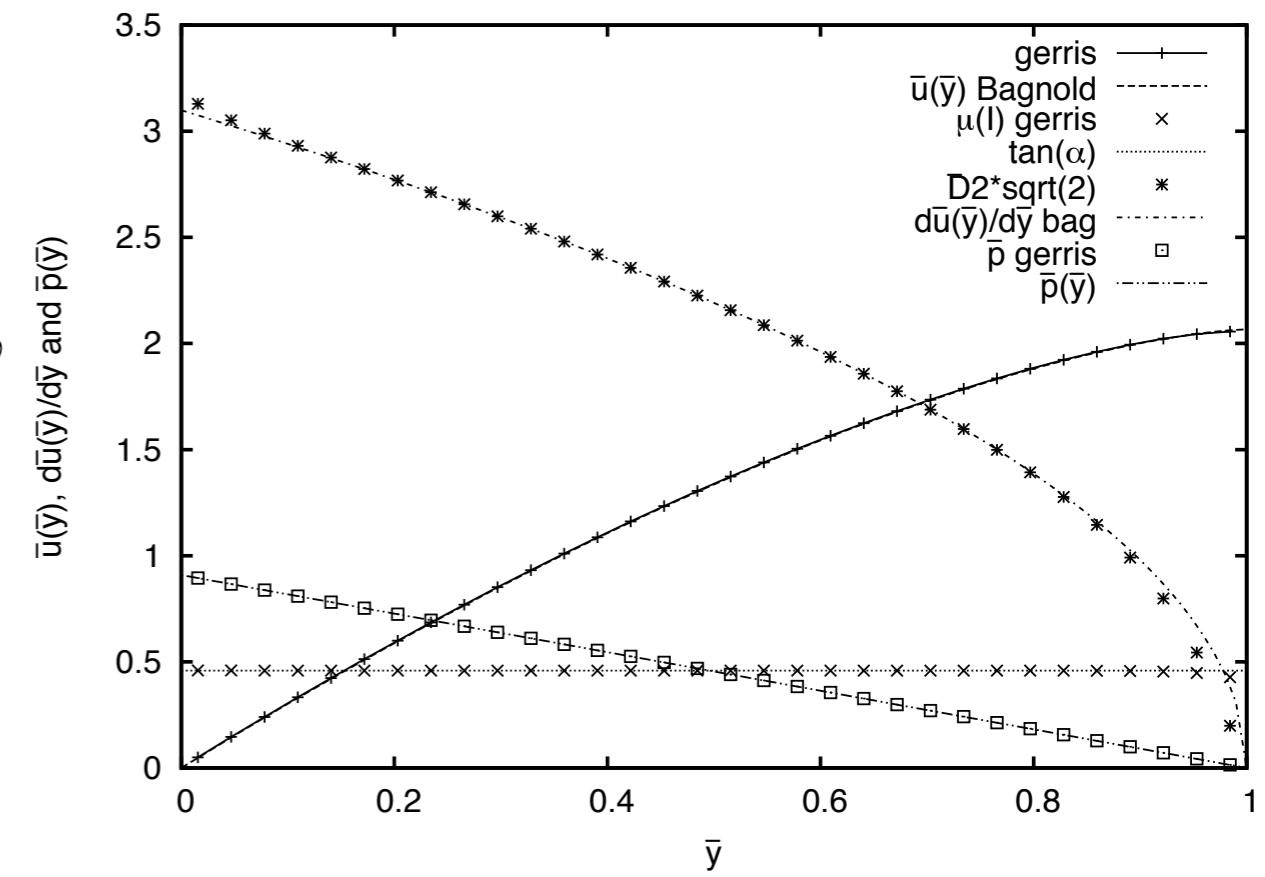
Test of the code: «Bagnold» avalanche



$$\mu(I) = \tan \alpha$$

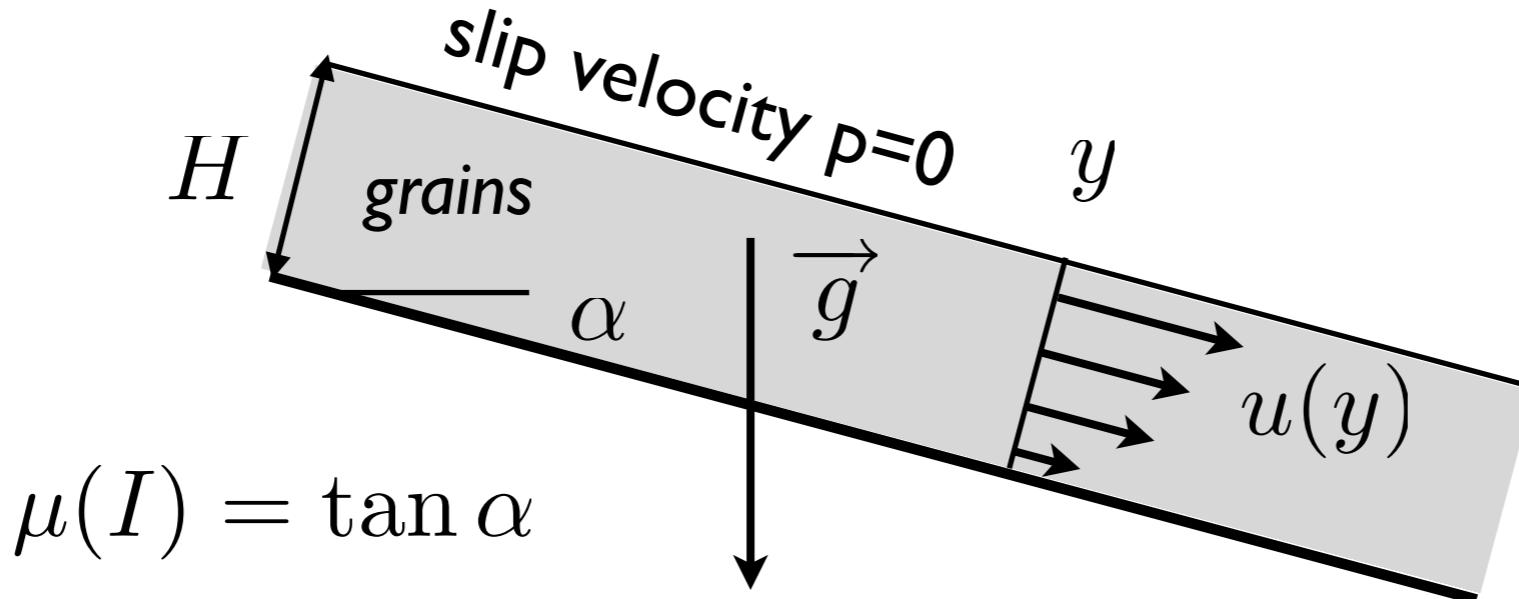
$$u = \frac{2}{3} I_\alpha \sqrt{gd \cos \alpha \frac{H^3}{d^3}} \left(1 - \left(1 - \frac{y}{H} \right)^{3/2} \right),$$

$$v = 0, \quad p = \rho g H \left(1 - \frac{y}{H} \right) \cos \alpha.$$





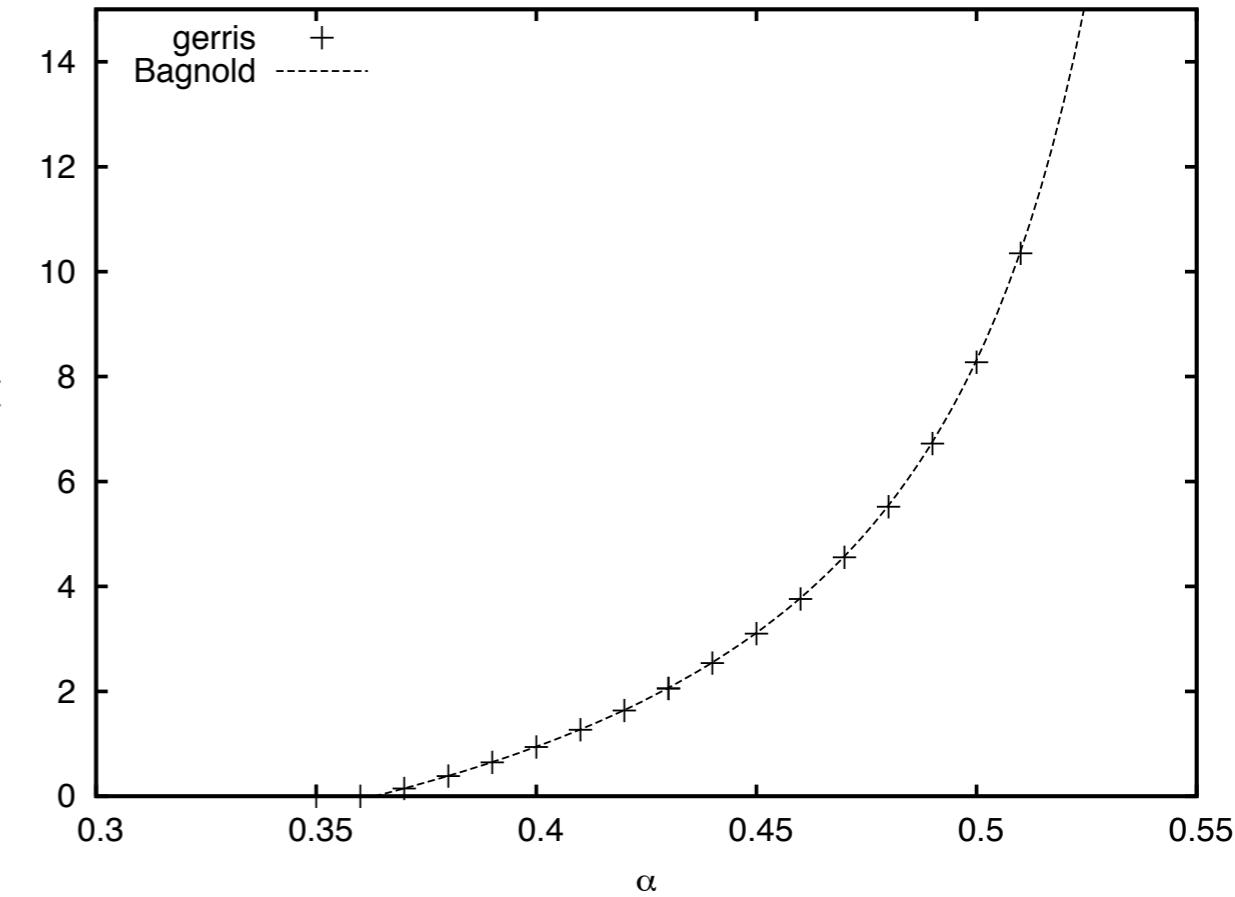
Test of the code: «Bagnold» avalanche



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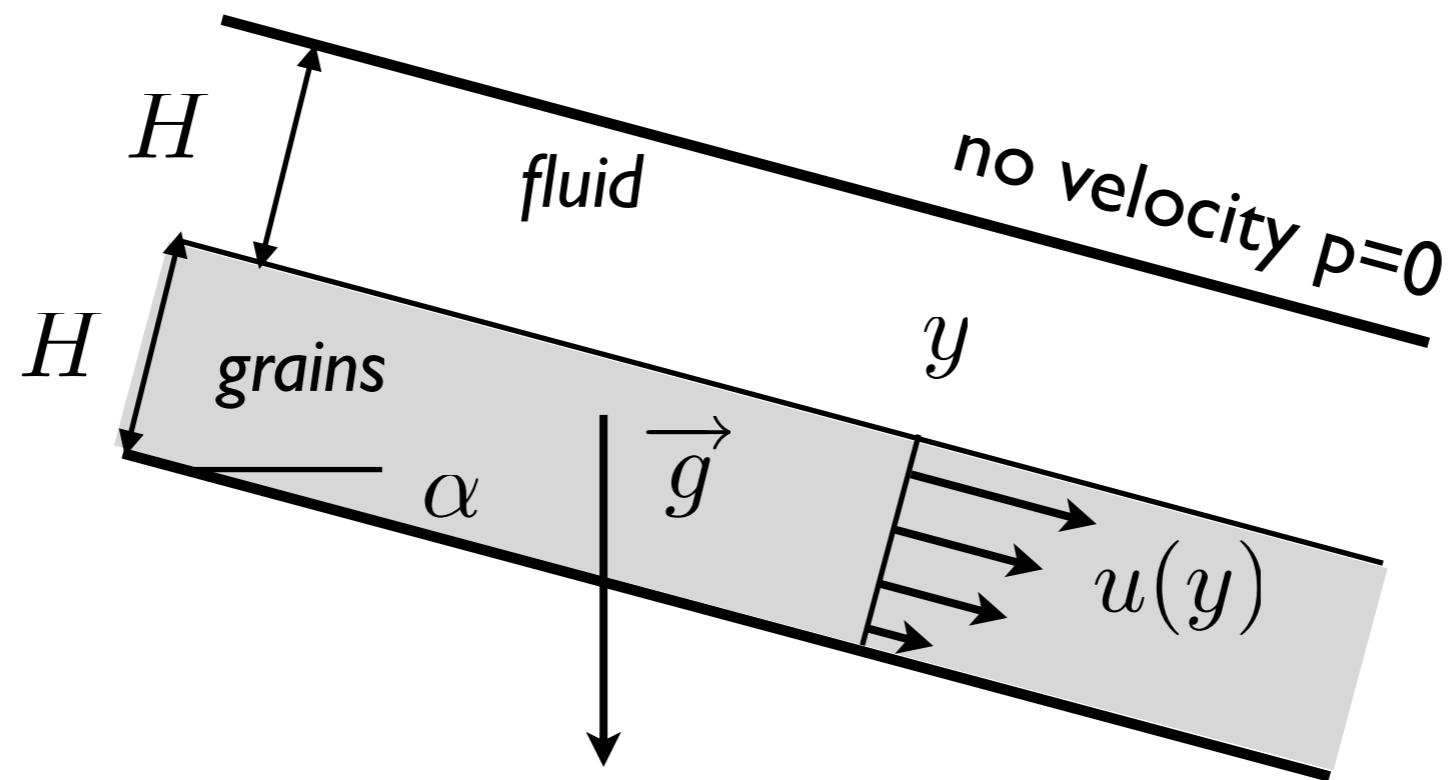
$$u = \frac{2}{3} I_\alpha \sqrt{gd \cos \alpha \frac{H^3}{d^3}} \left(1 - \left(1 - \frac{y}{H} \right)^{3/2} \right),$$

$$v = 0, \quad p = \rho g H \left(1 - \frac{y}{H} \right) \cos \alpha.$$





Test of the code: «Bagnold» avalanche



$$u(y) = \frac{(2H - y)(\rho_f gy \sin \alpha - 2\tau_0)}{2\mu_f}$$

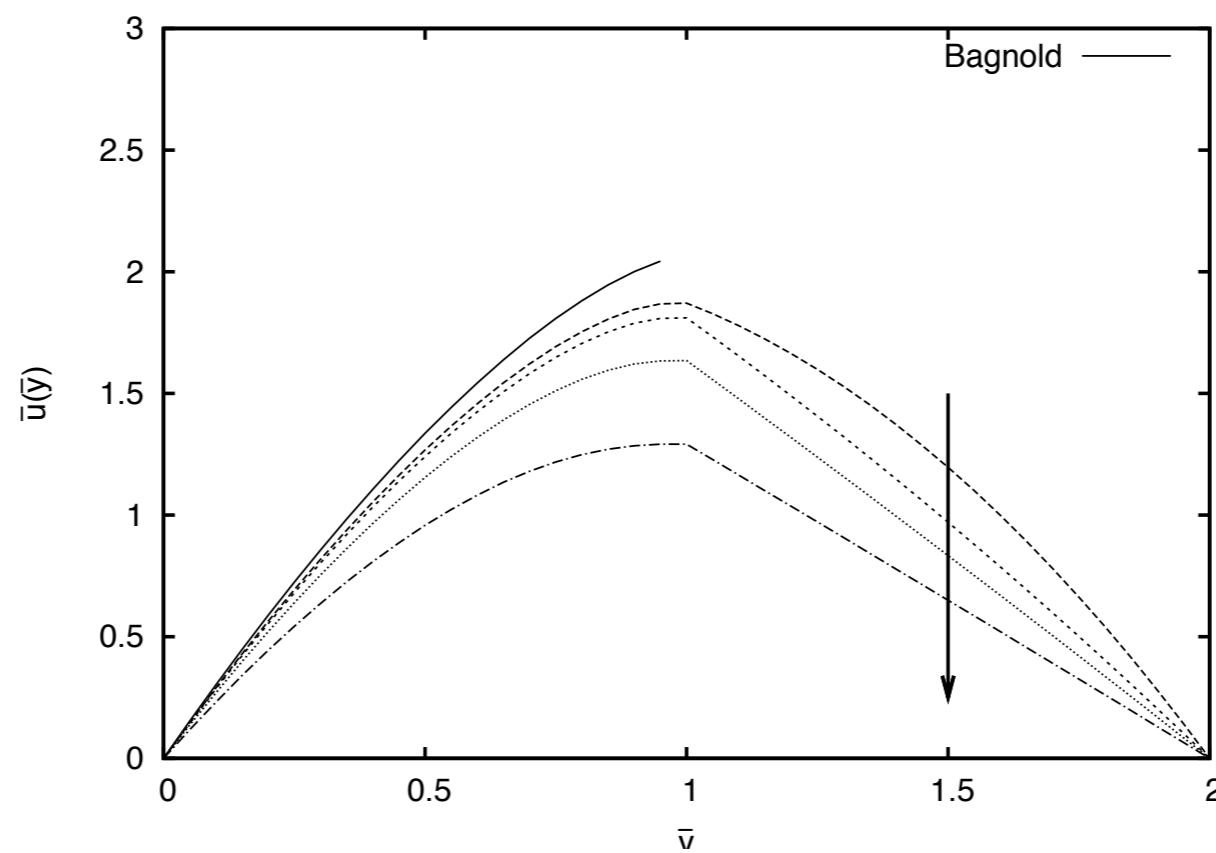
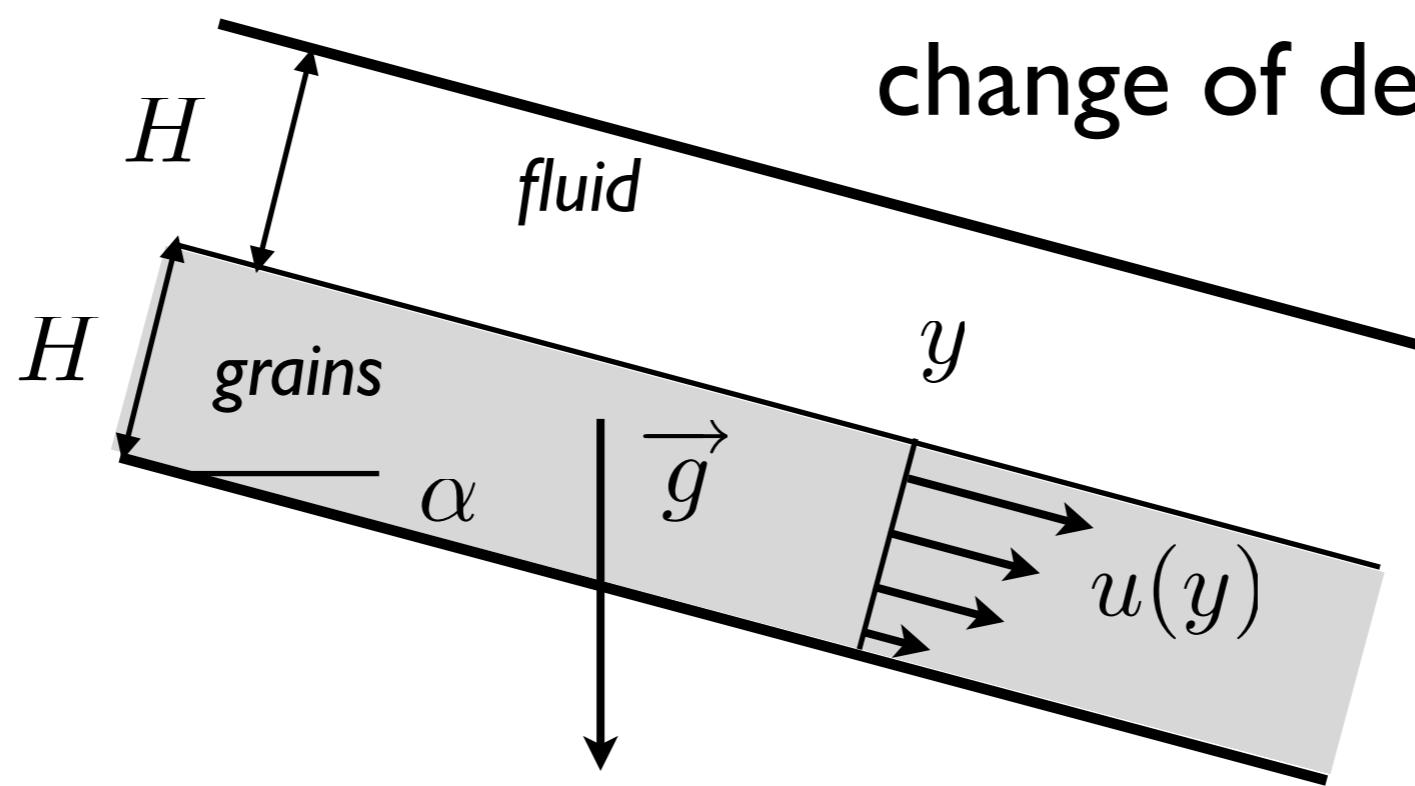
$$d\frac{\partial u}{\partial y} = \max \left[\sqrt{p_0/\rho + gH \left(1 - \frac{y}{H}\right) \cos \alpha} \times \mu^{-1} \left(\frac{\tau_0 + \rho g H \left(1 - \frac{y}{H}\right) \sin \alpha}{p_0 + \rho g H \left(1 - \frac{y}{H}\right) \cos \alpha} \right), 0 \right].$$

$$u(H^-) - u(H^+) = 0$$



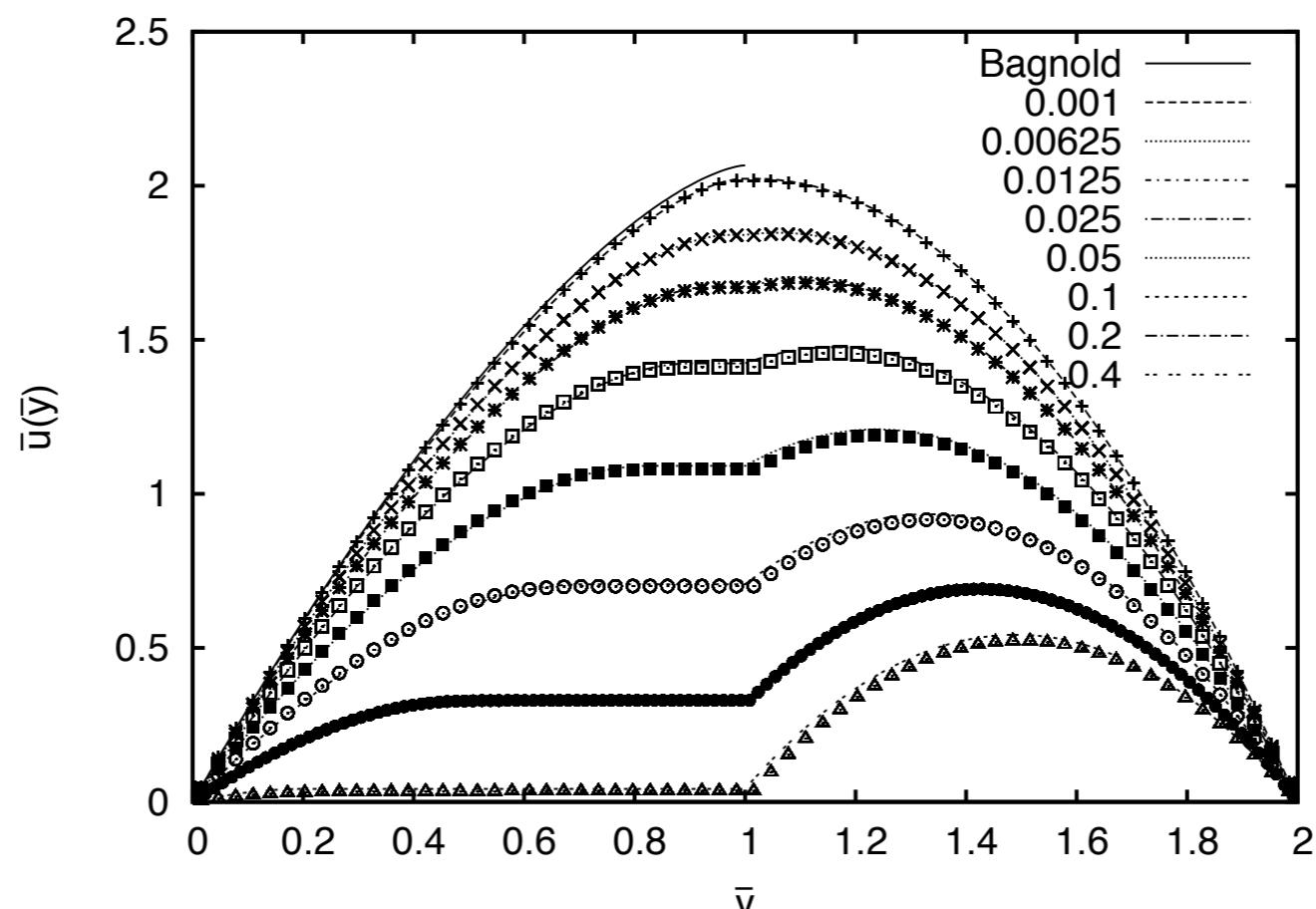
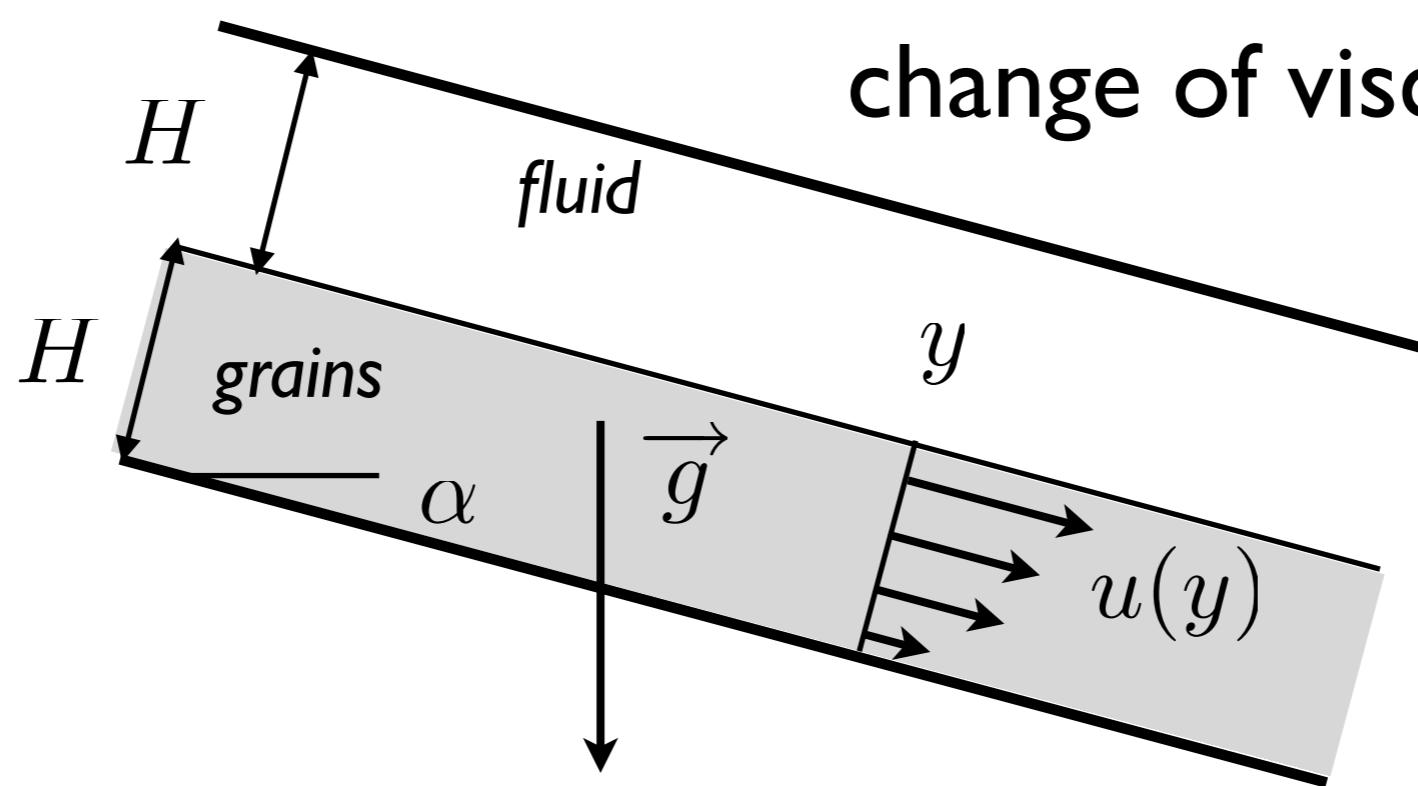
Test of the code: «Bagnold» avalanche

change of density of the fluid





Test of the code: «Bagnold» avalanche change of viscosity of the fluid





● Bagnold 3D

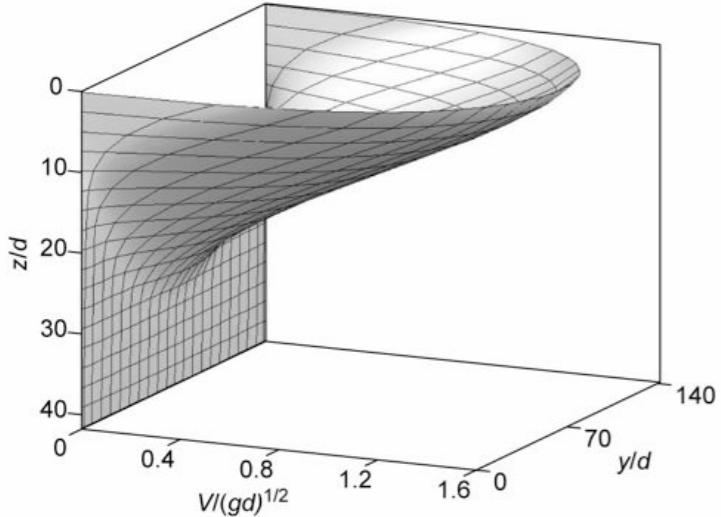


Figure 3: Typical 3D velocity profile predicted by the rheology ($W=142d$, $\theta=22.6^\circ$, $Q/d^{3/2}g^{1/2}=15.2$). For clarity only one quarter of the lines of the 71x80 computational grid is plotted.

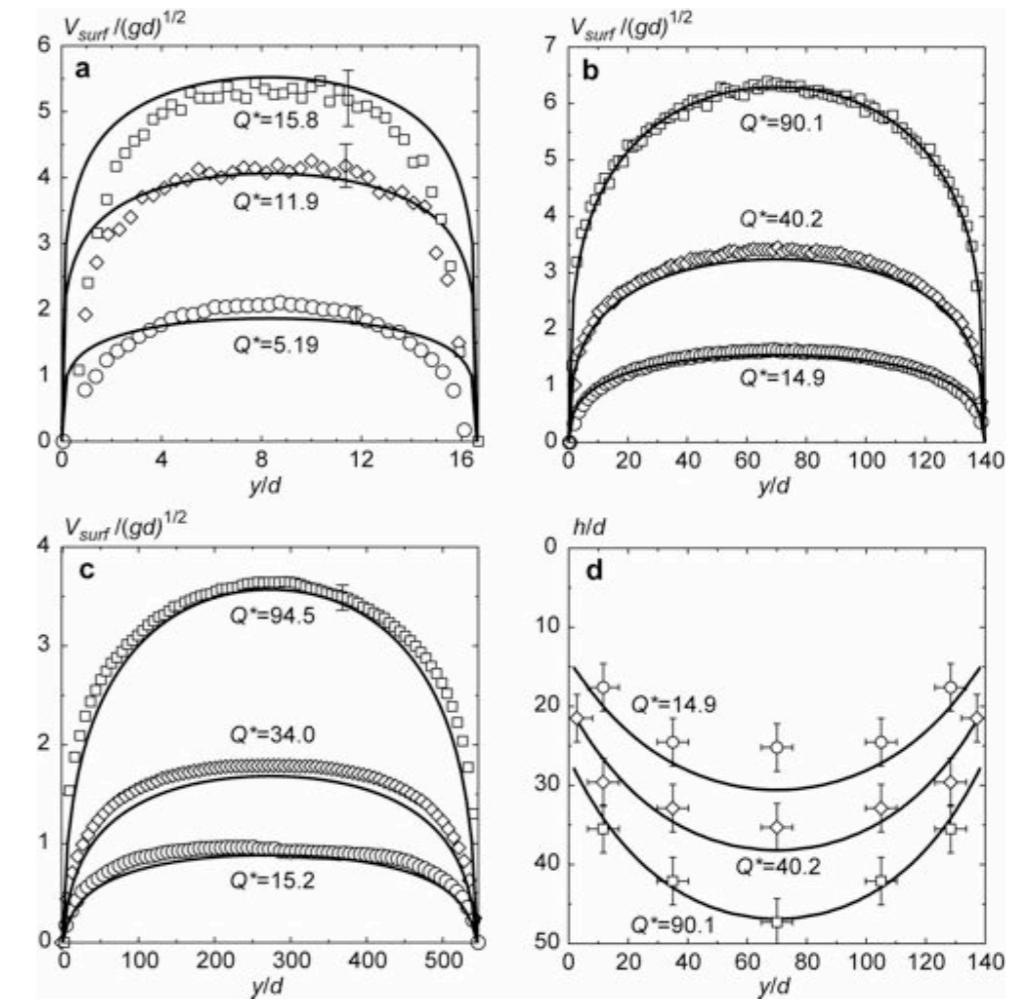
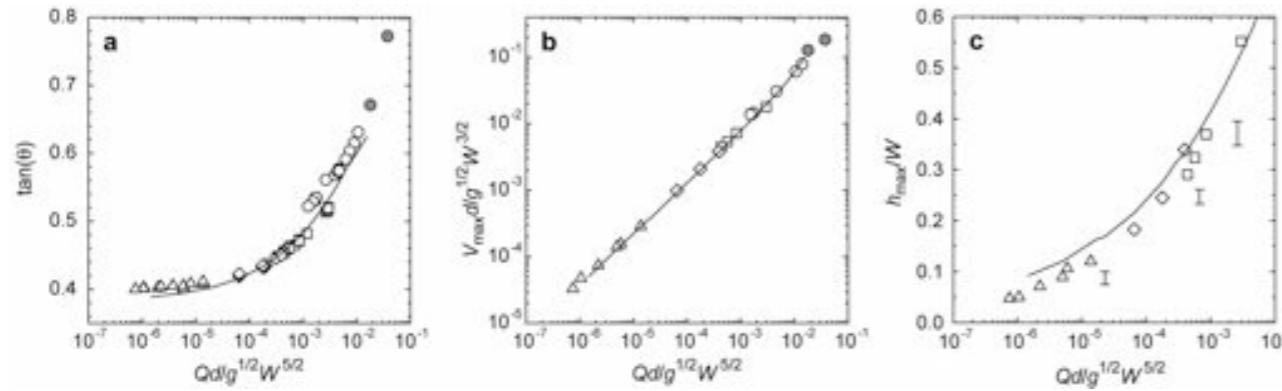


Figure 4: Comparison of 3D simulations (lines) and experimental results (symbols) for different flow rates ($Q^*=Q/d^{3/2}g^{1/2}$). **a**, **b**, **c**, Free-surface velocity profiles for channel $W=16.5d$ (**a**), $W=140d$ (**b**) and $W=546d$ (**c**). **d**, Depths of the flowing layer across the channel $W=140d$. The experimental and computational flow rates are equal within 2.5%. The error bars represent the dispersion of the measurements for different experiments.

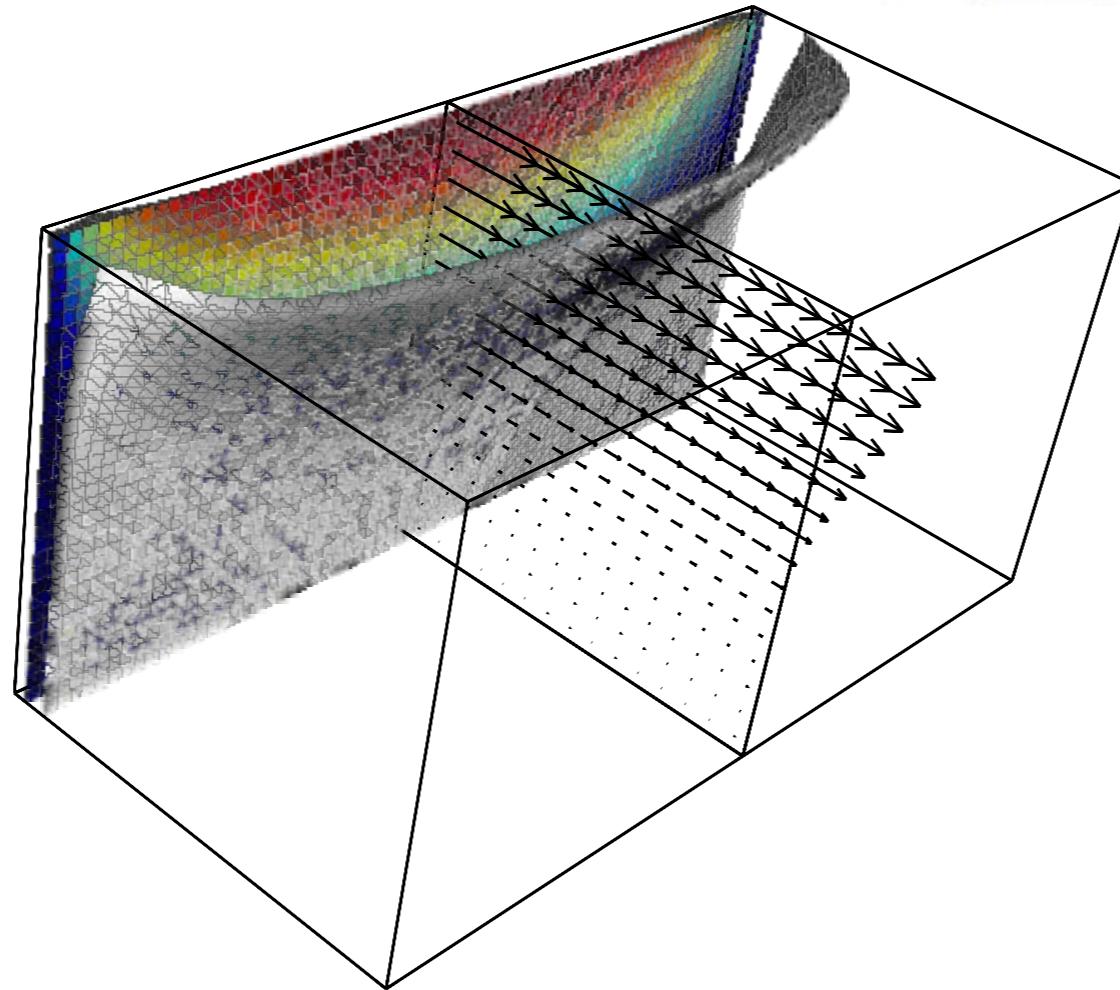
A constitutive law for dense granular flows

Pierre Jop¹, Yoël Forterre¹ & Olivier Pouliquen

nature 06



Gerris

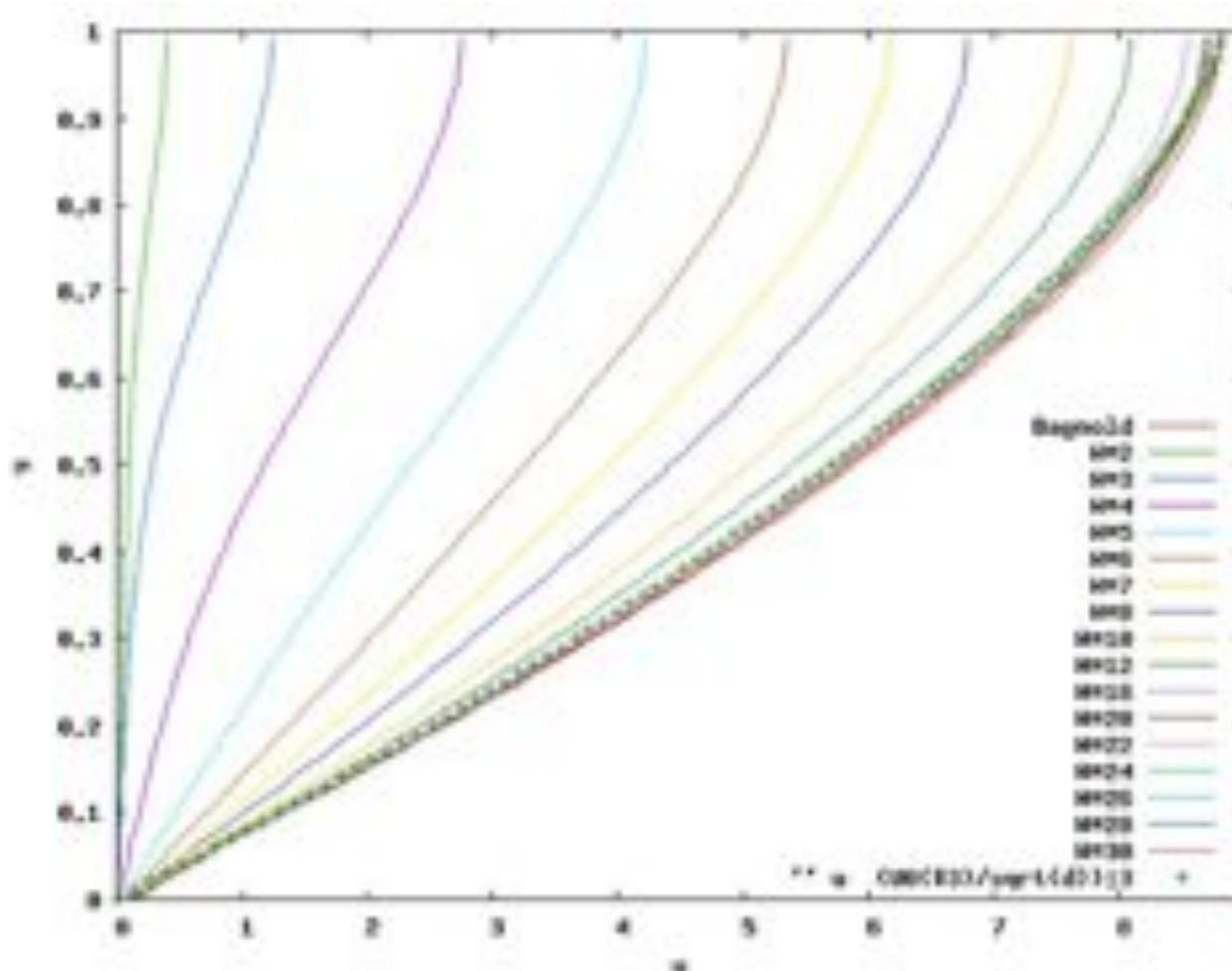


NS $\mu(l)$

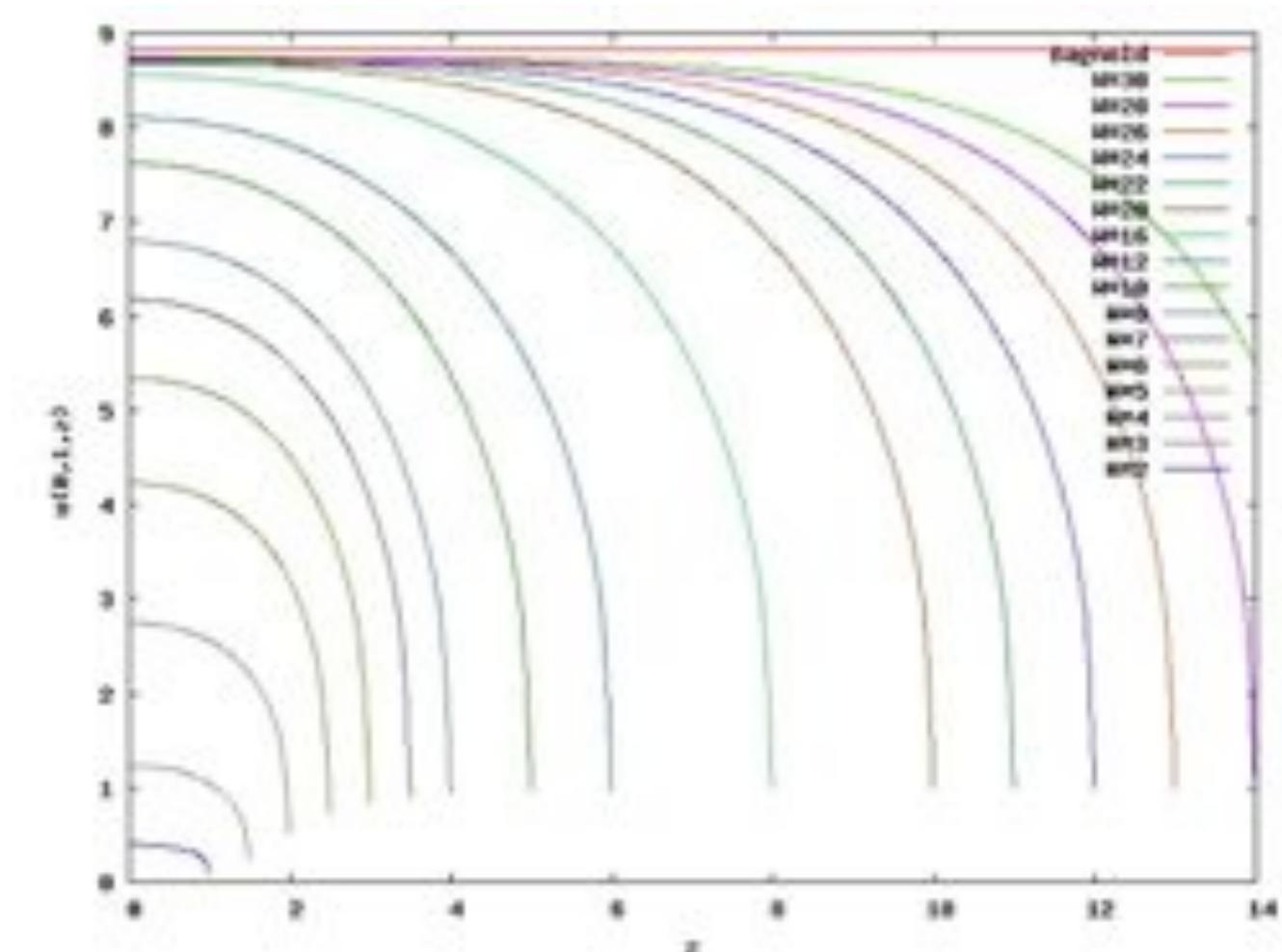


influence de la largeur sur l'écoulement: on passe de Bagnold à un écoulement en surface

Gerris



profils au centre en profondeur



profils en surface vus de dessus

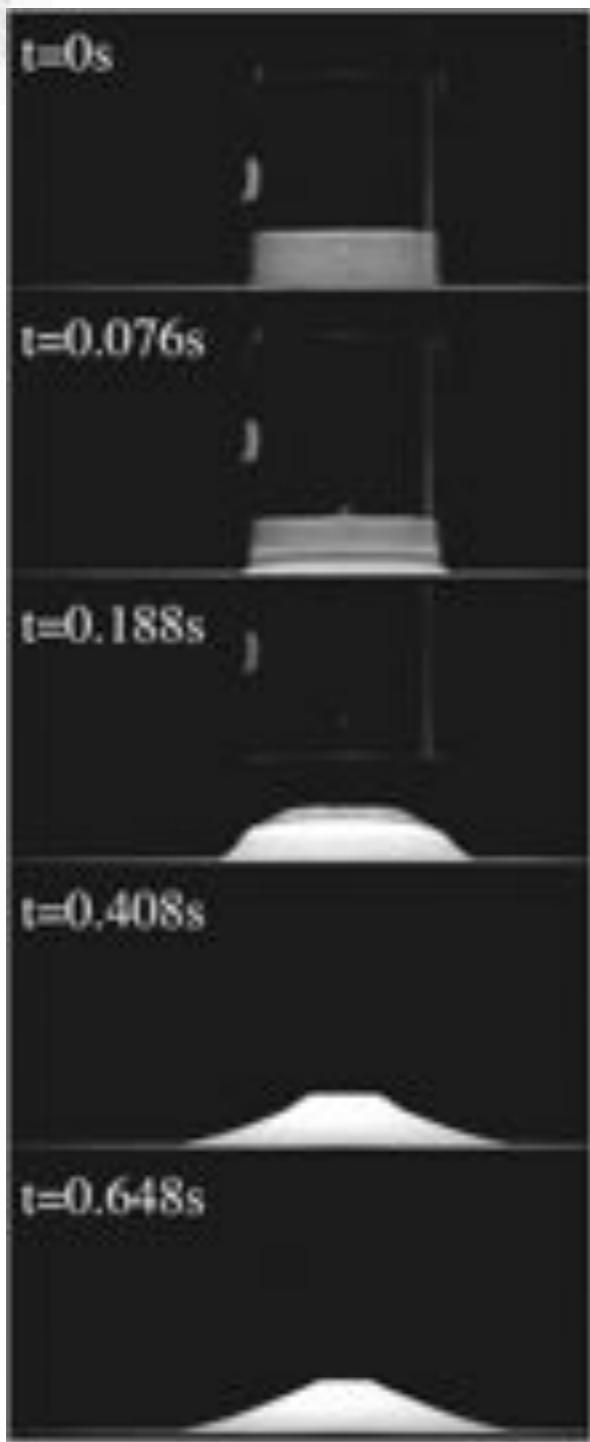


The sand pit problem: quickly remove the bucket of sand

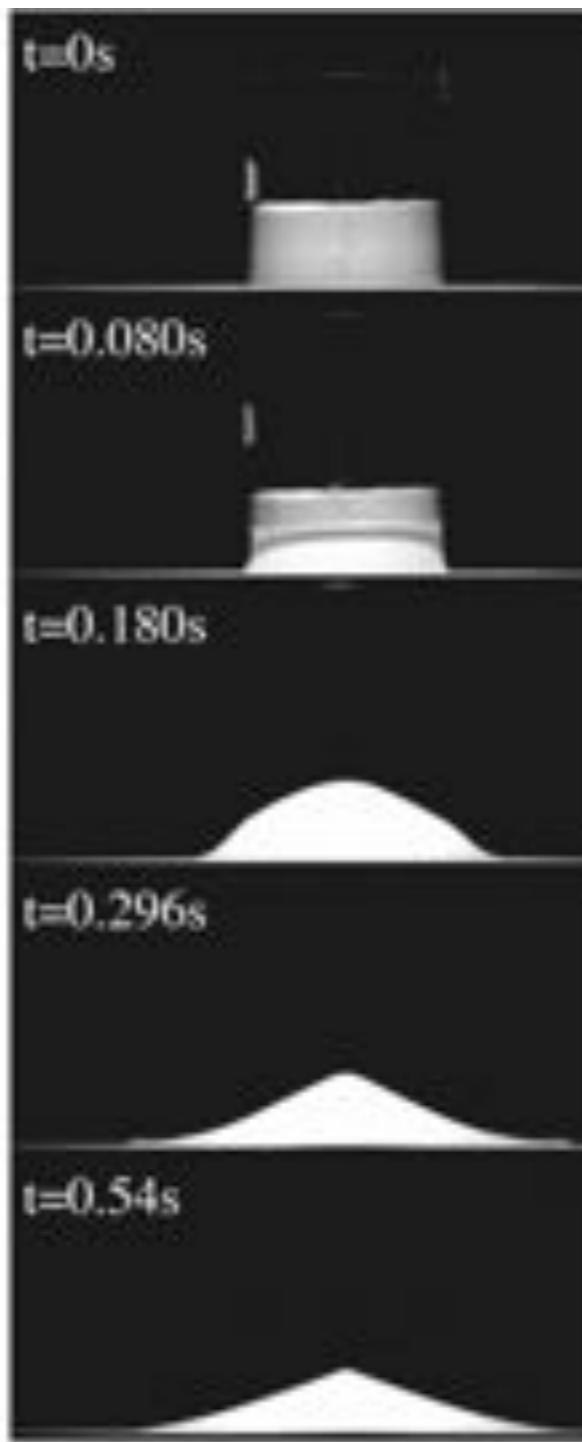
<http://www.mylot.com/w/photokeywords/pail.asp>



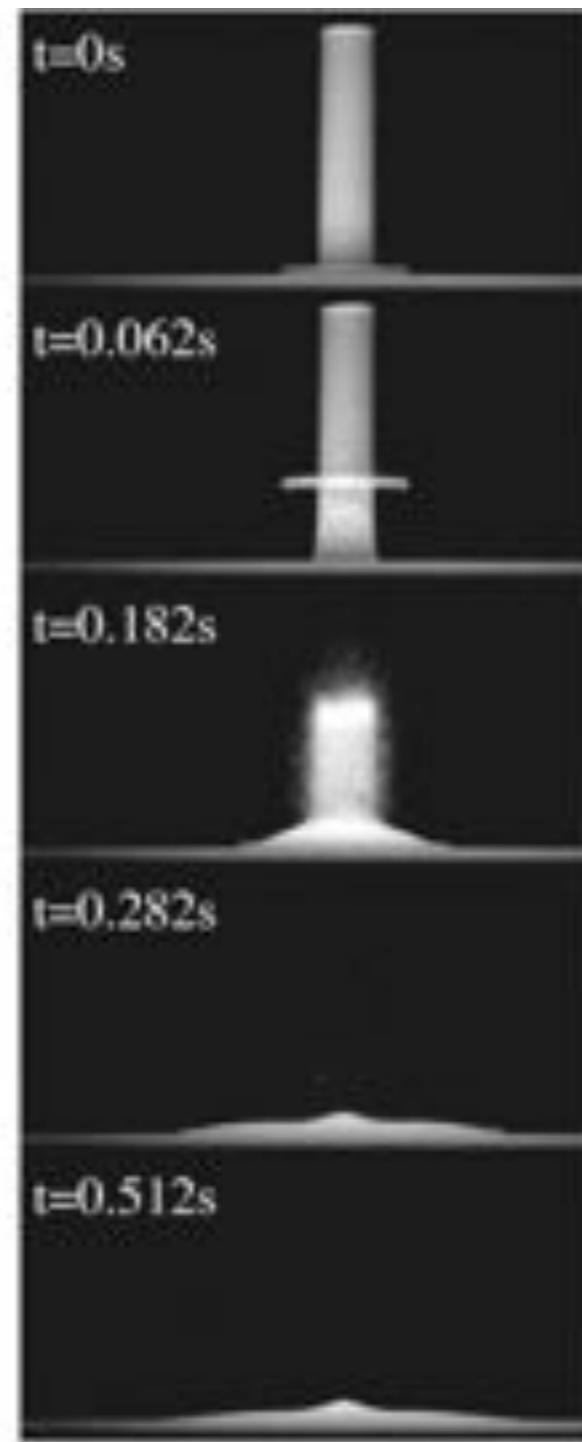
Granular Column Collapse



(a)



(b)



(c)



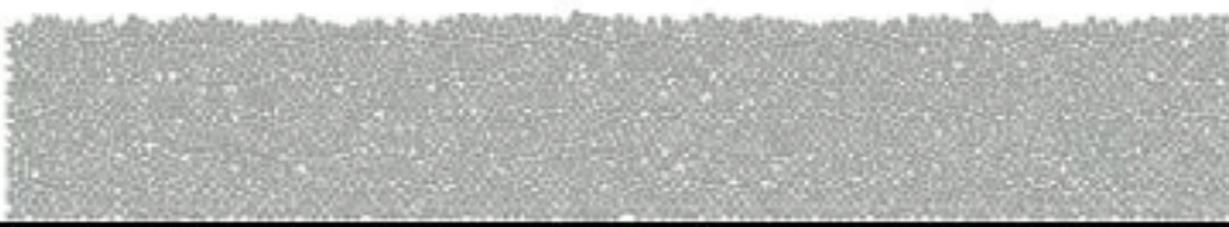
The sand pit problem: quickly remove the bucket of sand

<http://www.mylot.com/w/photokeywords/pail.asp>



Collapse of columns

a=0.37



Contact Dynamic
simulation Lydie Staron





Collapse of columns

a=0.90

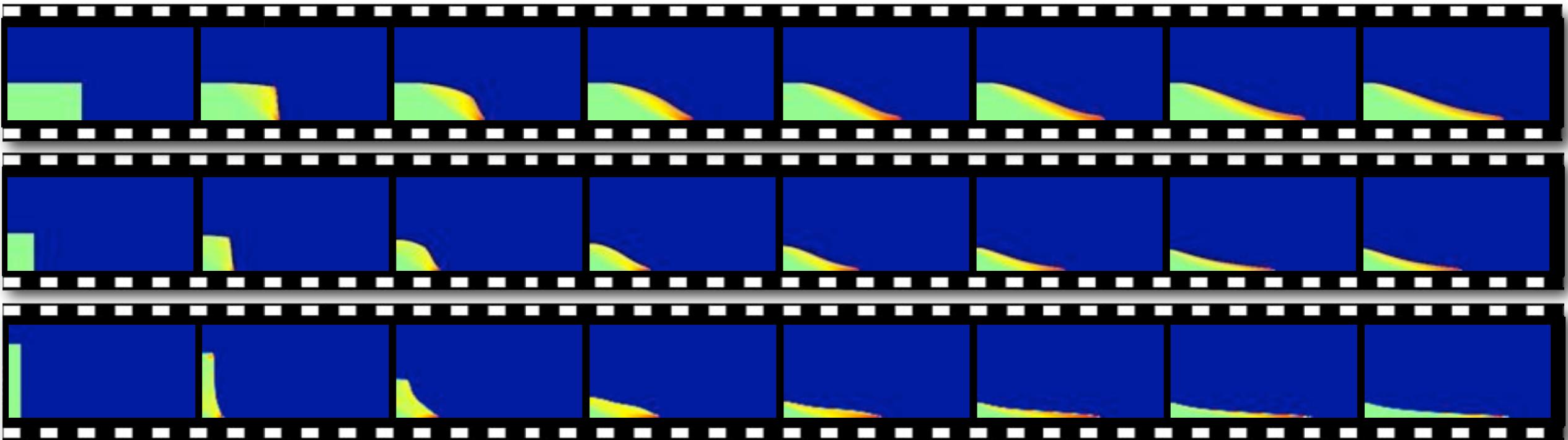


Contact Dynamic
simulation Lydie Staron





Collapse of columns simulation *Gerris* $\mu(l)$

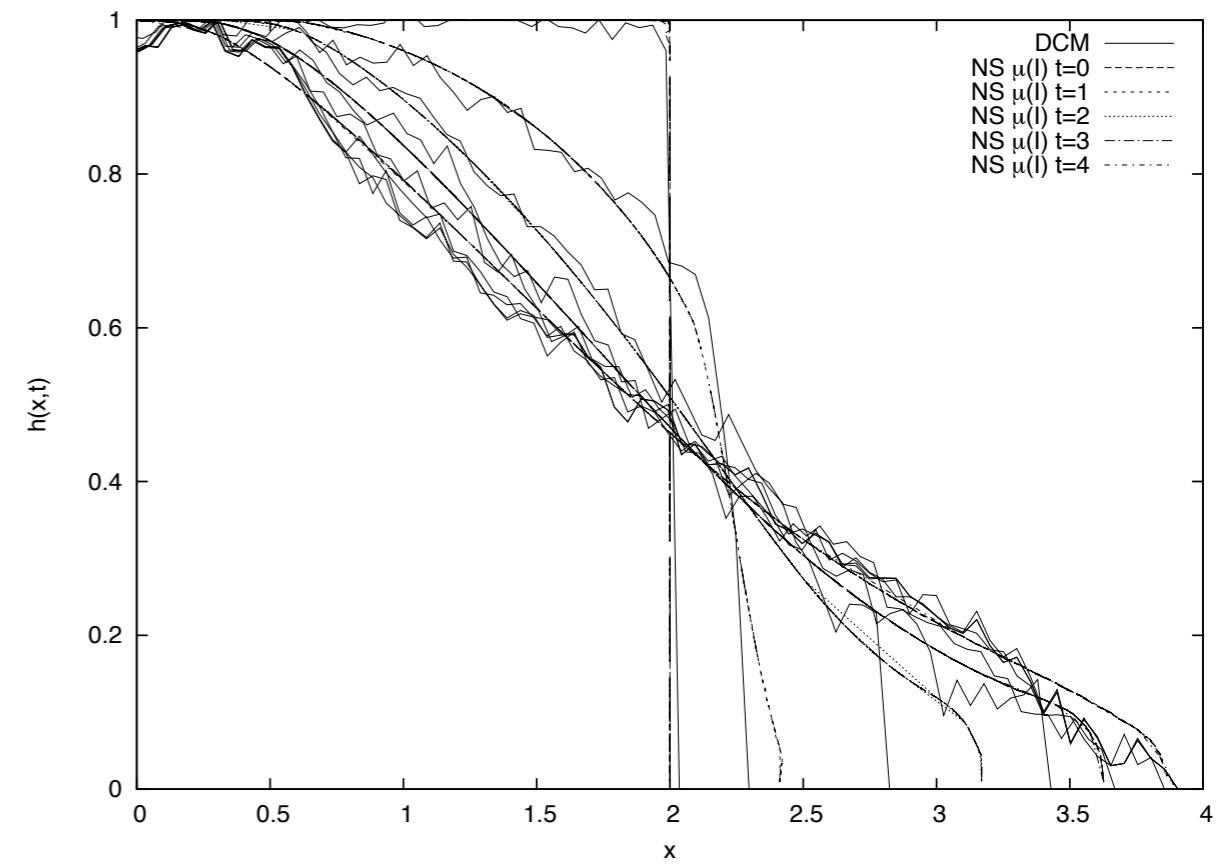
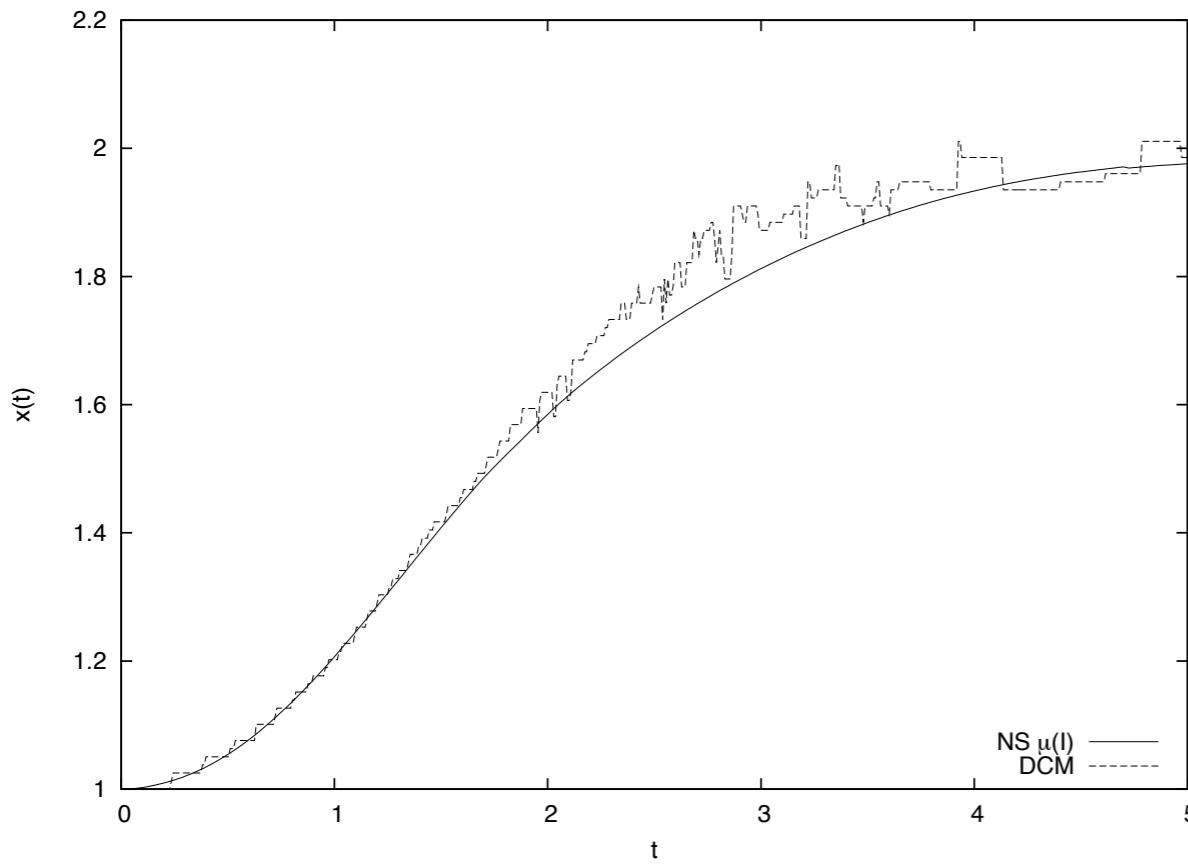


Snapshots of collapse of three columns of aspect ratio 0.5 1.42 and 6.26 (top to bottom)

Collapse of columns simulation *Gerris* $\mu(l)$



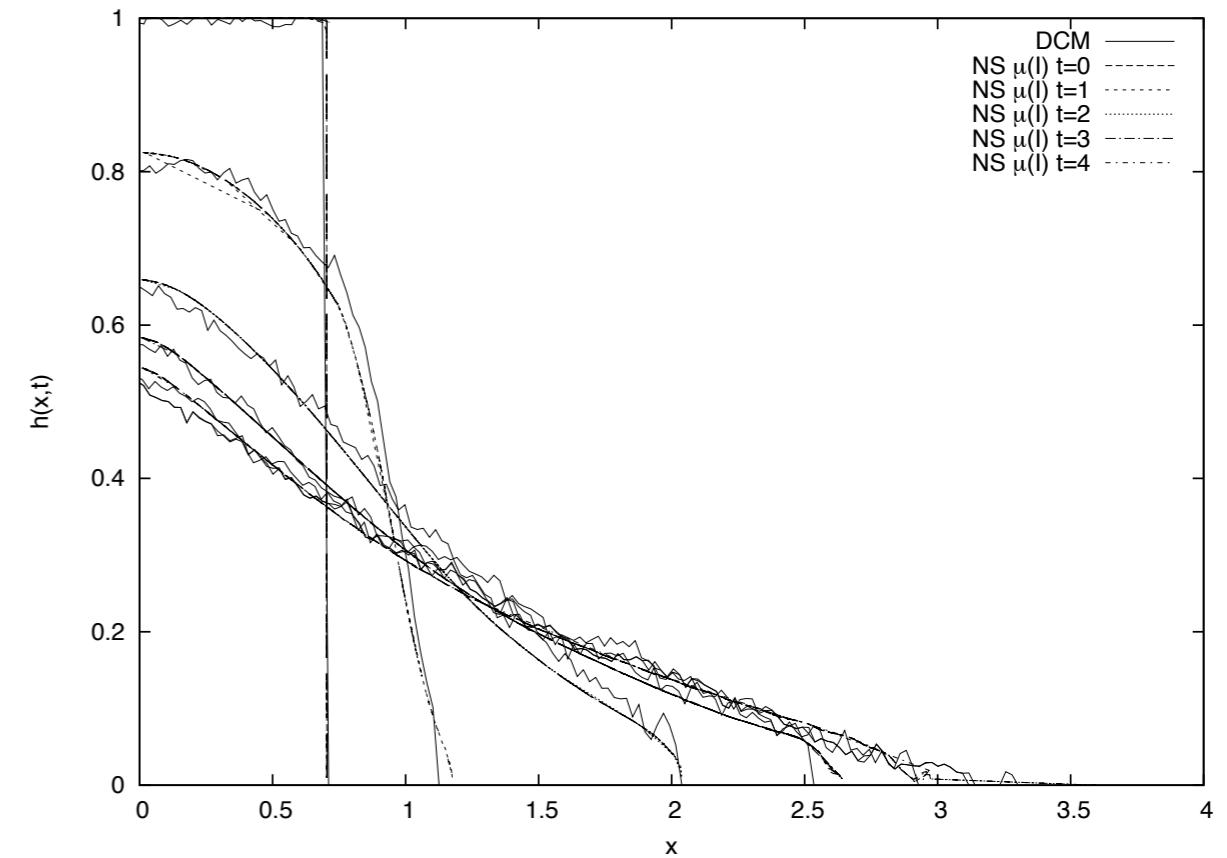
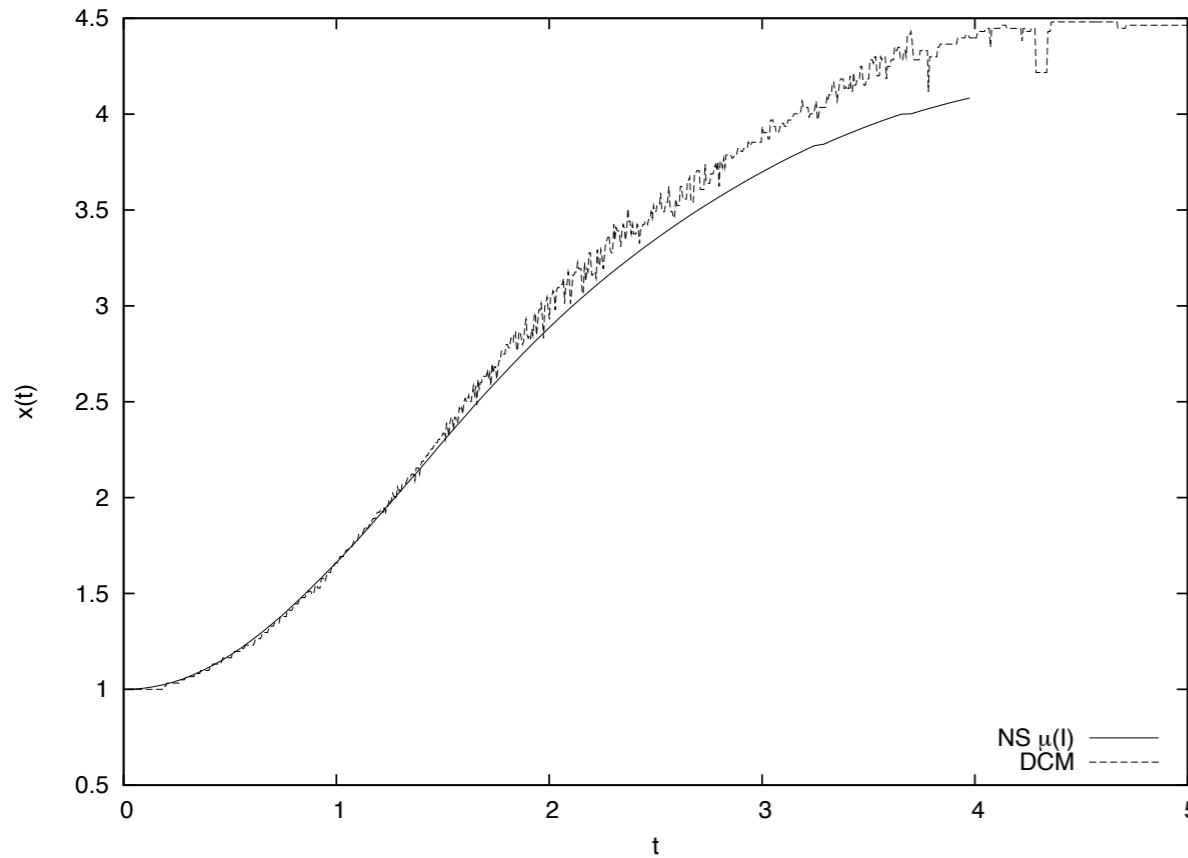
Collapse of columns of aspect ratio 0.5
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)



Collapse of columns simulation *Gerris* $\mu(l)$



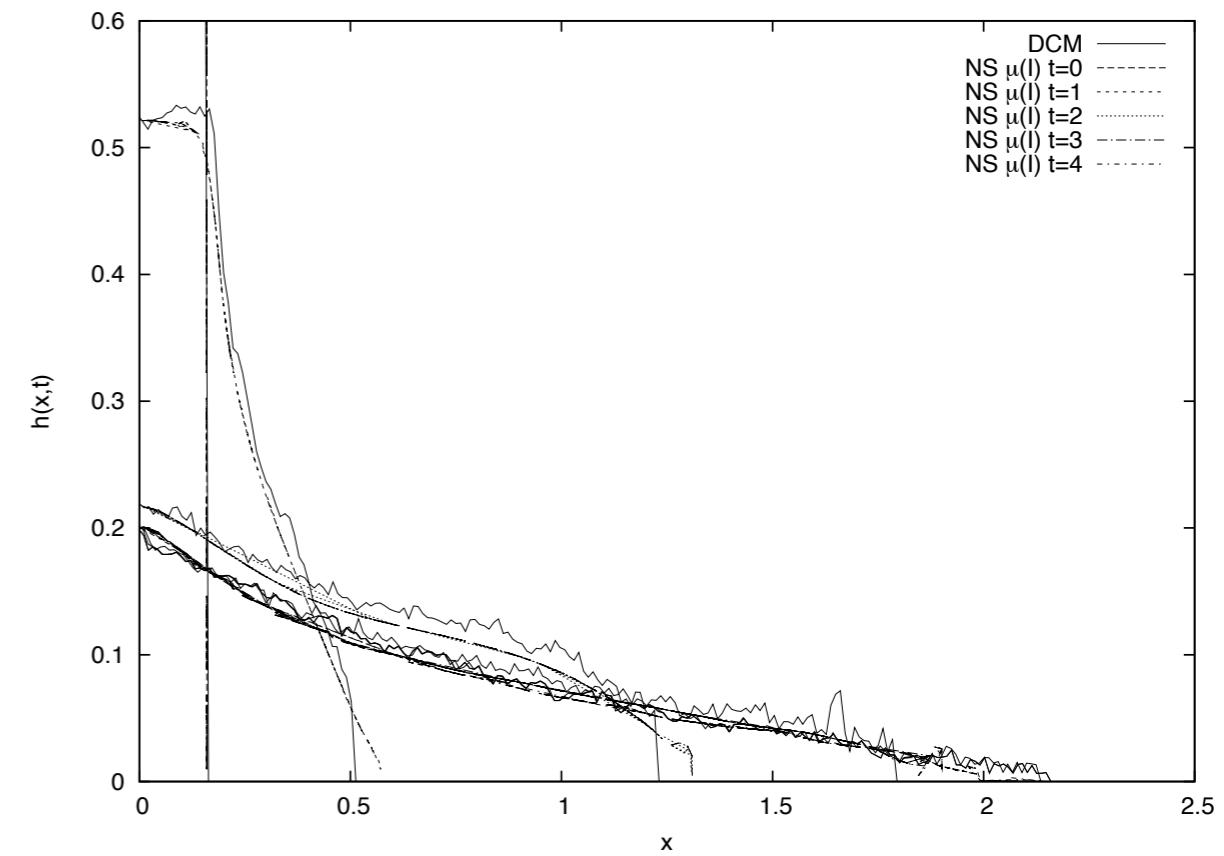
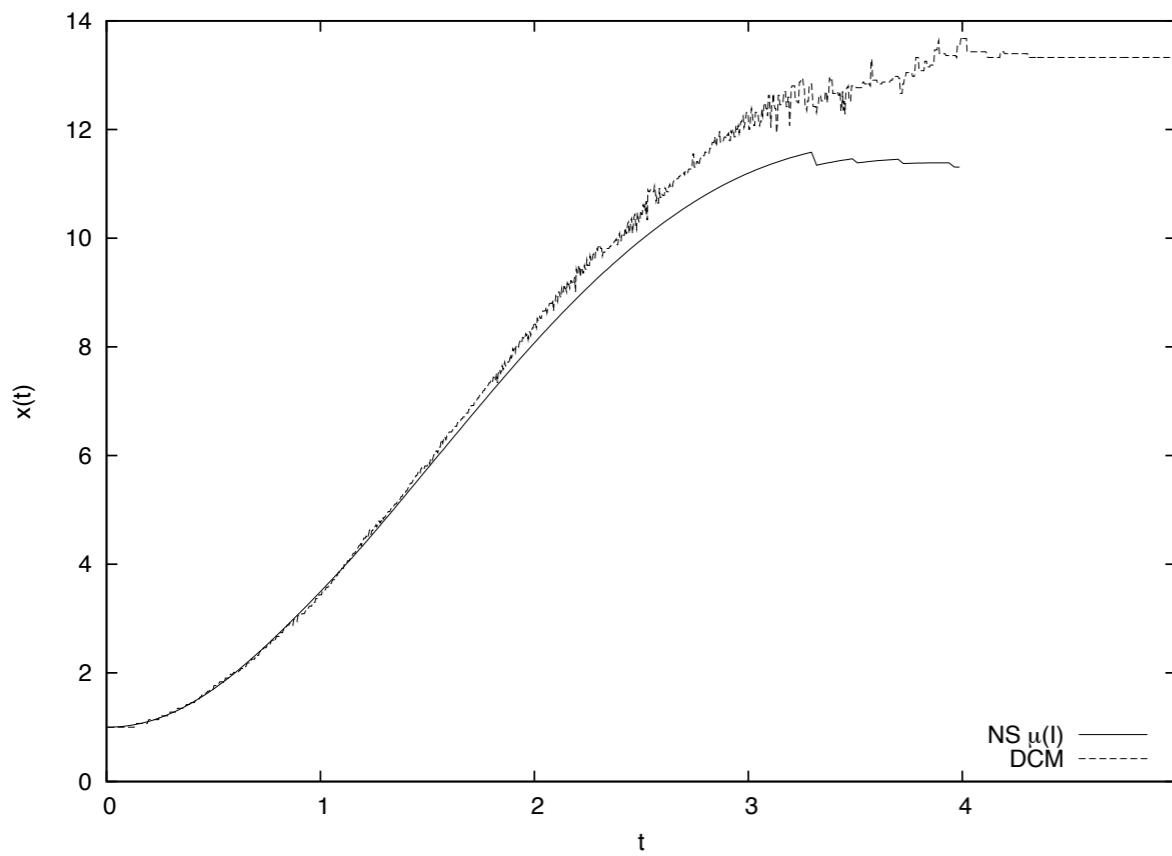
Collapse of columns of aspect ratio 1.42
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)





Collapse of columns simulation *Gerris* $\mu(l)$

Collapse of columns of aspect ratio 6.26
comparison of Discrete Simulation Contact
Method and Navier Stokes gerris, shape at time
0, 1, 2, 3, 4 and position of the front of the
avalanche as function of time (time measured
with $\sqrt{H_0/g}$ and space with aH_0)

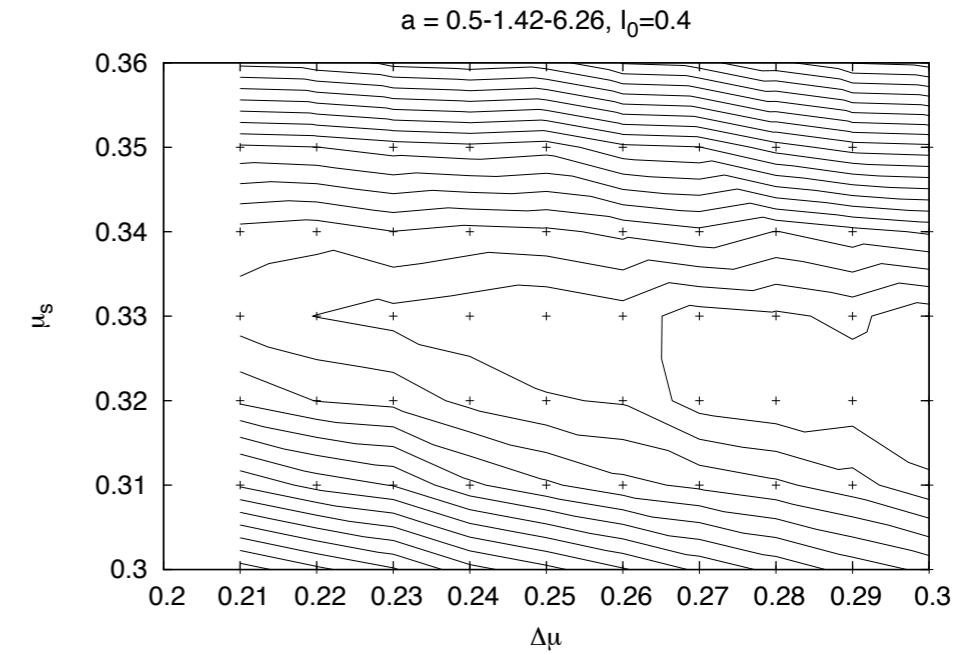
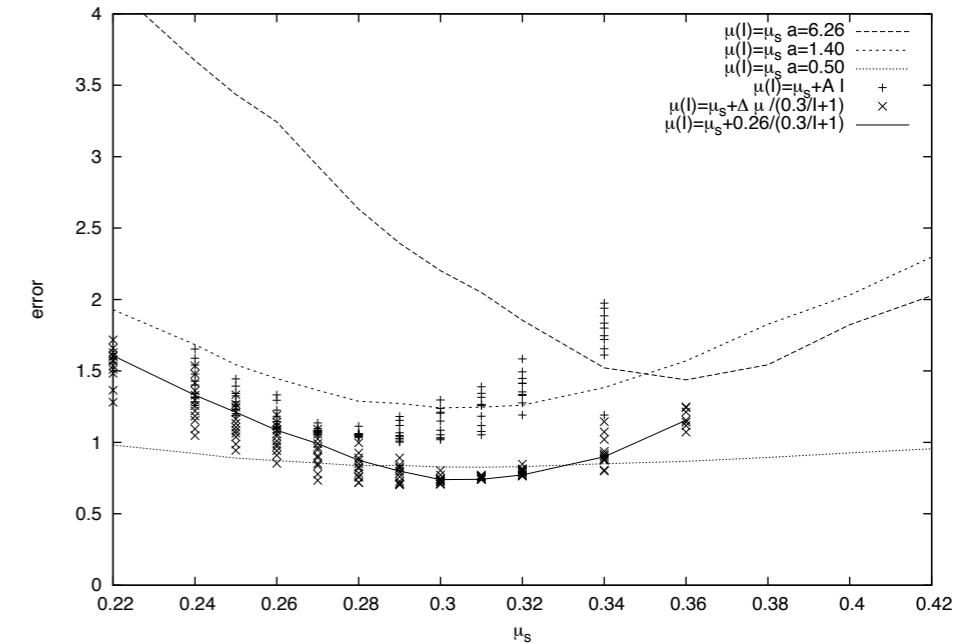


Collapse of columns simulation *Gerris* $\mu(I)$



optimisation

$$\mu(I) = \mu_s + \frac{\Delta\mu}{I_0 + 1}$$

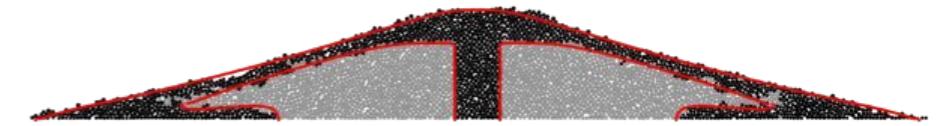
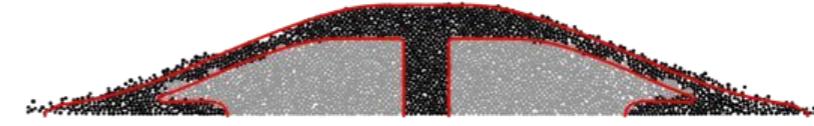
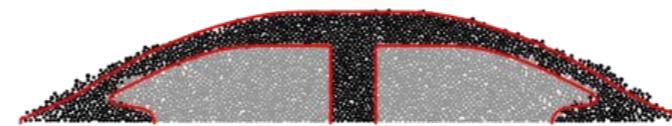
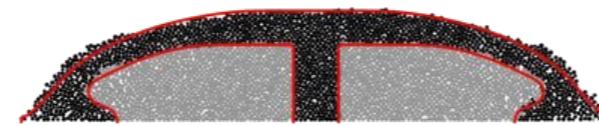
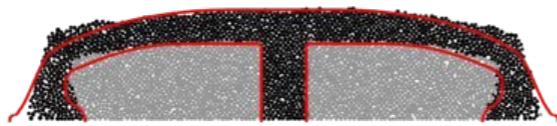
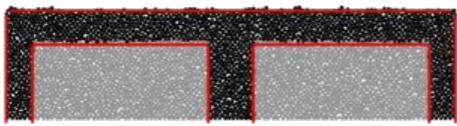


final values

$$\mu_s = 0.32 \quad \Delta\mu = 0.28 \quad I_0 = 0.4$$



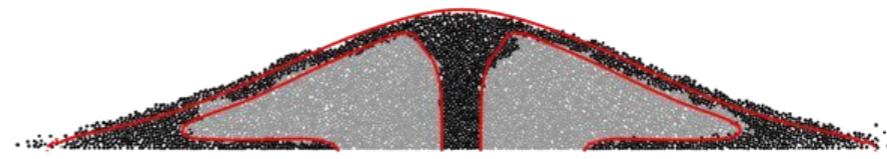
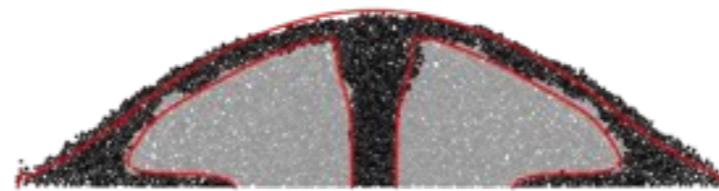
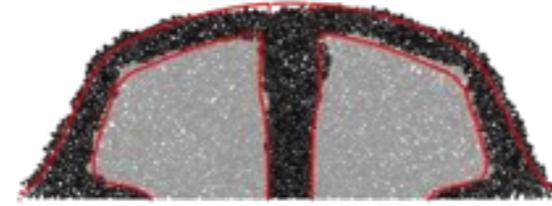
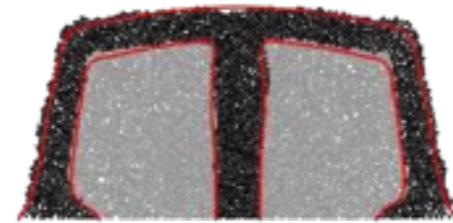
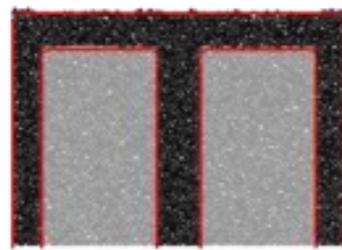
Collapse of columns simulation *Gerris* $\mu(l)$



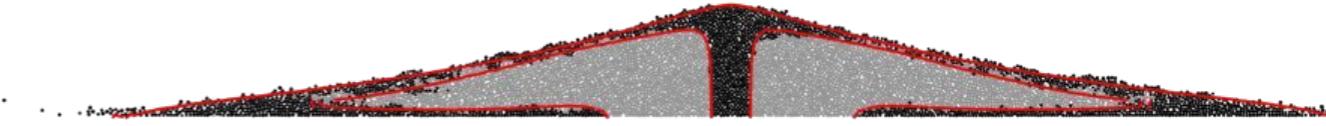
$a = 0.5$ DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

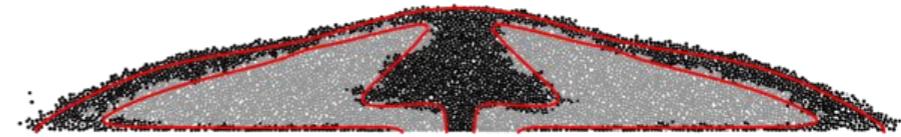
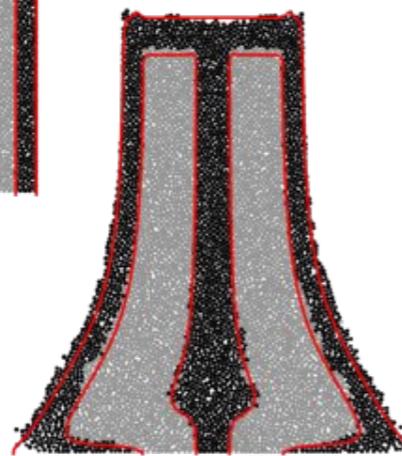
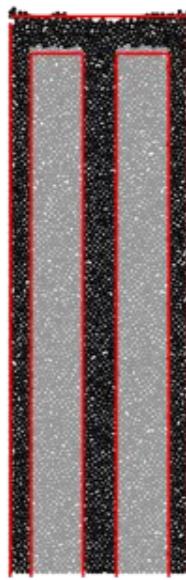


$a = 1.42$ DCM vs *Gerris* $\mu(l)$

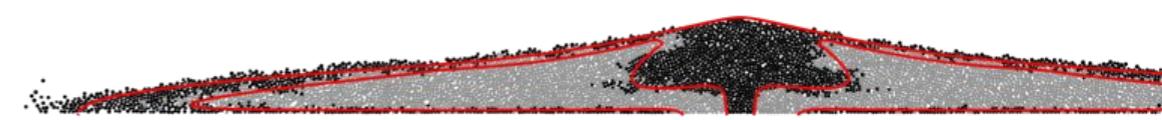




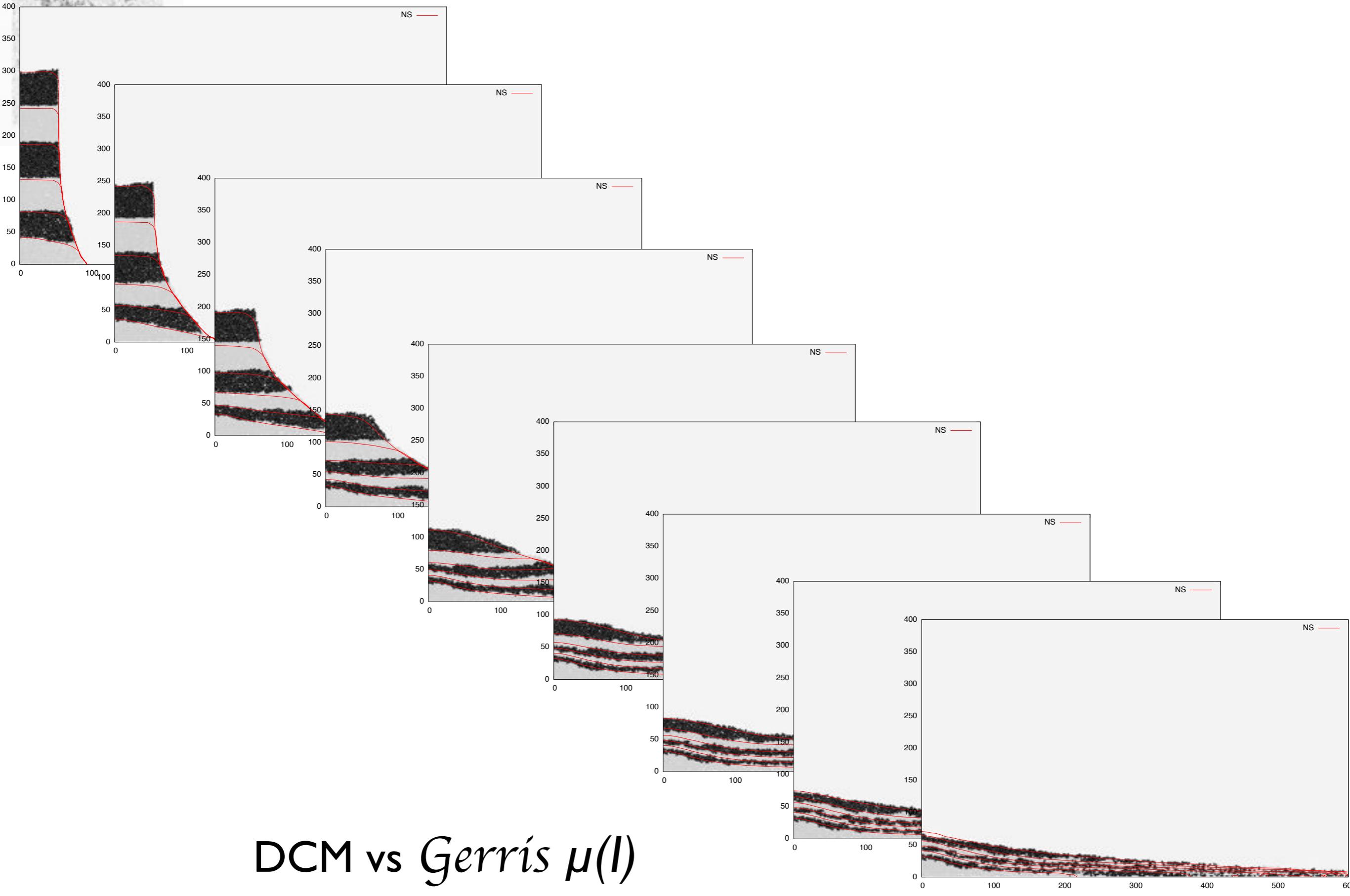
Collapse of columns simulation *Gerris* $\mu(l)$



$a = 6.6$ DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

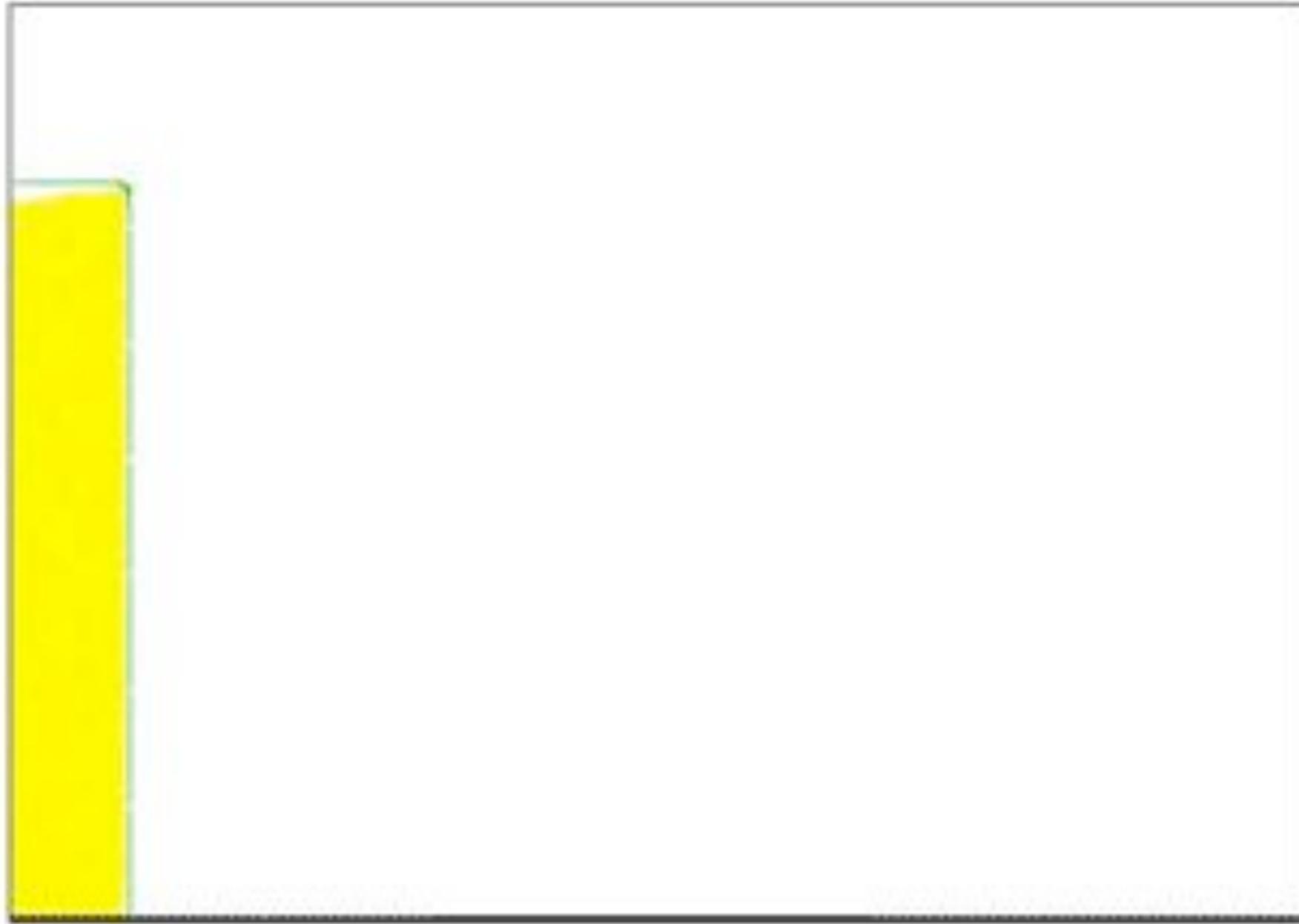


DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD $t=0.0190$



DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$



DCM vs *Gerris* $\mu(l)$



Collapse of columns simulation *Gerris* $\mu(l)$

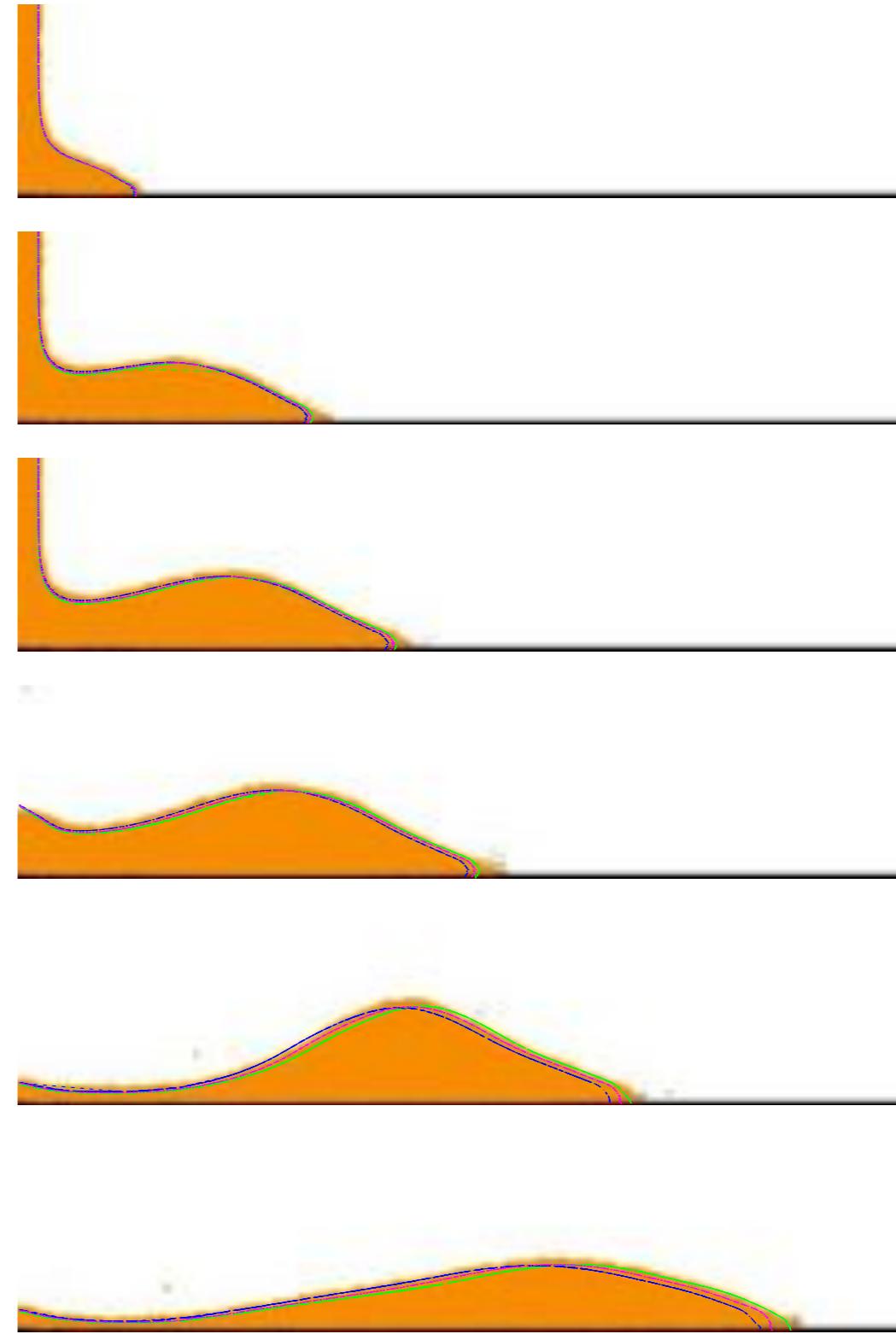
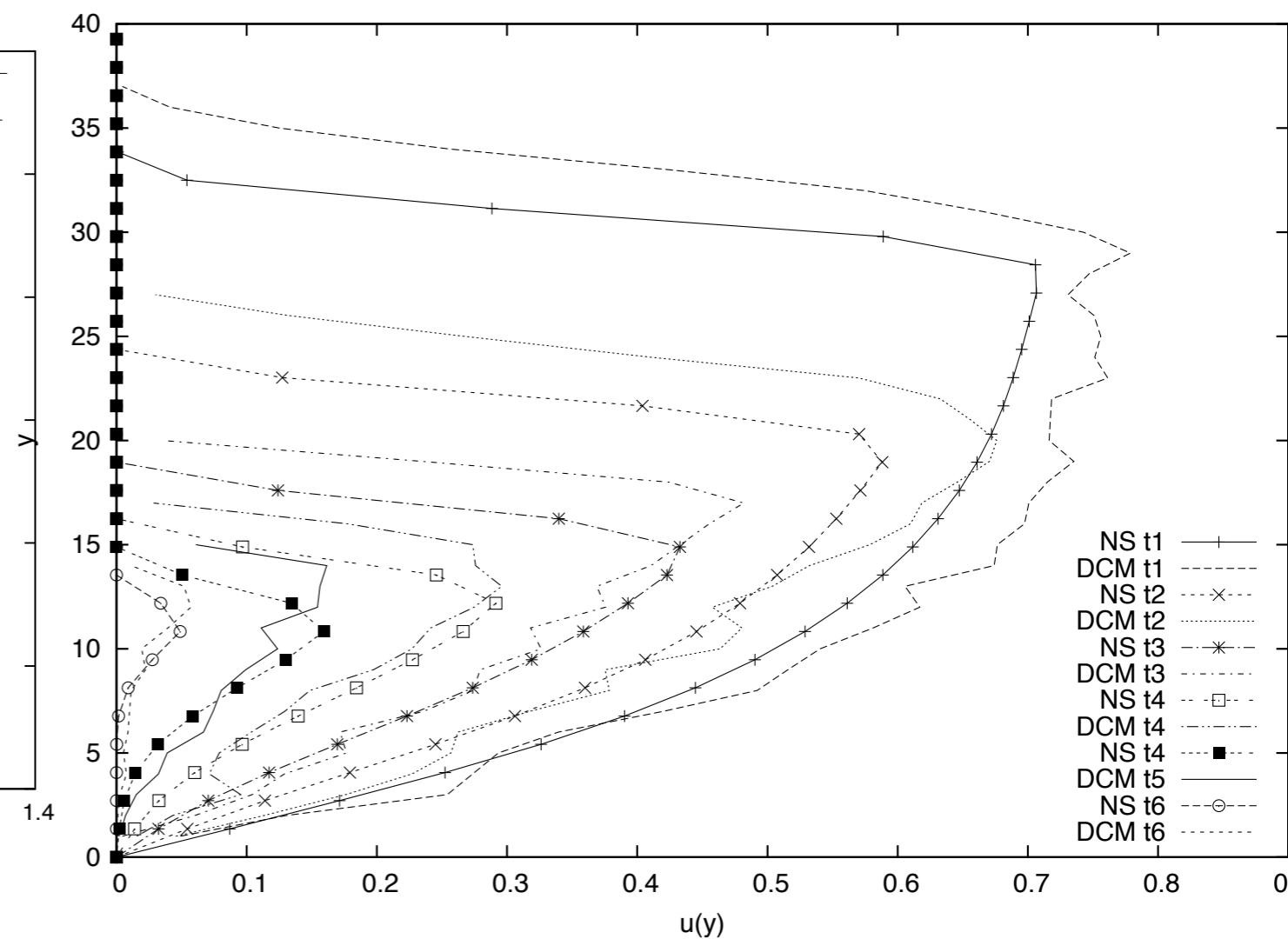
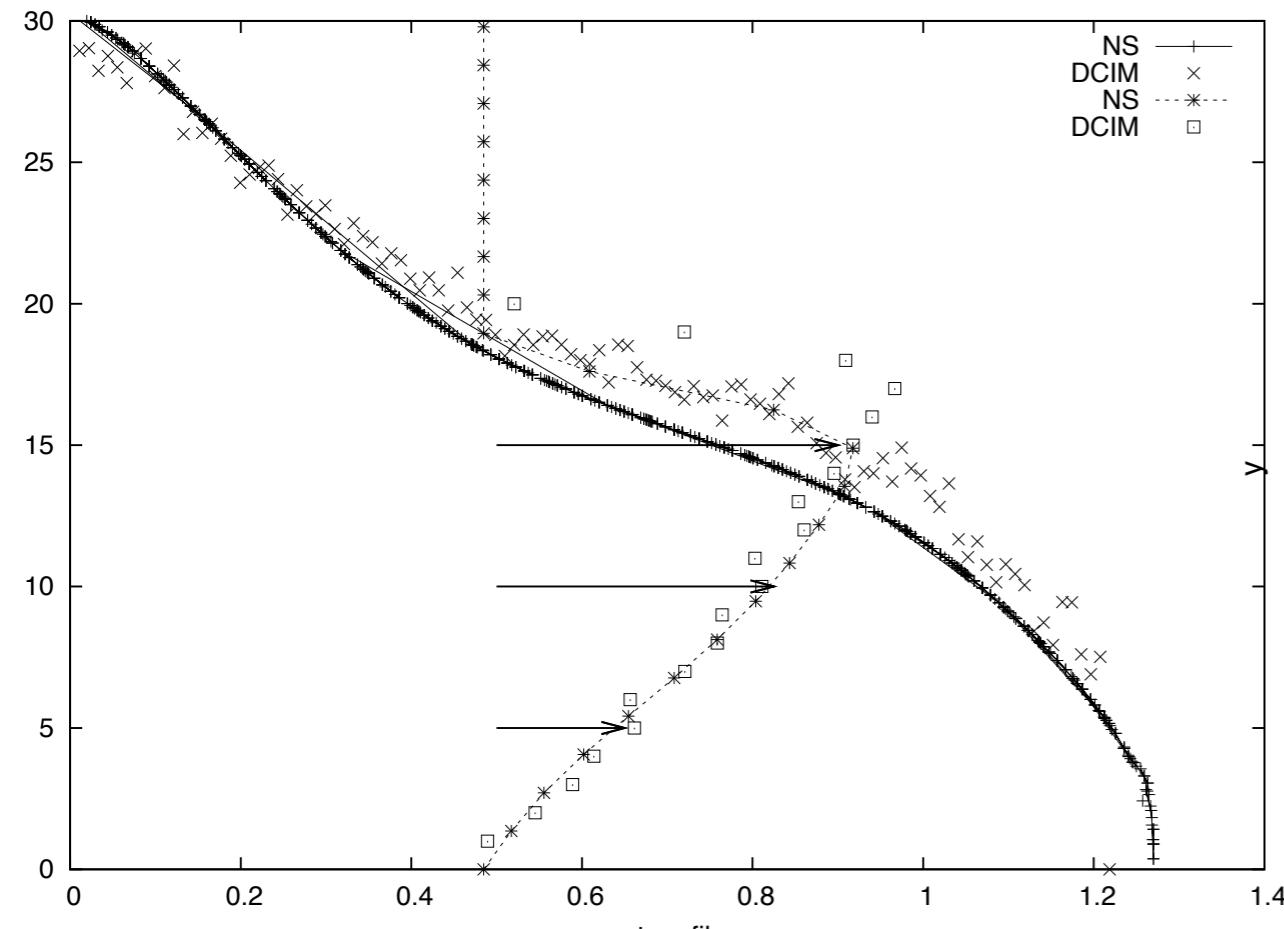


Figure 10: Strip representing a series of snapshots ($t = 0.5, 1.0, 1.2, 1.4, 1.7$, and 2.0) of a column collapse with aspect ratio $a = 68$. The most advanced curve (in green) corresponds to $\mu_s = 0.3$ $\Delta\mu = 0.26$ and $I_0 = 0.30$. the less advanced (in blue) $\mu_s = 0.32$ $\Delta\mu = 0.28$ and $I_0 = 0.30$ fits better the end of the heap. The curve in between (in cyan) corresponds to $\mu_s = 0.32$ $\Delta\mu = 0.28$ and $I_0 = 0.40$ and fits better the top of the surge.

DCM vs *Gerris* $\mu(l)$

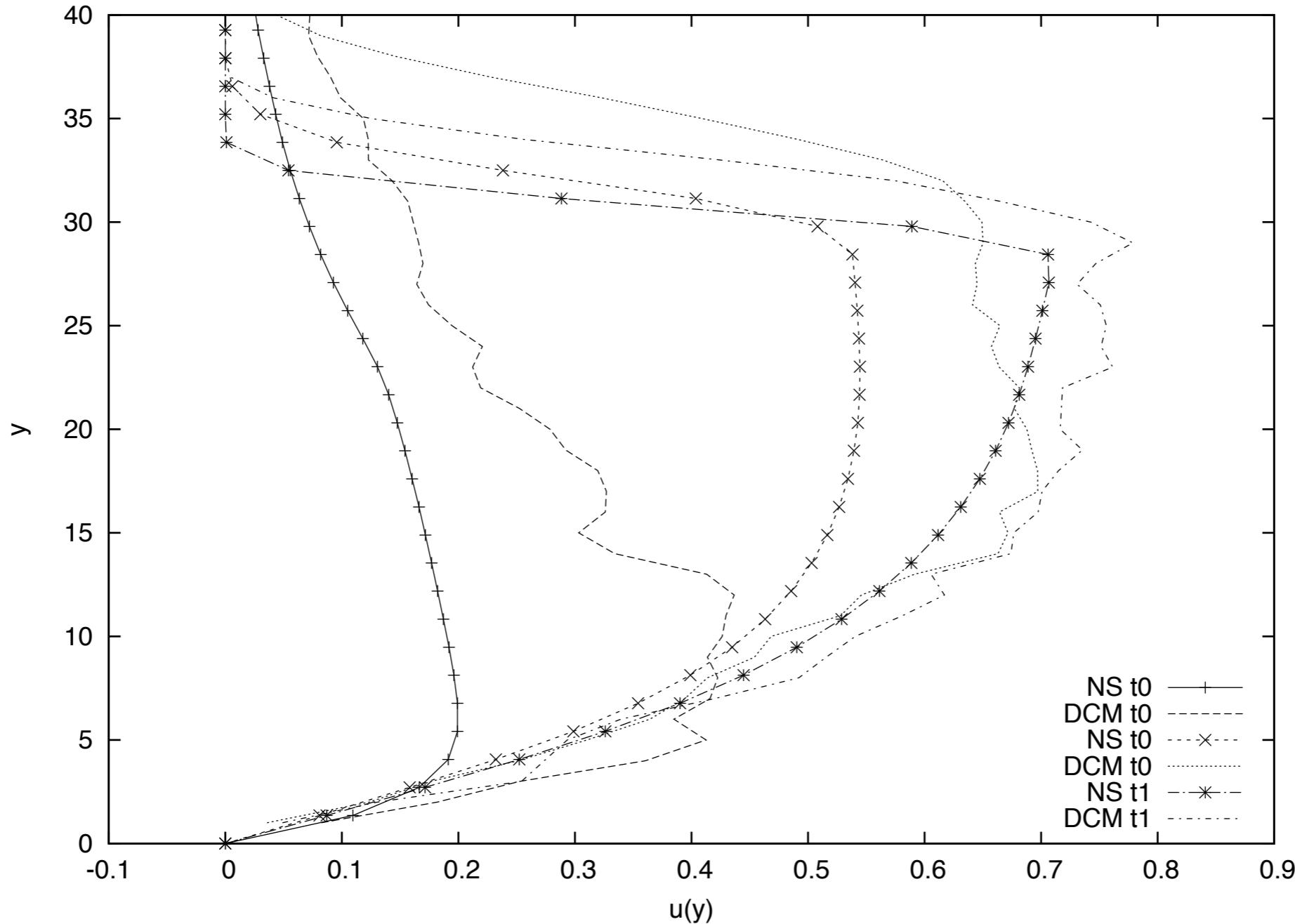
● comparaison de profiles de vitesse



DCM vs *Gerris* $\mu(I)$



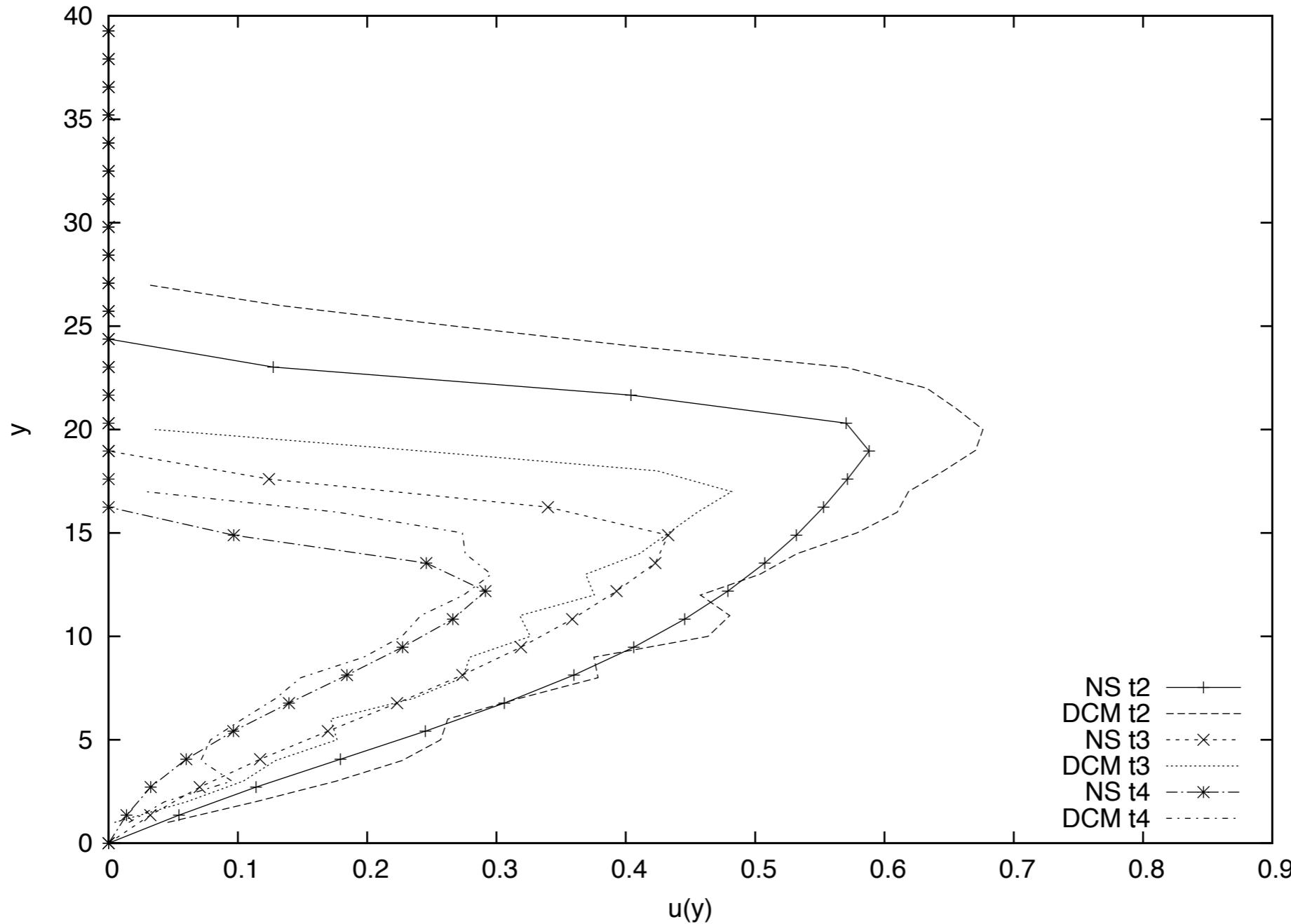
- comparaison of velocity profiles



DCM vs *Gerris* $\mu(l)$



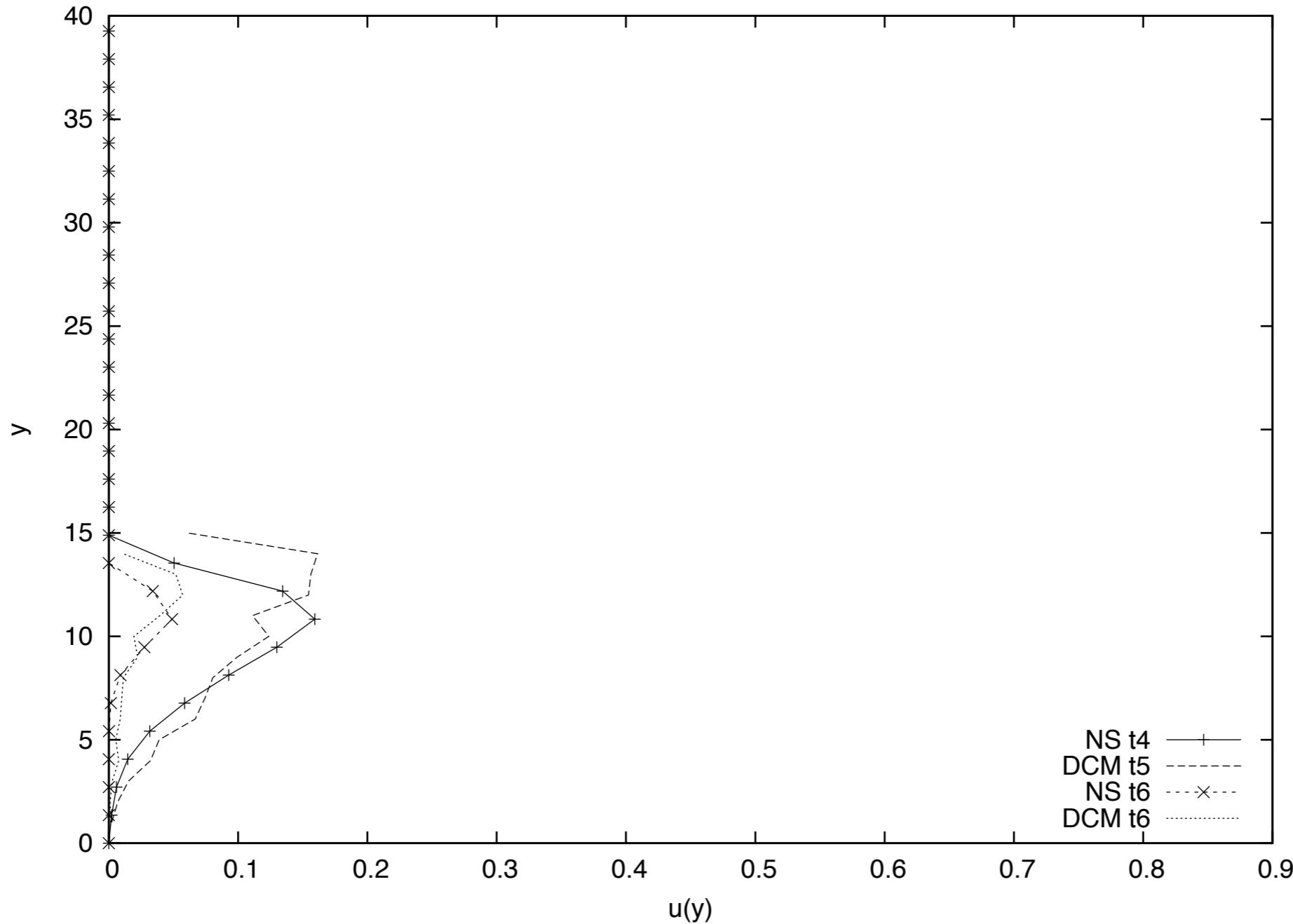
- comparaison of velocity profiles



DCM vs *Gerris* $\mu(l)$



- comparaison of velocity profiles

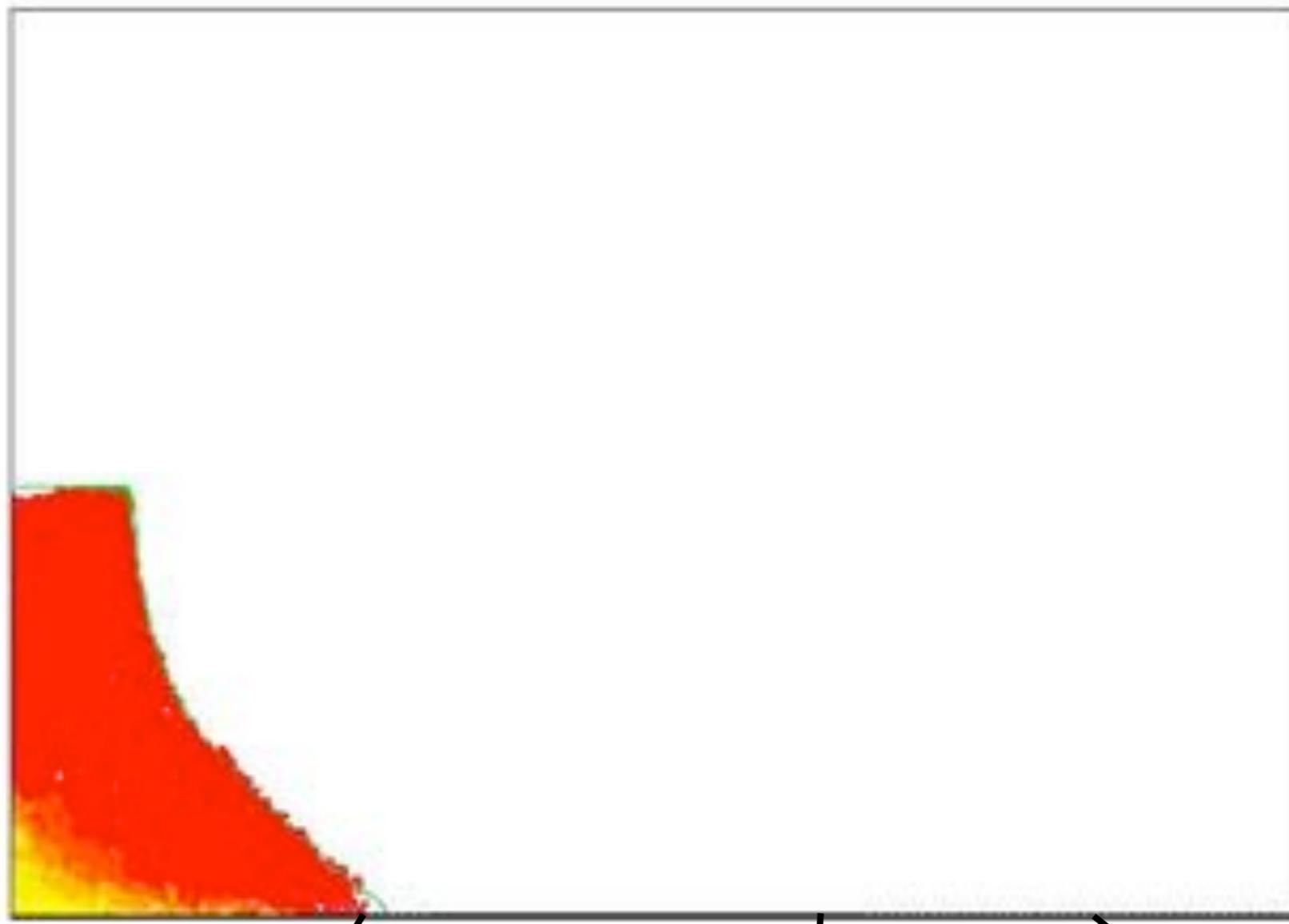


DCM vs *Gerris* $\mu(l)$

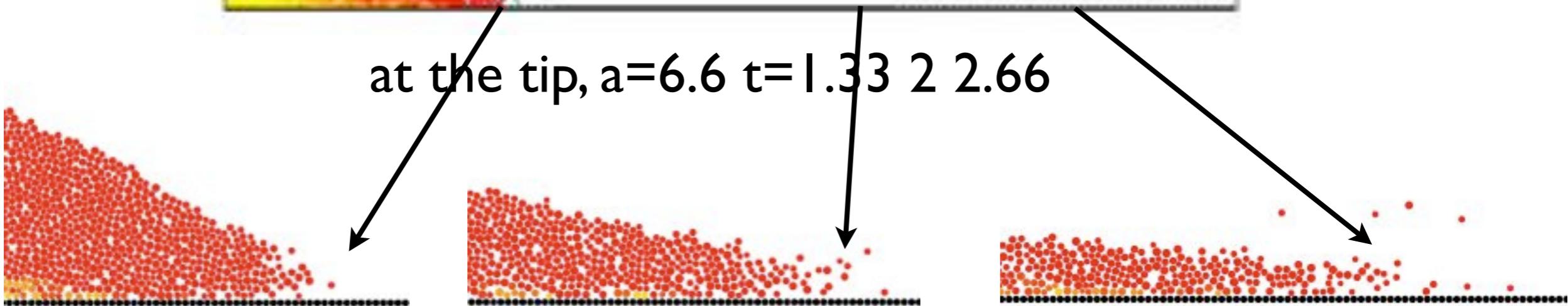


Collapse of columns simulation *Gerris* $\mu(l)$

NS/CD t=8.9318

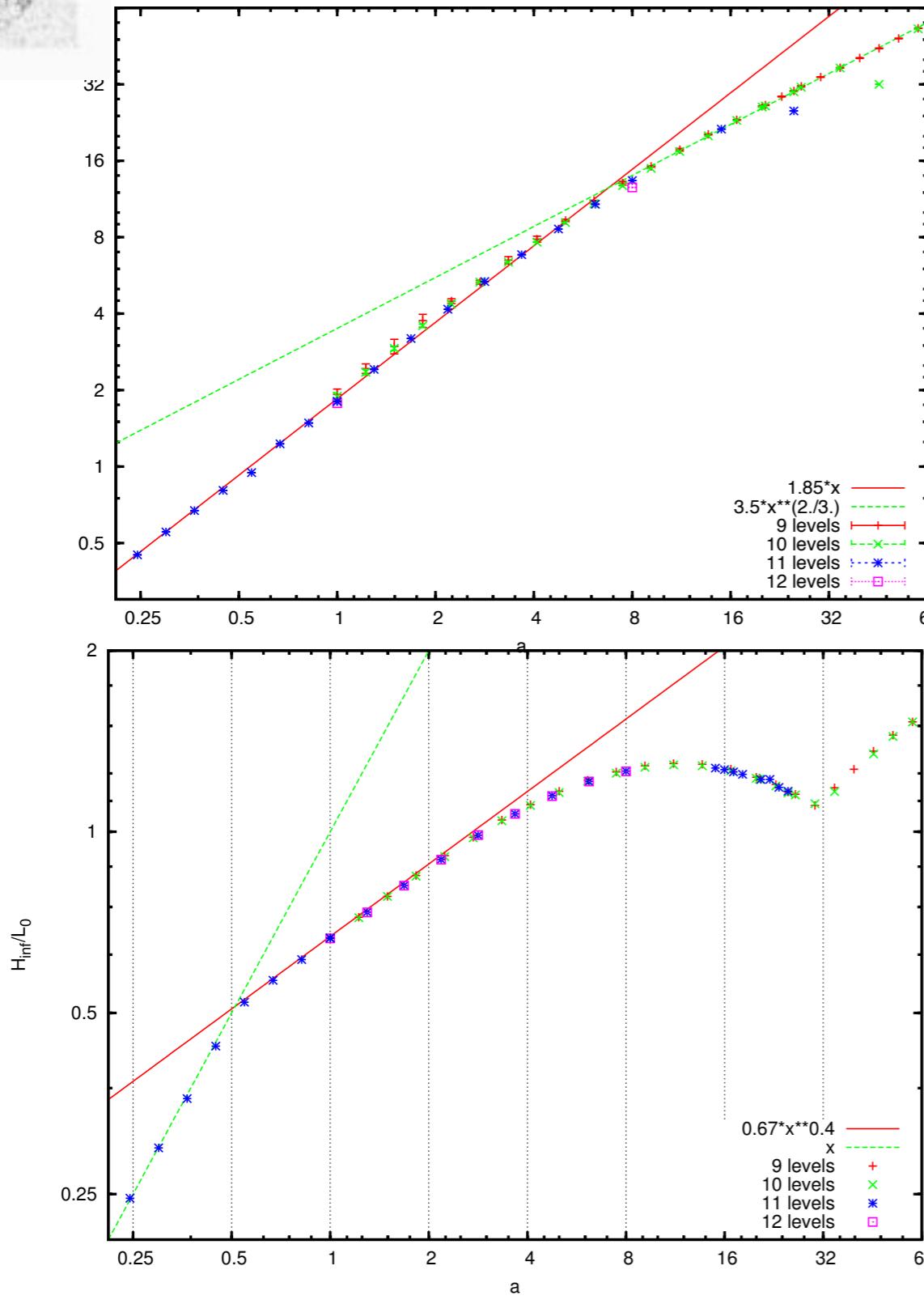


at the tip, $a=6.6$ $t=1.33$ 2 2.66



DCM vs *Gerris* $\mu(l)$

Collapse of columns simulation *Gerris* $\mu(I)$



Normalised final deposit extent as a function of aspect ratio a .

Well-defined power law dependencies with exponents of 1 and $2/3$ respectively.

We recover the experimental scaling [Lajeunesse et al. 04] and [Staron et al. 05]. Differences between the values of the prefactors are due to the difficulties to obtain the run out length: friction in the Navier Stokes code tends to underestimate it, whereas direct simulation shows that the tip is very gaseous, it can no longer be explained by a continuum mechanic description.



conclusion

- $\mu(l)$ obtained from experimental flows of dry granular flows [Jop et al. 06], implemented it in *Gerris*
- test case: analytical solution of steady avalanche (Bagnold solution)
- collapse of granular columns (shape as function of time compared to Discrete Simulations).
- The experimental trends of the scaling of the run out are reobtained
- Saint Venant Savage Hutter to be compared with.
- complete spectra: discrete grains/ Saint Venant/ Navier Stokes

This opens the door to systematic studies of granular flows using this continuum approach.



références:

- P. Jop, Y. Forterre, O. Pouliquen, (2006) "A rheology for dense granular flows", *Nature* 441, pp. 727-730 (2006)
- P.-Y. Lagrée, L. Staron and S. Popinet (to appear J.F.M.) "The granular column collapse as a continuum: validity of a two-dimensional Navier–Stokes model with a $\mu(I)$ -rheology"
- E. Lajeunesse, A. Mangeney-Castelnau, and J.-P. Vilotte, (2004) "Spreading of a granular mass on an horizontal plane», *Phys. Fluids*, 16(7), 2371-2381.
- L. Staron & E. J. Hinch (2005) "Study of the collapse of granular columns using two-dimensional discrete-grain simulation", *J. Fluid Mech.* (2005), vol. 545, pp. 1–27.