SEMINAR ON WHITEHEAD TORSION

Bonn, Winter semester 2006-2007

Consider pairs (K, L) of finite CW complexes for which there exists a strong deformation retraction $K \to L$. The *Whitehead group* $\mathrm{Wh}(L)$ of a finite CW complex L consists of the equivalence classes $[K, L]$ of such pairs under the relation $(K, L) \sim (K', L)$ if there is a *formal deformation* from K to K' relative to L, with the sum defined by $[K, L] + [K', L] = [K \cup_L K', L]$. In fact the Whitehead group $\mathrm{Wh}(L)$ depends only on the fundamental group of the connected components of L, and can also be defined (and sometimes computed) by algebraic means. The *Whitehead torsion* $\tau(f) \in \mathrm{Wh}(L)$ of a (cellular) homotopy equivalence $f : L' \to L$ is zero if and only if f is a *simple-homotopy equivalence*, a notion weaker to f being a homeomorphism.

The aim of this seminar is to follow the book of Cohen [Co] to study simple-homotopy theory, the Whitehead group and the Whitehead torsion in details, ending with applications such as the s-cobordism Theorem and the classification of lens spaces. For practical information please visit the web-page for the seminar (`http://www.math.uni-bonn.de/people/ausoni/seminar-ws06-07.html`).

List of talks

1. Introduction.
Homotopy equivalence; Whitehead's combinatorial approach to homotopy theory; CW-complexes.
[Co, Chapter 1]

2. A geometric approach to homotopy theory, I.
Formal deformations; mapping cylinders and deformations.
[Co, Chapter 2, §4, §5]

3. A geometric approach to homotopy theory, II.
The Whitehead group of a CW complex; simplifying a homotopically trivial CW pair.
[Co, Chapter 2, §6, §7]

4. A geometric approach to homotopy theory, III.
Matrices and formal deformations.
[Co, Chapter 2, §8]

5. The Whitehead group, I.
Algebraic conventions; the groups K_0R and K_1R of a ring; the Whitehead group $\mathrm{Wh}(G)$ of a group.
[Mi, §1, Appendix 2] and [Co, §9, §10]

6. The Whitehead group, II.
Some information about Whitehead groups; complexes with preferred bases.
[Co, §11, §12] and [Mi, §6]

7. Milnor's definition of torsion, I.
Acyclic chain complexes; stable equivalence of acyclic chain complexes; definition of the torsion of an acyclic complex.
[Co, §13, §14, §15]

8. Milnor's definition of torsion, II.
Milnor's definition of torsion; characterization of the torsion of a chain complex; changing rings.
[Co, §16, §17, §18] and [Mi, §3]

9. Whitehead torsion in the CW category, I.
The torsion of a CW pair – definition; fundamental properties.
[Co, §19, §20]

10. Whitehead torsion in the CW category, II.
The natural equivalence of $\mathrm{Wh}(L)$ and $\oplus_j \mathrm{Wh}(\pi_1 L_j)$; the torsion of a homotopy equivalence; product and sum theorems.
[Co, §21, §22, §23]

11. Whitehead torsion in the CW category, III.
The relationship between homotopy and simple-homotopy; invariance of torsion, h-cobordism and the Hauptvermutung.
[Co, §24, §25] and [Hu, Part 2]

12. Lens spaces, I.
Definition of lens spaces; the 3-dimensional spaces $L_{p,q}$; cell structure and homology groups; homotopy classification (1).
[Co, §26, §27, §28, §29.1 – §29.3]

13. Lens spaces, II.
Homotopy classification (2); simple-homotopy equivalence of lens spaces; the complete classification.
[Co, §29.4 – §29.6, §30, §31].

14/15. Additional topics.
Topological invariance of Whitehead torsion; the Wall finiteness obstruction.
[Co, Appendix], [Ro, §1.7].

References

[Co] M. M. Cohen, *A course in simple-homotopy theory*, Graduate Texts in Mathematics, vol. 10, Springer, 1973.

[Hu] J. F. P. Hudson, *Piecewise linear topology*, University of Chicago Lecture Notes prepared with the assistance of J. L. Shaneson and J. Lees, Benjamin, 1969.

[Mi] J. Milnor, *Whitehead torsion*, Bull. Amer. Math. Soc. **72** (1966), 358–426.

[Ro] J. Rosenberg, *Algebraic K-theory and its applications*, Graduate Texts in Mathematics, vol. 147, Springer, 1996 (second edition).

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