EXERCISES FOR TOPOLOGY III, WS05/06

Sheet 1, 21 October 2005

Solutions due on Monday, 31st of October.

Exercise 1.1. Prove the Eilenberg-Steenrod Axioms for cohomology. Prove also the sum and the weak homotopy axioms. You may use the corresponding results for homology, as well as the ingredients of their proof.

Exercise 1.2. Use the Eilenberg-Steenrod Axioms for cohomology to compute $H^*(S^n, R)$, $n \geqslant 0$.

Exercise 1.3. Compute the cohomology groups of the complex projective space $\mathbb{C}P^n$ with integral and with mod 2 coefficients. First, do it by using a CW-structure of $\mathbb{C}P^n$ and the corresponding cellular complex. Then do it by using the Universal Coefficients Theorem $\big($you will have to compute $\operatorname{Ext}^1_{\mathbb{Z}}(\mathbb{Z}, \mathbb{Z})\big)$.

Exercise 1.4. Same exercise as the previous one, replacing $\mathbb{C}P^n$ by $\mathbb{R}P^n$. Here, you will need in addition to compute $\operatorname{Ext}^1_{\mathbb{Z}}(\mathbb{Z}/2, \mathbb{Z})$.