EXERCISES FOR TOPOLOGY III, WS05/06

Sheet 3, November 3th 2005

Solutions due on Thursday, 10th of November.

Exercise 3.1. Let $\ell, m, n \geqslant 2$ be integers. Compute the following Ext-groups.
(a) $\mathrm{Ext}^*_{\mathbb{Z}}(\mathbb{Z}/m; \mathbb{Z})$,
(b) $\mathrm{Ext}^*_{\mathbb{Z}}(\mathbb{Z}/m; \mathbb{Z}/n)$,
(c) $\mathrm{Ext}^*_{\mathbb{Z}/\ell}(\mathbb{Z}/m, \mathbb{Z}/n)$ when defined,
(d) $\mathrm{Ext}^*_{\mathbb{Z}}(\mathbb{Q}, \mathbb{Z})$.
For (d), it might be useful to consider the short exact sequence of $\mathbb{Z}$-modules

$$0 \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{Q}/\mathbb{Z} \to 0$$

and the decomposition of $\mathbb{Q}/\mathbb{Z}$ as a sum over the prime numbers p of the Prüfer groups $\mathbb{Z}/p^{\infty}$.

Exercise 3.2. Repeat the exercise above, replacing Ext^*_R by Tor^R_*.

Exercise 3.3. Explain how the cohomology mod p Bockstein homomorphisms interact with the cup product.