COHOMOLOGY OF GROUPS : EXERCISES

Sheet 5, 4 June 2007

Exercise 5.1. Let G be a group, H be a subgroup of finite index, and let $\mathbb{Z}$ have trivial G-action. Give a formula for the transfer $\operatorname{tr}_H^G : H_1(G;\mathbb{Z}) \to H_1(H;\mathbb{Z})$ as a homomorphism $G_{\text{ab}} \to H_{\text{ab}}$ on the abelianizations of G and H.

Exercise 5.2. Write a detailled proof of the Cartan-Eilenberg double coset formula, using the hints given in the lecture.

Exercise 5.3. Let $\mathcal{S}_3$ be the group of permutations of a set with three elements. Let $\mathbb{Z}$ have the trivial $\mathcal{S}_3$-action. Compute

$$H^*(\mathcal{S}_3;\mathbb{Z})$$

using subgroups and invariants.

Exercise 5.4. Prove that if $\{G_i\}_{i\in I}$ is a directed system of groups, then for any $\mathbb{Z}[G]$-module M the natural map

$$\operatorname*{colim}_I H_*(G_i;M) \to H_*(\operatorname*{colim}_I G_i;M)$$

is an isomorphism.

Exercise 5.5. Let G be a group and H be a subgroup of finite index. Prove the transfer formula for the cup-product in cohomology given in the lecture, namely

$$\operatorname{tr}_H^G\big((\operatorname{res}_H^G u)\cup v\big) = u \cup (\operatorname{tr}_H^G v).$$

Exercise 5.6. Let $n \in \mathbb{N}$ be an even natural number and C_n be the cyclic group of order n. Compute $H^*(C_n;\mathbb{F}_2)$ and $H_*(C_n;\mathbb{F}_2)$ as $\mathbb{F}_2$-algebras, with the cup and Pontrjagin products, respectively.