## COHOMOLOGY OF GROUPS : EXERCISES

Sheet 7, 9 July 2007

**Exercise 7.1.** Let $p \geqslant 3$ be a prime, let $C_p = \langle t | t^p = 1\rangle$ and $C_{p^2} = \langle s | s^{p^2} = 1\rangle$ be cyclic groups of order $p$ and $p^2$. We let $C_p$ act on $C_{p^2}$ by $t(s) = s^{1+p}$. Let $E = C_{p^2} \rtimes C_p$ be the corresponding extension

$$1 \to C_{p^2} \to E \to C_p \to 1.$$

Compute $H^*(E; \mathbb{F}_p)$ as an algebra.
*Hint:* Consider the LHS spectral sequence associated to this extension. Show that it has a non-trivial $d_2$ differential by computing $H^2(E; \mathbb{F}_p)$ and comparing it with $E_2^{0,2} \oplus E_2^{1,1} \oplus E_2^{2,0}$. To compute $H^2(E; \mathbb{F}_p)$, you can for example use an explicit resolution, or first compute $H^i(E; \mathbb{Z})$ for $i = 2, 3$ with the LHS spectral sequence and apply universal coefficients. Show that it collapses at the $E_3$-term. Show that there are no multiplicative extensions.

**Exercise 7.2.** Compute $H^*(D_8; \mathbb{F}_2)$ as an $\mathbb{F}_2$-algebra.
*Hint:* Prove that there is an isomorphism $D_8 \cong C_2 \wr C_2$.