Real algebraic K-theory and trace methods

Auditorium, Maison des Sciences de l'Homme, Aubervilliers

27-31.05.2024

MONDAY

13:00 – 13:45	*Registration*	
13:45 – 14:45	Lior Yanovski:	*Higher descent and cyclotomic redshift in algebraic K-theory.*
15:00 – 16:00	Fabian Hebestreit:	*Karoubi-Grothendieck-Witt spectra.*
16:00 – 16:45	*Coffee Break*	
16:45 – 17:45	Doosung Park:	*Syntomic cohomology and real topological cyclic homology.*
18:00 – 19:00	Jeremy Hahn:	*Algebraic K-theory of Morava K-theory.*
19:00 – 20:30	*Dinner buffet*	

TUESDAY

09:00 – 10:00	Tom Bachmann:	*Unstable real étale homotopy theory.*
10:00 – 10:45	*Coffee break*	
10:45 – 11:45	Markus Land:	*Signatures of local systems via Grothendieck-Witt theory.*
11:45 – 13:30	*Lunch buffet*	
13:45 – 14:45	Dustin Clausen:	*The Galois descent conjecture of Ausoni-Rognes.*
15:00 – 16:00	Maxime Ramzi:	*On the polynomial functoriality of TR and TC.*
16:00 – 16:45	*Coffee Break*	
16:45 – 17:45	Kirsten Wickelgren:	*$\mathbb{A}^1$-chains and logarithmic zeta functions.*
18:00 – 19:00	*Discussion and questions*	

WEDNESDAY

09:00 – 10:00	Iuliia Semikina:	*Cut-and-paste K-theory of manifolds and cobordisms.*
10:15 – 11:15	Jordan Levin:	*Purity in L-theory of rings with involution.*
11:15 – 12:00	*Coffee break*	
12:00 – 13:00	Mura Yakerson:	*Motivic Adams conjecture.*

THURSDAY

09:00 – 10:00	Marco Schlichting:	*On the presentation of the Grothendieck-Witt group of symmetric bilinear forms over local rings.*
10:00 – 10:30	*Coffee break*	
10:30 – 11:30	Foling Zou:	*Equivariant Loday construction.*
11:30 – 12:45	*Lunch break*	
13:00 – 14:00	Matthew Morrow:	*Motivic cohomology of singular schemes.*
14:15 – 15:15	Ran Azouri:	*Enumerating motivic nearby cycles at a singularity.*
15:15 – 16:00	*Coffee Break*	
16:00 – 17:00	Lucy Yang:	*A real Hochschild–Kostant–Rosenberg theorem.*
18:00 – 21:00	*Conference Dinner*	

FRIDAY

08:45 – 09:45	Mona Merling:	*Topological homology of rings with twisted group action.*
09:45 – 10:30	*Coffee break*	
10:30 – 11:30	Marc Levine:	*Non-$\mathbb{A}^1$-invariant motivic homotopy theory, slice filtrations and the BMS filtration.*
11:45 – 12:45	Irakli Patchkoria:	*On the geometric fixed points of the real topological cyclic homology.*

ABSTRACTS

Ran Azouri (Paris 13) : *Enumerating motivic nearby cycles at a singularity.*

We will present motivic enhancements for classical results on nearby cycles. Milnor's work on the Milnor fibre and the Picard-Lefschetz formula on the monodromy, in the setting of complex manifolds, have been adapted by Deligne to algebraic geometry with the formalism of nearby cycles. In the talk we will further adapt those results to $\mathbb{A}^1$-homotpy theory, through which we can deal with various cohomology theories, such as hermitian K-theory. Using the quadratic Euler characteristic we obtain a quadratic version for Milnor's formula, based on a work by Levine-Pepin Lehalleur-Srinivas. Then we compute the motivic mondromy around a singular point, obtaining a Picard-Lefschetz formula for motives, in a work in progress with Emil Jacobsen. Both results use Ayoub's motivic nearby cycles on a semistable reduction, and apply to semi-quasi-homogeneous singularities.

Tom Bachmann (Universität Mainz) : *Unstable real étale homotopy theory.*

I will explain joint work with Asok-Elmanto-Hopkins clarifying the relationship between motivic homotopy theory and semialgebraic topology in an unstable setting: we show that (in an appropriate sense) semialgebraic topology (i.e. the Version of algebraic topology built using a real closed field in place of the real numbers, and real algebraic varieties instead of topological spaces) is obtained from motivic homotopy theory by identifying the two circles S^1 and G_m.

Dustin Clausen (IHES) : *The Galois descent conjecture of Ausoni-Rognes.*

In the context of their redshift program, Ausoni-Rognes predicted that the $T(n+1)$-local K-theory of $K(n)$-local commutative rings should satisfy Galois descent. I will discuss the proof of this conjecture, as well as its surprising connection with other aspects of redshift and chromatic homotopy theory. This talk will be based on joint works with Akhil Mathew, Niko Naumann, and Justin Noel, as well as a joint work with Robert Burklund.

Jeremy Hahn (MIT) : *Algebraic K-theory of Morava K-theory.*

I will discuss a few things that we know, and a few things that we don't know, about the algebraic K-theory of $(2p^n-2)$-periodic Morava K-theory. Perhaps surprisingly, since $K(n)$ is not commutative or even E_2, the methods of syntomic cohomology can be used to make precise calculations. Qualitative consequences include redshift, Lichtenbaum–Quillen, the telescope conjecture, etc., with redshift in particular seeming difficult to establish via simpler techniques. This is joint work in progress with Gabriel Angelini-Knoll and Dylan Wilson.

Fabian Hebestreit (Universität Bielefeld) : *Karoubi-Grothendieck-Witt spectra.*

Joint work with B.Calmès, E.Dotto, Y.Harpaz, K.Moi, M.Land, D.Nardin, T.Nikolaus and W.Steimle.

I will explain an analogue of the fundamental fibre sequence relating connective K-, Grothendieck-Witt- and L-spectra that involves non-connective K-theory and the Karoubi-Grothendieck-Witt-spectra of the title. It is this version of Grothendieck-Witt spectra that satisfy Zariski (even Nisnevich) descent, and as part of the identification of the appropriate version of L-spectra, I will give a unification of the Bass-Heller-Swan and Shaneson splittings in K- and L-theory, respectively, that also allows for the computation of Grothendieck-Witt groups of Laurent polynomial rings.

Markus Land (LMU München) : *Signatures of local systems via Grothendieck-Witt theory.*

I will begin with a brief introduction to the non-multiplicativity of the signature of various types of bundles. I will then explain how to calculate the signatures of local systems of forms over stably framed base manifolds via $GW(\mathbb{Z})$ in its various flavors. For local systems arising from smooth manifold bundles, this recovers divisibility results previously established via index theory.

Jordan Levin (Paris 13) : *Purity in L-theory of rings with involution.*

A number of recent advances in chromatic homotopy theory and K-theory have led to the proof of the famous Redshift Conjecture of Ausoni-Rognes. Among those Land-Mathee-Meier-Tamme have proved various *purity* results for chromatically localized algebraic K-theory, for instance, $L_{T(n)}K(R)$ depends only on $L_n^f(R)$. In this talk, we build on work of Land and the foundational work in Hermitian trace methods of Harpaz-Nikolaus-Shah to deduce a similar property for the L-theory of an arbitrary associative ring spectrum with involution. As a consequence, we deduce a *no redshift theorem* in a number of cases. Finally, we shall discuss under what conditions L-theory might satisfy *whiteshift*, that is, strict preservation of chromatic height.

Marc Levine (Universität Duisburg-Essen) : *Non-$\mathbb{A}^1$-invariant motivic homotopy theory, slice filtrations and the BMS filtration.*

There has been a flurry of activity in constructing non-$\mathbb{A}^1$-invariant extensions of Voevodsky's $\mathbb{A}^1$-invariant motivic stable homotopy category. This is a report on two such constructions, one by Annala-Hoyois-Iwasa, another by Binda-Park-Østvaer. In both constructions, one has representability of (Bass delooped) algebraic K-theory, topological cyclic homology and many other non-$\mathbb{A}^1$-invariant theories currently in use. There is also a version of Voevodsky's slice filtration available, and in some settings, a comparison of this filtration on topological cyclic homology with the BMS filtration. In positive characteristic and assuming resolution of singularities, this is shown by Binda-Park-Østvaer to be an isomorphism.

Mona Merling (University of Pennsylvania) : *Topological homology of rings with twisted group action.*

Topological Hochschild homology, an invariant of ring spectra, is the realization of a cyclic object defined using Connes' cyclic category and carries an action of the circle. Real topological Hochschild homology, an invariant of ring spectra with involution, is the realization of a dihedral object used using the dihedral category and carries an action of $O(2)$. In this talk, we describe a simultaneous generalization of these constructions, a topological version of homology associated with crossed simplicial groups, which are categories that generalize the cyclic and dihedral category. In particular, we will discuss the example of a crossed simplicial group whose associated homology takes as input rings with twisted group action, which generalize rings with involution. This is joint work with Gabriel Angelini-Knoll and Maximilien Péroux.

Mathew Morrow (Université Paris-Saclay) : *Motivic cohomology of singular schemes.*

One output of the motivic homotopy theory of Morel and Voevodsky is a theory of $\mathbb{A}^1$-invariant motivic cohomology of any qcqs scheme (in fact, it yields a priori several candidates for such a theory, but they are shown to coincide in some joint work with Bachmann and Elmanto which I will not have the time to discuss in detail); this is related to the homotopy-invariant K-theory of the scheme via an Atiyah-Hirzebruch spectral sequence. At least in the case of schemes defined over a field, I will explain how one can use trace methods to deform this $\mathbb{A}^1$-invariant motivic cohomology into a new theory of not-necessarily-$\mathbb{A}^1$-invariant motivic cohomology, related to the actual algebraic K-theory of the scheme. Furthermore, some of the deepest properties of K-theory, such as the Weibel vanishing range proved by Kerz-Strunk-Tamme, may be upgraded to the level of this motivic cohomology. We will also see some relations to algebraic cycles on singular varieties, and a new, cohomological proof of an old conjecture of Murthy about K_0. Joint work with Elmanto.

Doosung Park (Bergische Universität Wuppertal) : *Syntomic cohomology and real topological cyclic homology.*

In this talk, I will show that real topological cyclic homology admits a complete exhaustive filtration whose graded pieces are equivariant suspensions of syntomic cohomology. Combined with the announced results of Antieau-Krause-Nikolaus and Harpaz-Nikolaus-Shah, this would lead to the computation of the equivariant slices of the real K-theory of $\mathbb{Z}/p^n$ after a certain suspension. The key ingredients of the proof are a real refinement of the Hochschild-Kostant-Rosenberg filtration and the computation of real topological Hochschild homology of perfectoid rings in my joint work with Hornbostel.

Irakli Patchkoria (University of Aberdeen) : *On the geometric fixed points of the real topological cyclic homology.*

In this talk we will give an overview of recent calculations of geometric fixed points of the real topological cyclic homology (TCR). First we will give a general equaliser formula for calculating it. Then we apply this formula to compute the geometric fixed points of TCR for spherical monoid rings, fields of characteristic 2 and the ring of integers. This is all joint with Dotto and Moi. We will also indicate how these calculations agree with the recent theorem of Harpaz-Nikolaus-Shah which states that the geometric fixed points of TCR agrees with the normal L-theory. At the end we will mention a recent result of Read which says that the geometric fixed points of TCR is not 4-periodic for $\mathbb{Z}/4$.

Maxime Ramzi (University of Copenhagen) : *On the polynomial functoriality of TR and TC.*

I will discuss joint work with Thomas Nikolaus in which we prove that (the underlying spaces of) TR and TC can be made functorial in polynomial functors of stable ∞-categories, mirroring the situation for K-theory.

Marco Schlichting (University of Warwick) : *On the presentation of the Grothendieck-Witt group of symmetric bilinear forms over local rings.*

We prove a Chain Lemma for inner product spaces over commutative local rings R with residue field other than $\mathbb{F}_2$ and use this to show that the usual presentation of the Grothendieck-Witt group of symmetric bilinear forms over R as the zero-th Milnor-Witt K-group holds provided the residue field of R is not $\mathbb{F}_2$. This is joint work with Robert Rogers.

Iuliia Semikina (Université de Lille) : *Cut-and-paste K-theory of manifolds and cobordisms.*

The generalized Hilbert's third problem asks about the invariants preserved under the scissors congruence operation: given a polytope P in $\mathbb{R}^n$, one can cut P into a finite number of smaller polytopes and reassemble these to form Q. Kreck, Neumann and Ossa introduced and studied an analogous notion of cut and paste relation for manifolds called the SK-equivalence ("schneiden und kleben" is German for "cut and paste"). In this talk I will explain the construction that will allow us to speak about the "K-theory of manifolds" spectrum. The zeroth homotopy group of the constructed spectrum recovers the classical groups SK_n. I will show how to relate the spectrum to the algebraic K-theory of integers, and how this leads to the Euler characteristic and the Kervaire semicharacteristic when restricted to the lower homotopy groups. Further I will describe the connection of our spectrum with the cobordism category.

Kirsten Wickelgren (Duke University) : $\mathbb{A}^1$*-chains and logarithmic zeta functions.*

André Weil proposed a beautiful connection between algebraic topology and the number of solutions to equations over finite fields in a celebrated paper from 1948: the zeta function of a variety over a finite field is simultaneously a generating function for the number of solutions to its defining equations and a product of characteristic polynomials of endomorphisms of cohomology groups. The ranks of these cohomology groups are the Betti numbers of the associated complex manifold. We enrich the logarithmic derivative of the zeta function to a power series with coefficients in the Grothendieck–Witt group of stable isomorphism classes of unimodular bilinear forms, using traces of powers of Frobenius in $\mathbb{A}^1$-homotopy theory. Building off of work of Morel–Sawant and Bondarko, we construct a symmetric monoidal chain functor from smooth schemes to bounded complexes of homotopy modules. We show the quadratically enriched logarithmic zeta function to be connected to the Betti numbers of the associated real manifold under restrictions. This is joint work in progress with Tom Bachmann and joint work with Margaret Bilu, Wei Ho, Padma Srinivasan and Isabel Vogt.

Mura Yakerson (Oxford University/IMJ-PRG) : *Motivic Adams conjecture.*

The well-known Adams conjecture in topology is a theorem about compactifications of real vector bundles on CW-complexes, which has important implications for analyzing stable homotopy groups of spheres. In the talk we will discuss an algebro-geometric version of this statement, which tackles algebraic vector bundles on smooth algebraic varieties. This is joint work with Alexey Ananyevskiy, Elden Elmanto and Oliver Röndigs.

Lucy Yang (Columbia University) : *A real Hochschild–Kostant–Rosenberg theorem.*

Grothendieck–Witt and real K-theory are enhancements of K-theory in the presence of duality data. Similarly to ordinary K-theory, real K-theory admits homological approximations, known as real trace theories. We will see how C_2-genuine equivariant algebra is the natural setting for these theories and introduce enhancements of the cotangent and de Rham complexes to this involutive setting. We will identify a filtration on real Hochschild homology and compute the associated graded in terms of the involutive de Rham complex. This work is both inspired by and builds on that of Raksit.

Lior Yanovski (University of Jerusalem) : *Higher descent and cyclotomic redshift in algebraic K-theory.*

Algebraic K-theory is very hard to compute largely because it fails to have good descent properties. Recent groundbreaking works related to chromatic redshift show that good descent properties can be recovered if one localizes the K-theory spectrum at a specific chromatic height. In this talk, I will discuss a work of Ben-Moshe, Carmeli, Schlank, and myself showing that these local pieces of algebraic K-theory satisfy much stronger *higher descent* properties. I will also outline an application of these results to certain Galois (hyper)descent properties in a particular example of importance to the resolution of the celebrated long-standing telescope conjecture by Burklund, Hahn, Levy, and Schlank.

Foling Zou (Chinese Academy od Science) : *Equivariant Loday construction.*

The Loday construction of a commutative ring or ring spectrum on a simplicial set is the tensoring of the commutative ring with the simplicial set. We propose an equivariant version of a Loday construction for G-Tambara functors or commutative G-spectra where G is an arbitrary finite group. We relate our construction to other constructions such as the twisted cyclic nerve and the Real topological Hochschild homology.