Publication List

Georg Biedermann

Publications:

1. "Truncated resolution model structures"
   Biedermann
   Journal of Pure and Applied Algebra, Volume 208, Issue 2, Feb 2007, 591–601

This article contains results from my PhD thesis. It provides technical statements for my next article that are valid in the general context of Bousfield's resolution model structures. The basic observation is that the localization technique by Bousfield/Friedlander can be categorically dualized and yields an easy way to construct notoriously difficult colocalizations (or right Bousfield localizations.)

2. "Interpolation categories for homology theories"
   Biedermann
   Journal of Pure and Applied Algebra, Volume 208, Issue 2, Feb 2007, 497–530

This paper contains the main results from my PhD thesis. Let E be a ring spectrum satisfying some standard technical assumptions. Then E_* is a homological functor from the triangulated E-local stable homotopy category to comodules over the Hopf algebroid (E_*, E_*E). I study the inverse moduli problem for realizing such comodules as spectra. As a conceptual answer I offer interpolation categories over which E_* factors and that correspond to the injective dimension. The result is inspired by Bousfield's insight that the injective dimension of the target category for complex K-theory localized at $p > 2$ is 2. My advisor Franke, and later independently Hovey/Sadofsky, proved that for the Johnson-Wilson spectra $E(n)$ the respective injective dimension is n. At the end of the paper an application to $E(n)$-local Picard groups is given.

3. "Calculus of functors and model categories"
   Biedermann, Boris Chorny, Oliver Röndigs
   Advances in Mathematics, 214, 2007, 92-115

Calculus of homotopy functors is an advanced tool devised by Thomas G. Goodwillie to extract information about unstable homotopy theory from stable knowledge. The Goodwillie tower of a homotopy functor is much like the Taylor tower of a smooth function. It serves both as a computational machinery and as way of conceptually organizing unstable homotopy theory. Goodwillie works entirely in the homotopy category of functors from (pointed) spaces to themselves or spectra. He pointed out that a formulation in terms of Quillen model structures is desirable. In this article we begin to describe such a refinement.

4. "On the homotopy theory of n-types"
   Biedermann *Homology, Homotopy and its Applications*, 10-1 (2008), 305–325

This paper contributes to the homotopy theory of simplicial (pre-)sheaves over a Grothendieck site developed by Jardine, Joyal, Morel, and Voevodsky, among many others. A Postnikov n-type is a space whose homotopy groups vanish above dimension n. It is classical that the homotopy category of 1-types is equivalent to the category of groupoids, that of connected 1-types, ie. $K(\pi_1, 1)$, to the category of groups. Using results from my first paper, I construct the homotopy theory of (pre-)sheaves of n-types. Then I provide a similar truncation for (pre-)sheaves of groupoids together with a Quillen equivalence between both sides. The paper is put into the context of "Local Homotopy Theory", a forthcoming Springer Monographs in Mathematics (Section 5.6) by Jardine, who explains some of my ideas.

5. "Homotopy nilpotent groups"
   Biedermann, William G. Dwyer
   Algebraic & Geometric Topology 10-1 (2010), 33–63

The joint project began with the following observation of mine: Let X be a path connected space and GX be Kan's simplicial loop group construction (modelling ΩX). Let $GX/\Gamma_{n+1}GX$ be universal n-nilpotent quotient, which is part of the lower central series of GX. Then the functor $X \mapsto B(GX/\Gamma_{n+1}GX)$ is n-excisive in Goodwillie's sense. Thus, it receives by universality a natural map from the Goodwillie tower of the identity functor. To better understand the connection we introduce the notion of "homotopy n-nilpotent group". It interpolates between infinite loop spaces ($n = 1$) and loop spaces ($n = \infty$), backwards! This nilpotence degree yields a new invariant for loop spaces that has since been proven to be closely connected with Lusternik-Schnirelmann cocategory by Costoya-Scherer-Viruel.

6. "Calculus of Functors and Model Categories II"
   Biedermann, Oliver Röndigs
   Algebraic & Geometric Topology 14-5 (2014), 2853–2913.

Here we continue our work on the model categorical foundations of Goodwillie's homotopy calculus. We provide in this context a complete description of the Goodwillie tower in the pointed sense. In particular, the relation between the n-homogeneous part and symmetric multilinear functors is carefully studied. Along the way we generalize his setting to functors between pointed simplicial categories satisfying mild technical assumptions.

7. "Duality and small functors"
   Biedermann, Boris Chorny
   Algebraic & Geometric Topology 15-5 (2015), 2607–2655.

Spanier-Whitehead duality is classically stated as a selfduality of the homotopy category of finite spectra. Christensen/Isaksen extend this to a Quillen equivalence between the opposite category of all spectra and the category of prospectra. We propose a different extension. We prove that the Yoneda embedding provides a Quillen equivalence between the opposite category of all spectra and the category of small endofunctors of spectra. As byproduct we exhibit a fibrant-projective model structure on small functors, which yields a model for homotopy functors different from the one considered in the previous paper. Further, we enhance the Bousfield/Friedlander localization technique to one that only assumes functoriality up to homotopy.

Accepted for publication:

8. "The realization space for an unstable coalgebra", monograph, 108 pages
   joint with G. Raptis and M. Stelzer, `arXiv:1409.0410`

Quillen and Sullivan give a complete description of the homotopy category of simply connected rational spaces in terms of simply connected rational differential graded cocommutative coalgebras. An associated deformation theory and moduli spaces of topological realizations of a rational dgcc are constructed in an unpublished work by Schlessinger/Stasheff.

We tackle the same problem for mod p coefficients. We describe a tower of moduli spaces converging to the moduli space of realization of an unstable coalgebra. Its fibers are completely described in terms André-Quillen cohomology. As a consequence we unify, generalize and sharpen obstruction theories for realizations obtained by Harper, Bousfield and Blanc since the 1970s.

Preprints:

9. "A generalized Blakers-Massey theorem"
   M. Anel, Biedermann, E. Finster, A. Joyal

The classical Blakers-Massey theorem (or sometimes called homotopy excision) concerns a (homotopy) pushout of topological spaces

$$\begin{tikzcd} A \arrow[r, "g"] \arrow[d, "f"'] & C \arrow[d] \\ B \arrow[r] & D. \end{tikzcd}$$

It says that if f is m-connected and g is n-connected then the cartesian gap map

$$(f, g) : A \to B \times^h_D C$$

is $(m + n - 1)$-connected. We generalize the notion of connectivity and the setting; our theorem holds in arbitrary ∞-topoi. We observe that n-connected maps form the left class of a unique homotopy factorization system and are closed under (homotopy) base change. We call this data a modality. Modalities generalize fiberwise localization and acyclic classes in the sense of Dror-Farjoun. To each modality on an ∞-topos we associate a Blakers-Massey theorem. Using the special case of the n-connected modalities we rederive the classical theorem and a recent generalization by Chacholski-Scherer-Werndli without much difficulty. Our arguments use only a tiny amount of homotopical machinery. Another result describes the behaviour of the cocartesian gap map of (homotopy) pullback. Further applications are given in the following paper.

10. "A Blakers-Massey theorem for the Goodwillie tower"
    M. Anel, Biedermann, E. Finster, A. Joyal

In this second article we observe that a left exact localization yields a modality. In particular, for each $n \geq 0$ Goodwillie's n-excisive approximation P_n gives the n-excisive modality whose left class consist of those natural transformations that become objectwise equivalences after applying P_n. We prove using the (derived) pushout product $\square$ that

$$P_m\text{-equivalences} \square P_n\text{-equivalences} \subset P_{m+n+1}\text{-equivalences}.$$

This theorem is our main new contribution to Goodwillie calculus. With it and our generalized Blakers-Massey theorem from the previous paper we can prove a conjecture by Goodwillie: in a homotopy pushout of functors

$$\begin{tikzcd} F \arrow[r, "\beta"] \arrow[d, "\alpha"'] & H \arrow[d] \\ G \arrow[r] & K, \end{tikzcd}$$

if α is a P_m-equivalence and g is a P_n-equivalence, then the cartesian gap map

$$(\alpha, \beta) : F \to G \times^h_K H$$

is a P_{m+n+1}-equivalence. Several known delooping theorems in the context of functor calculus follow including Goodwillie's theorem that homogeneous functors deloop infinitely.

In Preparation:

11. "Homotopy nilpotent groups and their associated functors"
    Biedermann

We continue to study homotopy nilpotent groups. To each such object we associate an n-excisive functor. The category of homotopy n-nilpotent groups admits an adjunction with the category of loopes n-excisive functors. In fact, the former becomes a retract of the latter. We compute the derivatives of the functors that we associate to homotopy nilpotent groups in terms of their first derivative.

12. "A new construction of the unstable Adams spectral sequecne"
    joint with G. Raptis and M. Stelzer

We give a new construction of Bousfield's unstable Adams spectral sequence for unpointed spaces. We initiate a deeper investigation of its negative part ($t - s \leq 0$). This relates closely to our obstruction theory for realizing unstable coalgbras over the Steenrod algebra and the associated moduli problem in our monograph above.