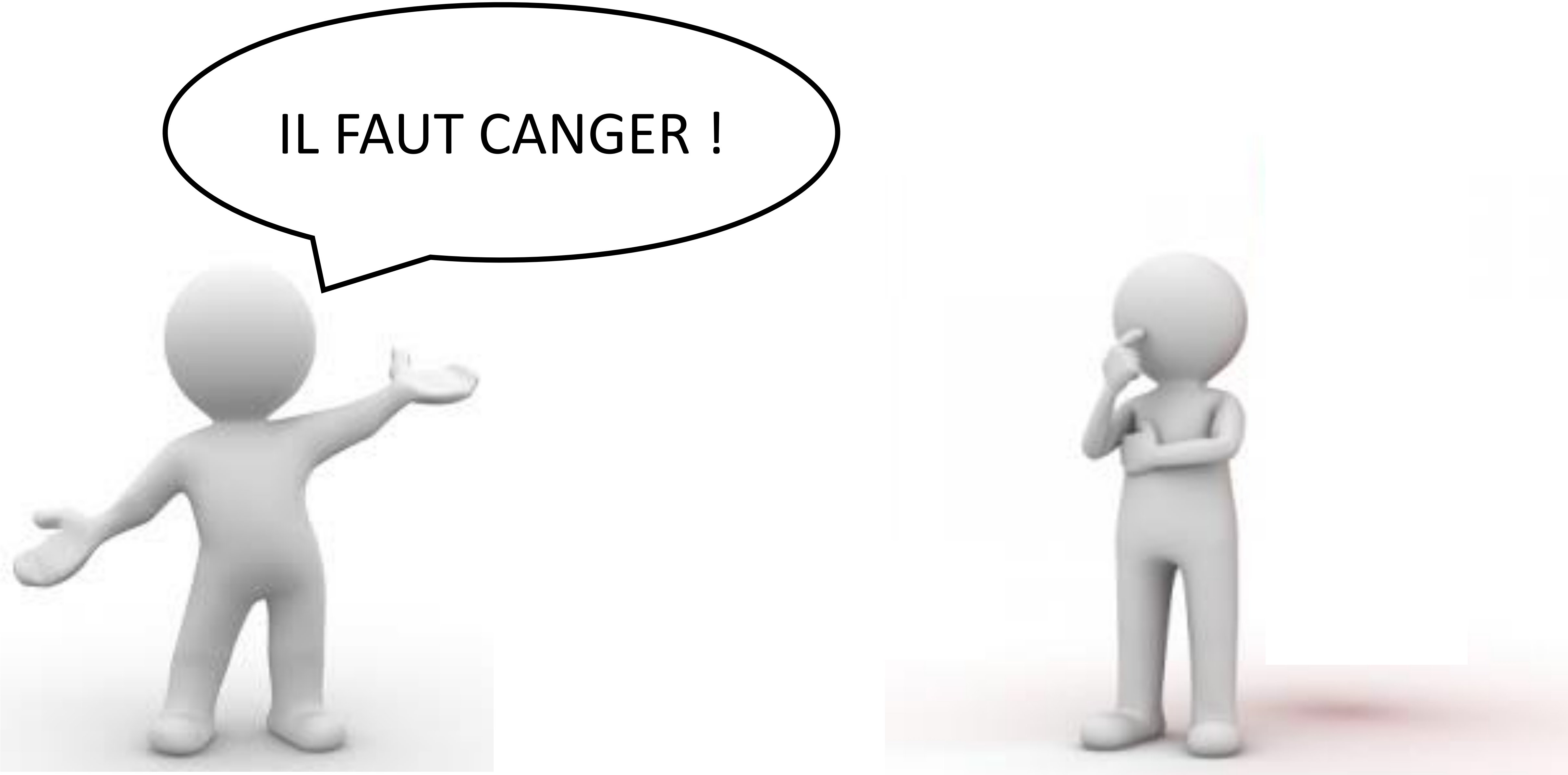
L'erreur dans la transmission d'informations numériques, comment la corriger ?

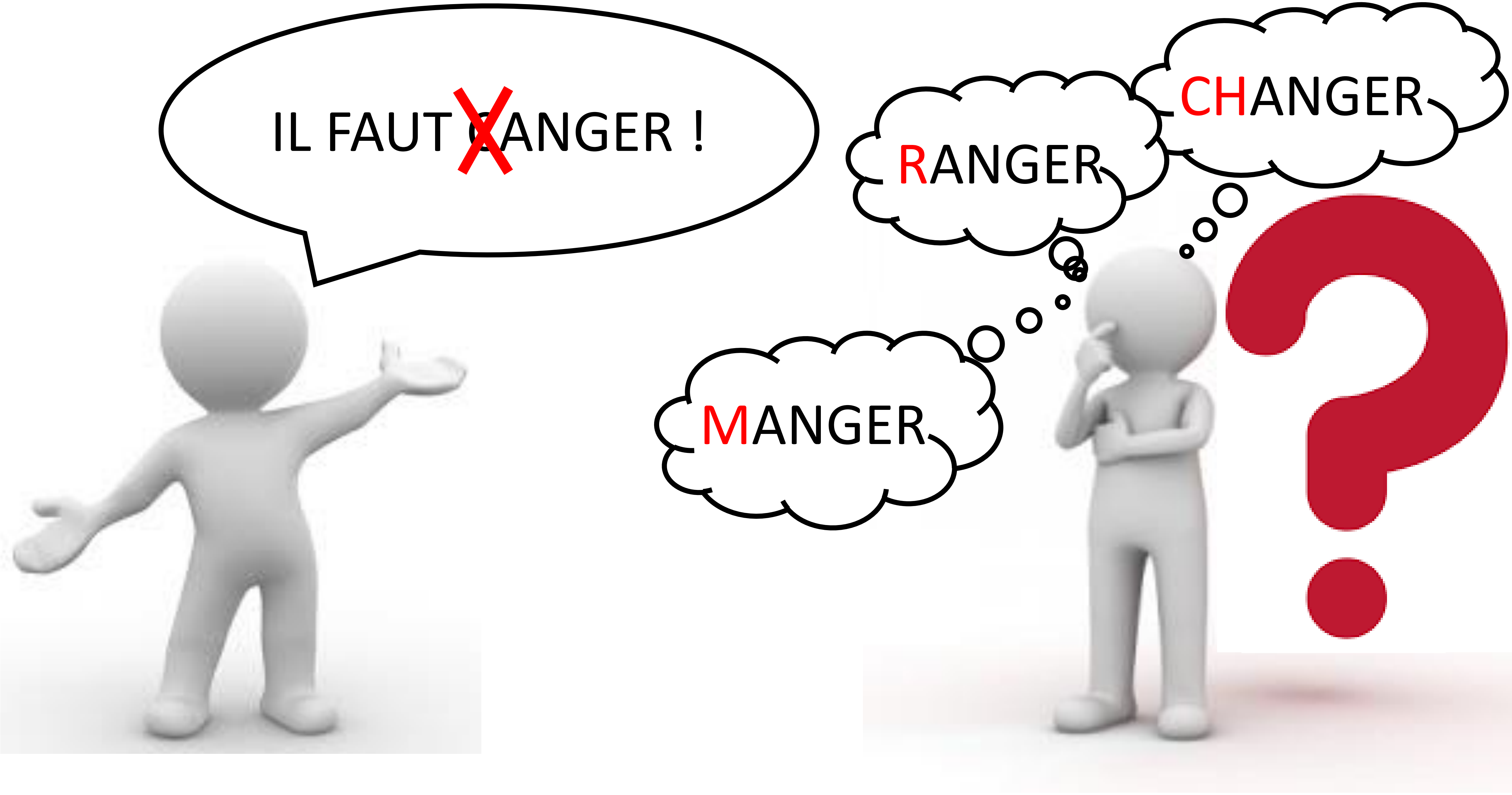
Martino Borello

(Paris 8 - LAGA)

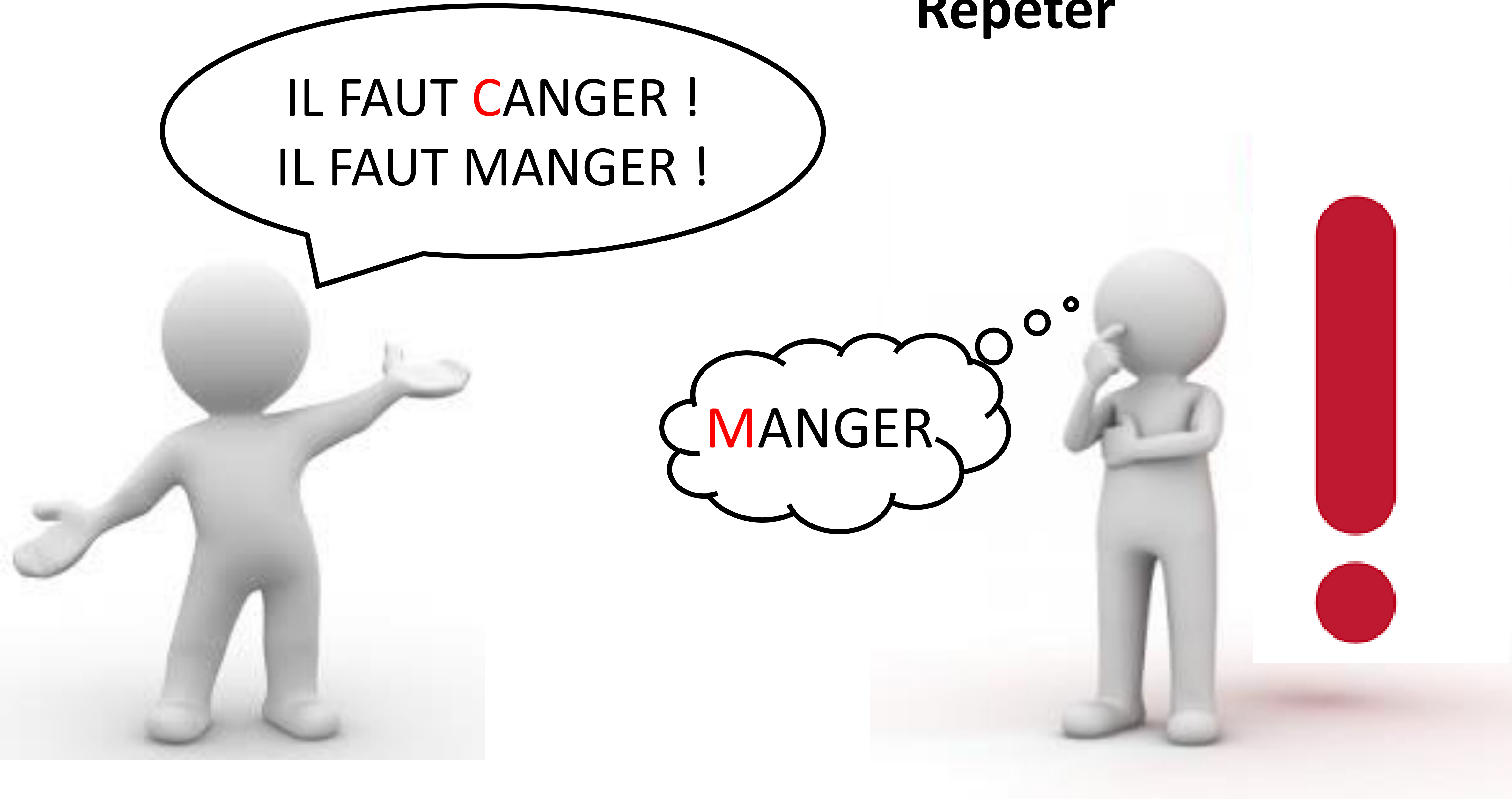
20/10/2017

UNIVERSITÉ PARIS 8 VINCENNES-SAINT-DENIS


IL FAUT CANGER !

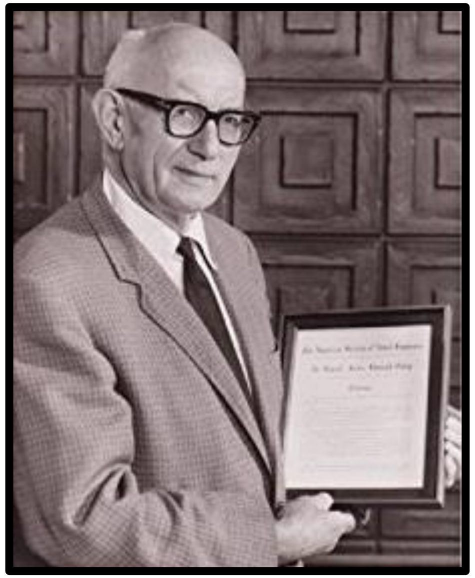

IL FAUT CANGER !
RANGER
CHANGER
MANGER

Répéter

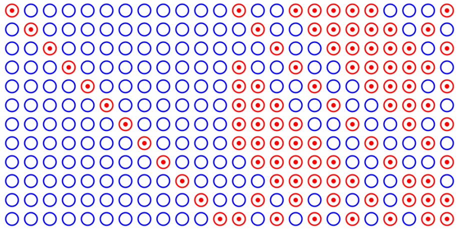


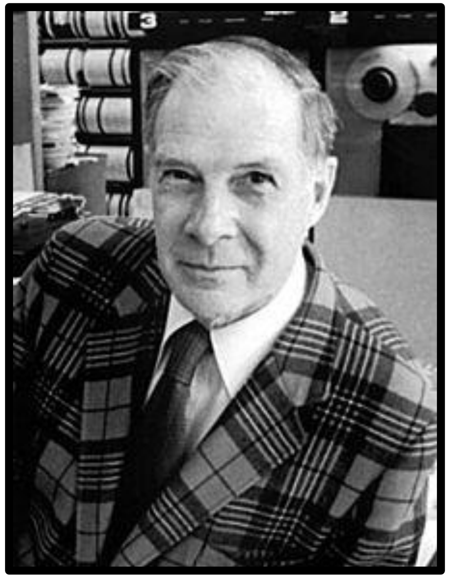
Ajouter des informations

Les pères fondateurs

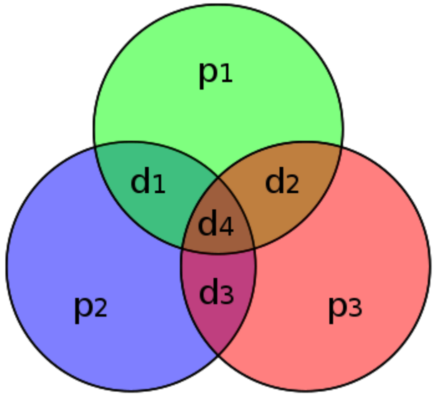


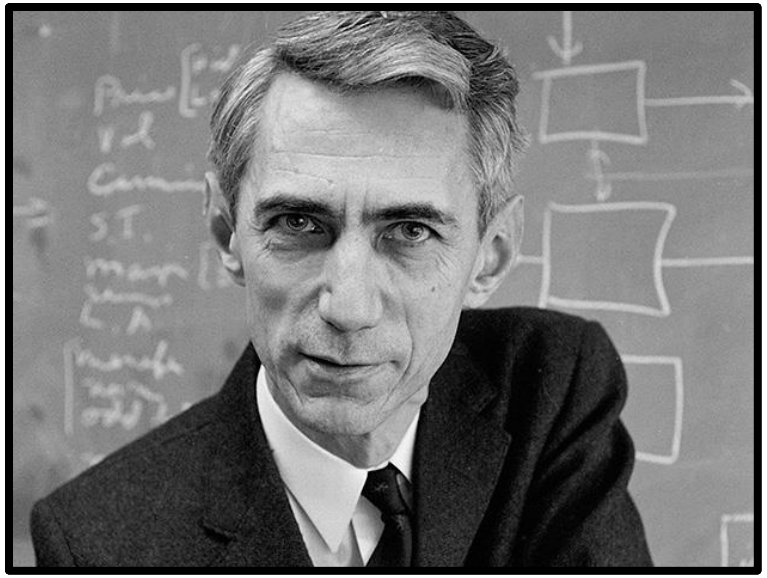
Marcel J. E. Golay
1902-1989
Suisse



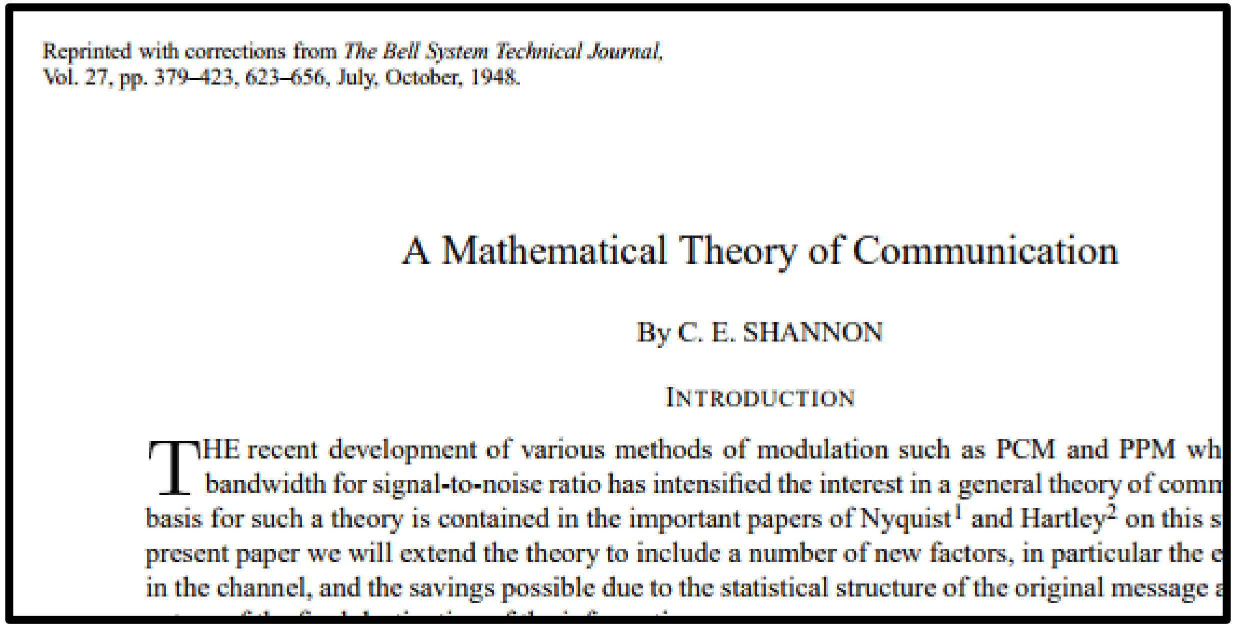


Richard W. Hamming
1915-1998
États-Unis





Claude E. Shannon
1916-2001
États-Unis



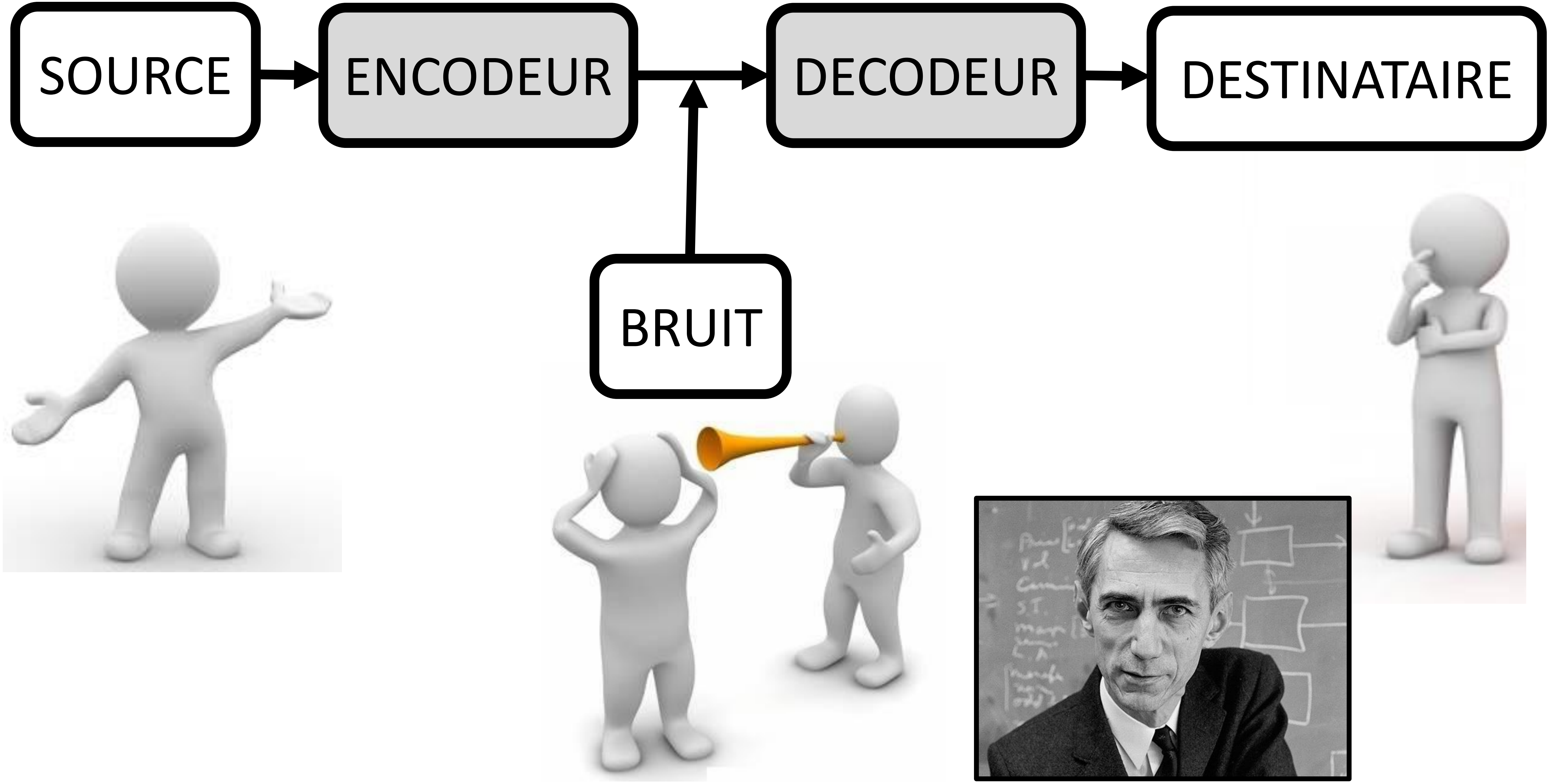
Reprinted with corrections from *The Bell System Technical Journal*,
Vol. 27, pp. 379–423, 623–656, July, October, 1948.

A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM wh
bandwidth for signal-to-noise ratio has intensified the interest in a general theory of comm
basis for such a theory is contained in the important papers of Nyquist[1] and Hartley[2] on this s
present paper we will extend the theory to include a number of new factors, in particular the e
in the channel, and the savings possible due to the statistical structure of the original message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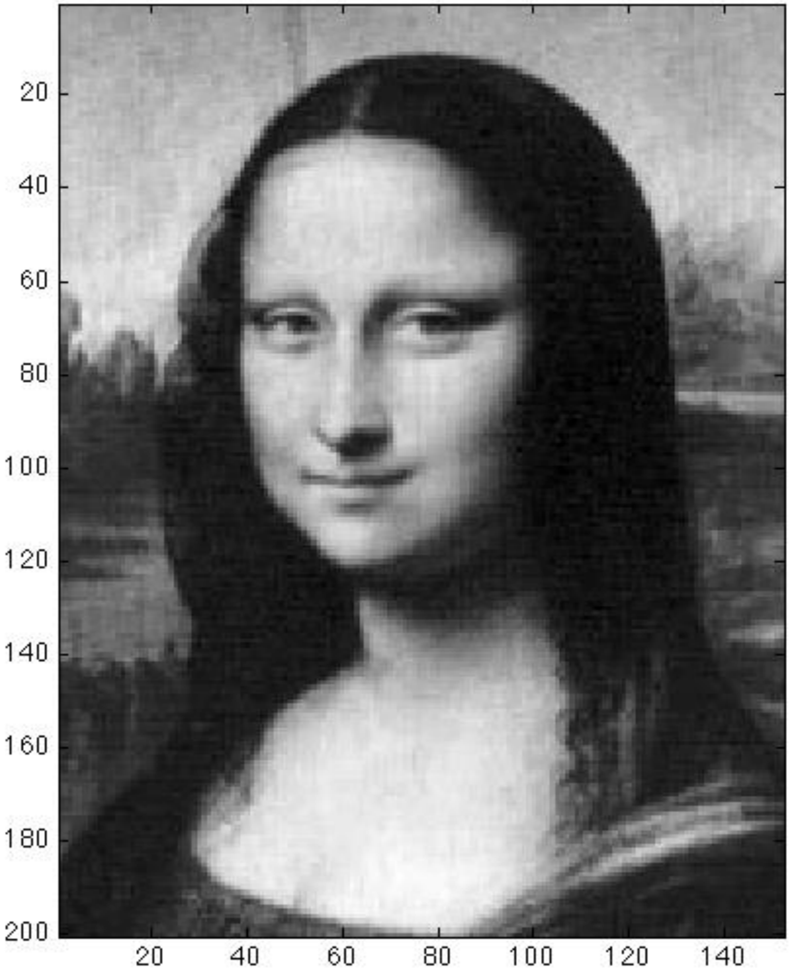
BRUIT

Image transmise de la terre



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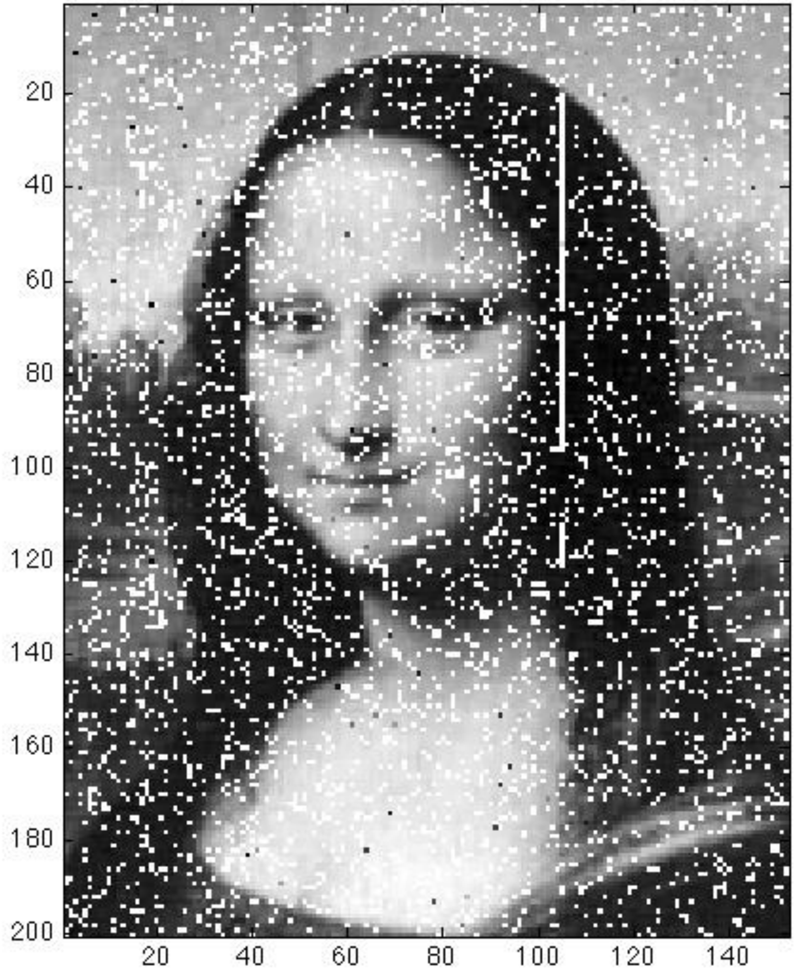
NASA's Lunar Reconnaissance Orbiter (LRO)

Image reçue (sans code)



BRUIT

NASA's Lunar Reconnaissance Orbiter (LRO)

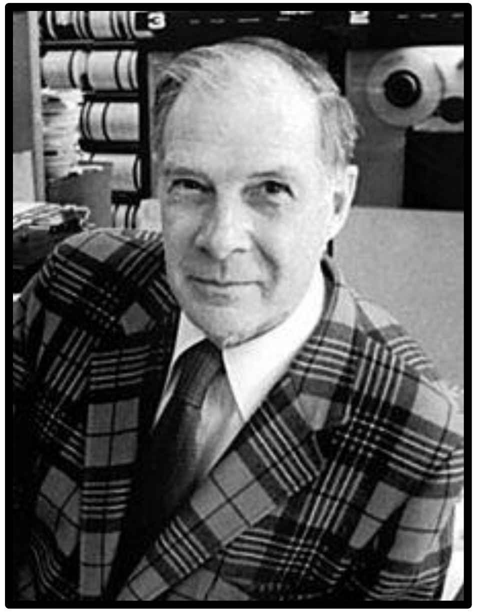
$$2 = (2 \times 1 + 9 \times 2 + 4 \times 3 + 0 \times 4 + 1 \times 5 + 9 \times 6 + 9 \times 7 + 6 \times 8 + 1 \times 9) \bmod 11$$
$$= 211 \bmod 11 = (19 \times 11 + 2) \bmod 11$$

Détecter

Ordinateur **modèle V** :

- Il occupait environ 100 mètres carrés ;
- Il pesait 10 tonnes ;
- Il pouvait résoudre des systèmes de 13 équations linéaires en moins de 4 heures ;
- chaque fois que l'ordinateur découvrait une erreur dans l'entrée, il abandonnait simplement le travail et prenait le suivant.

Deux week-ends de suite, je suis arrivé et j'ai trouvé qu’une erreur était survenue et rien n’avait été fait. J’était vraiment en colère et agacé parce que je voulais ces réponses et deux week-ends avaient été perdus. Et alors j'ai dit "Bon sang, si la machine peut <u>détecter</u> une erreur, pourquoi ne peut-elle pas <u>localiser la position</u> de l'erreur et la <u>corriger</u> ?"

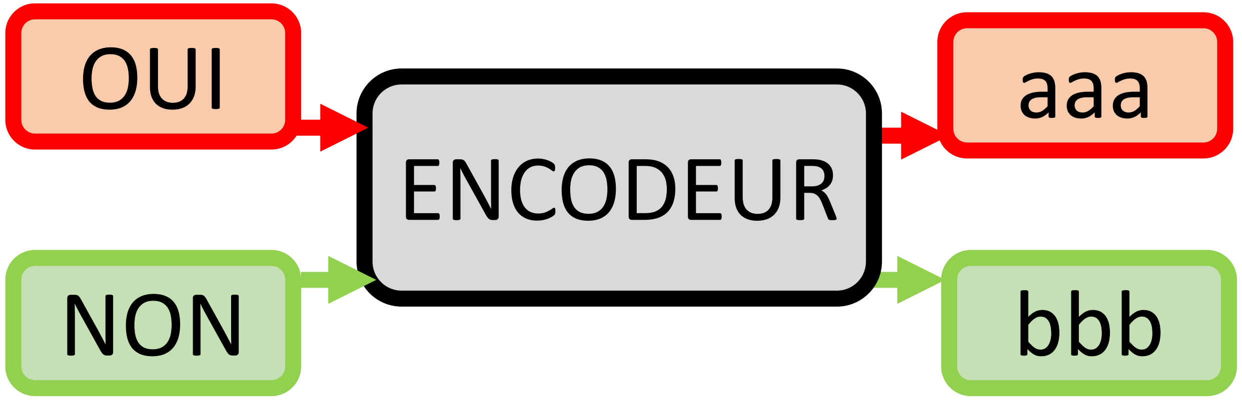


ENCODEUR

- A **alphabet**
- n **longueur**
- $C \subseteq A^n$ **code** (en blocs)
- $c \in C$ **mot** du code

ENCODEUR

- $A = \{ a, b \}$
- n = 3
- $A^3 = \{ aaa, aab, aba, abb, baa, bab, bba, bbb \}$
- $C = \{ aaa, bbb \}$



- $C = \{ aaa, bbb \}$

ENCODEUR

- pour $c, c' \in C$,

  $d(c, c') = \#\{\, i \mid c_i \neq c'_i \,\}$

  (**distance de Hamming**)

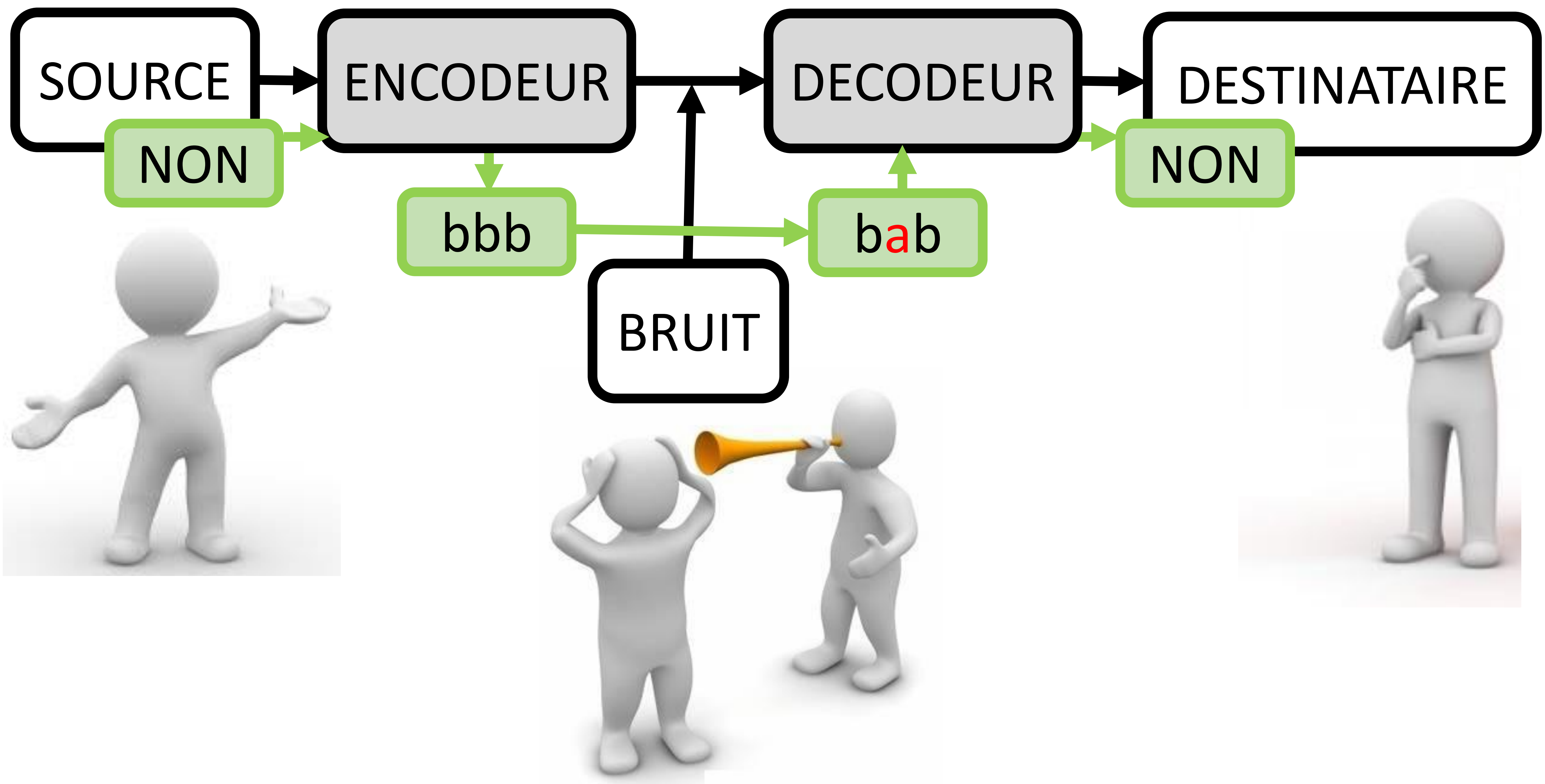
- **Distance minimale.**

ENCODEUR

- pour $c = aaa$ et $c' = bbb$

  $d(c, c') = 3$

- Distance minimale 3.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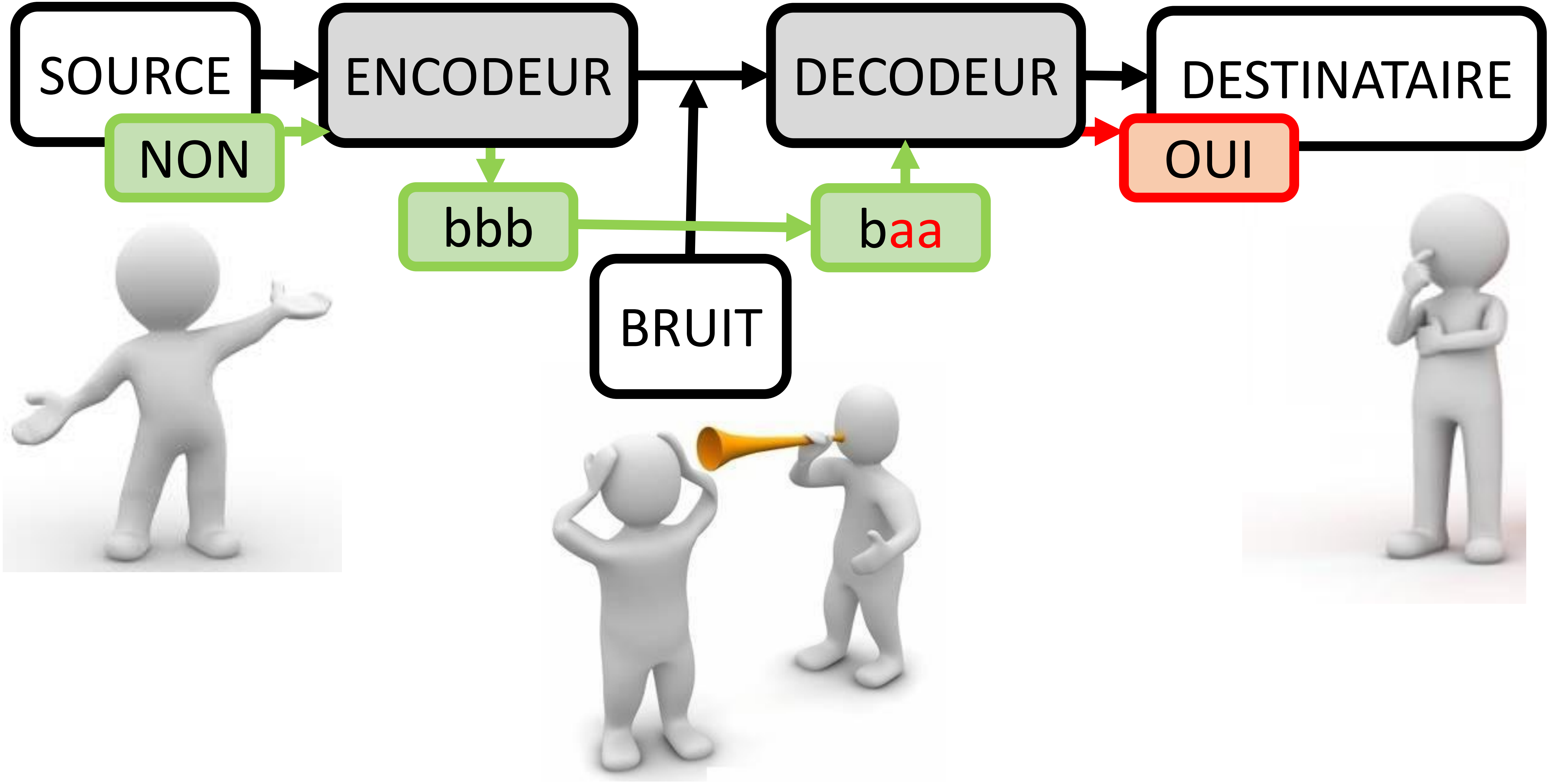
bab
NON


SOURCE
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DESTINATAIRE
NON
bbb
baa
OUI
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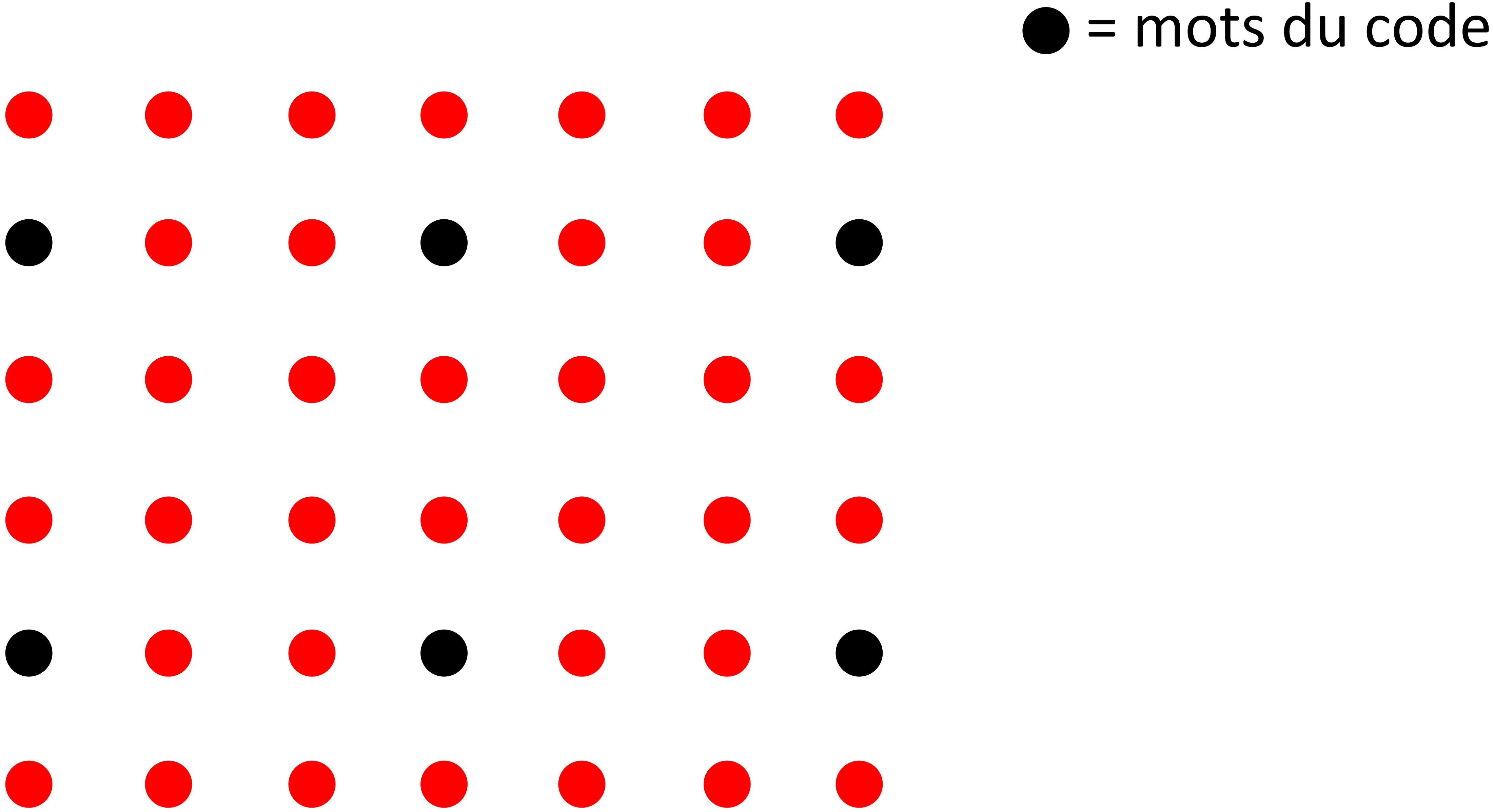
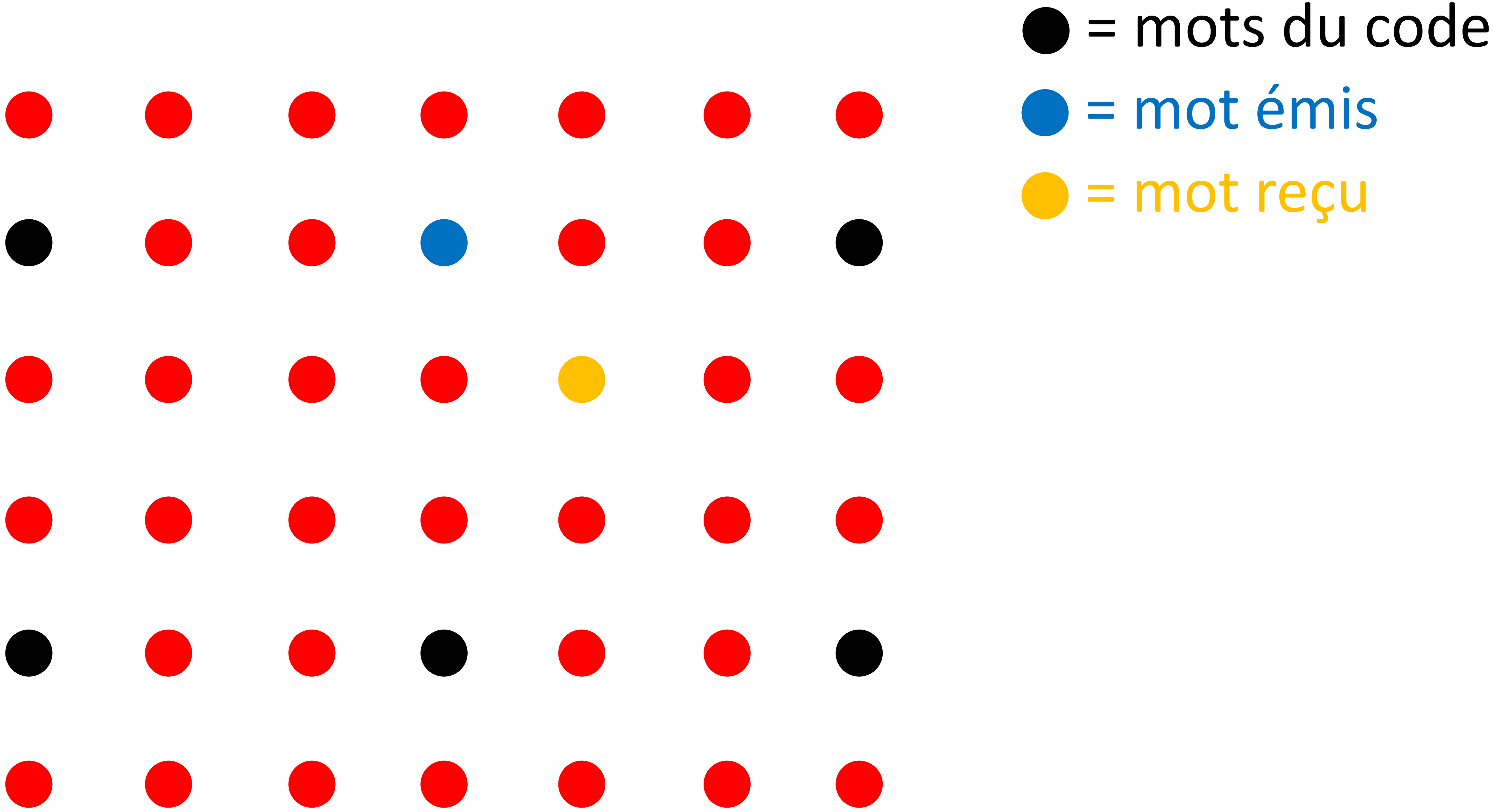
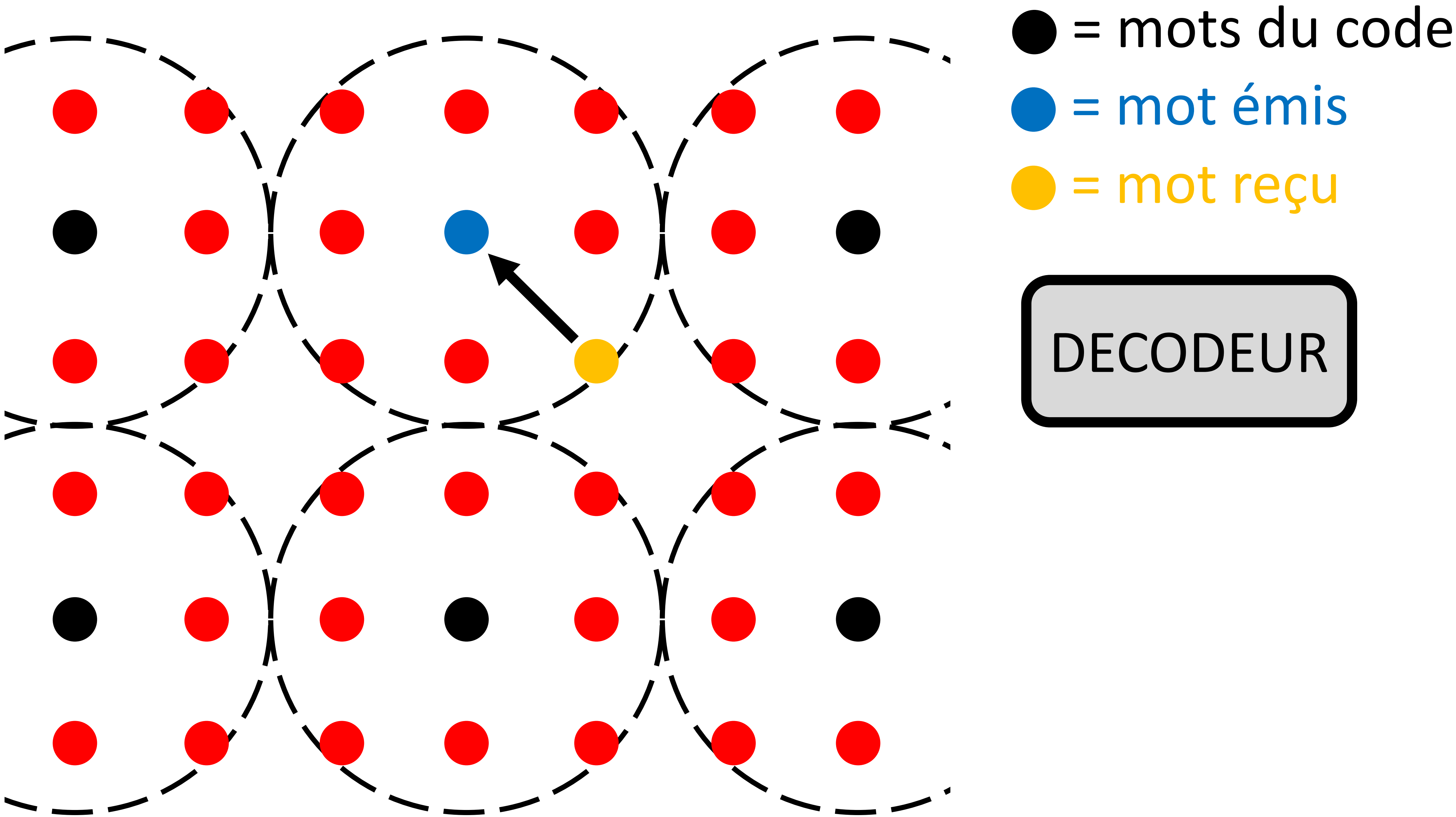

= mots du code


= mots du code
= mot émis
= mot reçu


= mots du code
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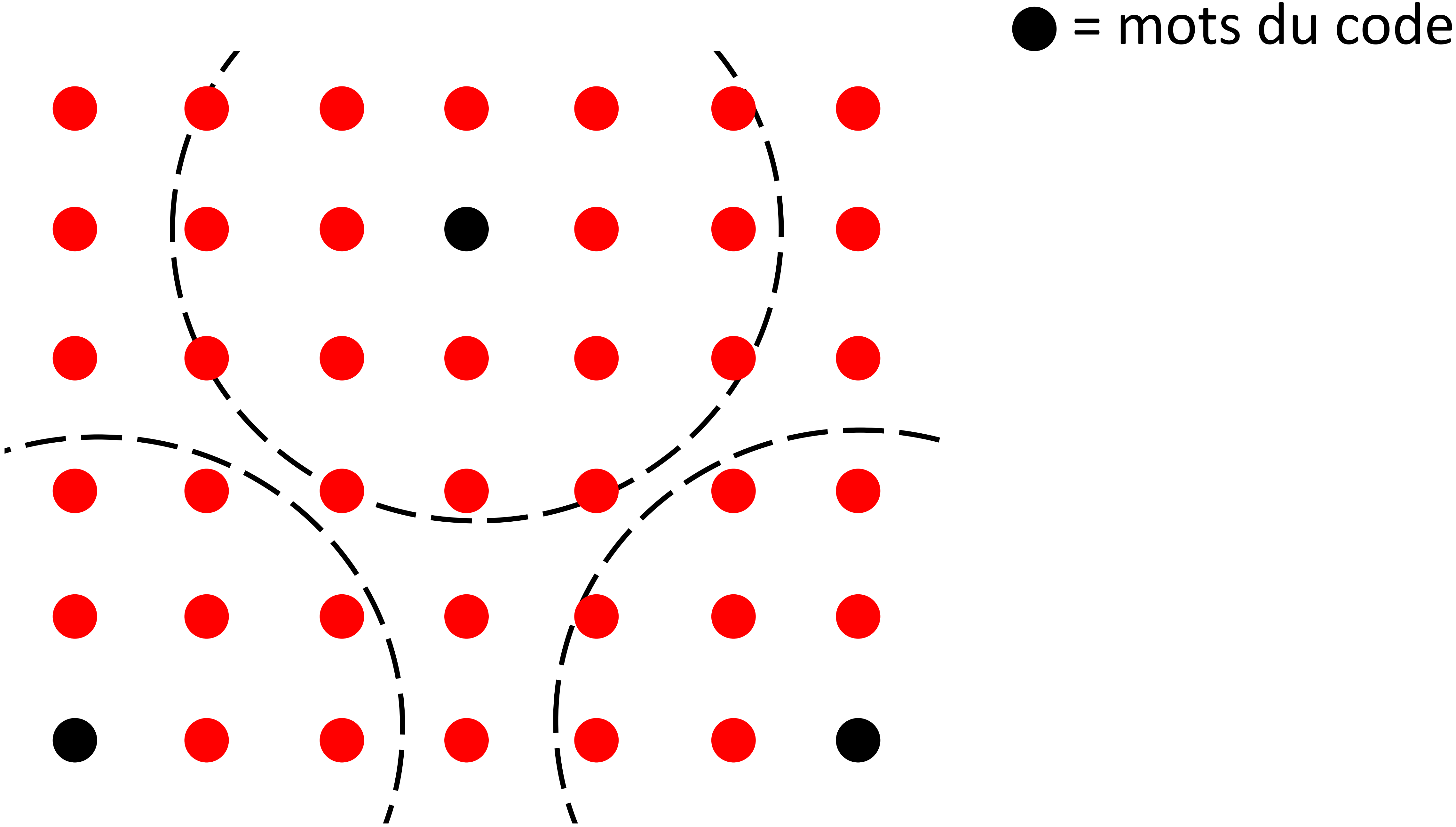
DECODEUR


= mots du code

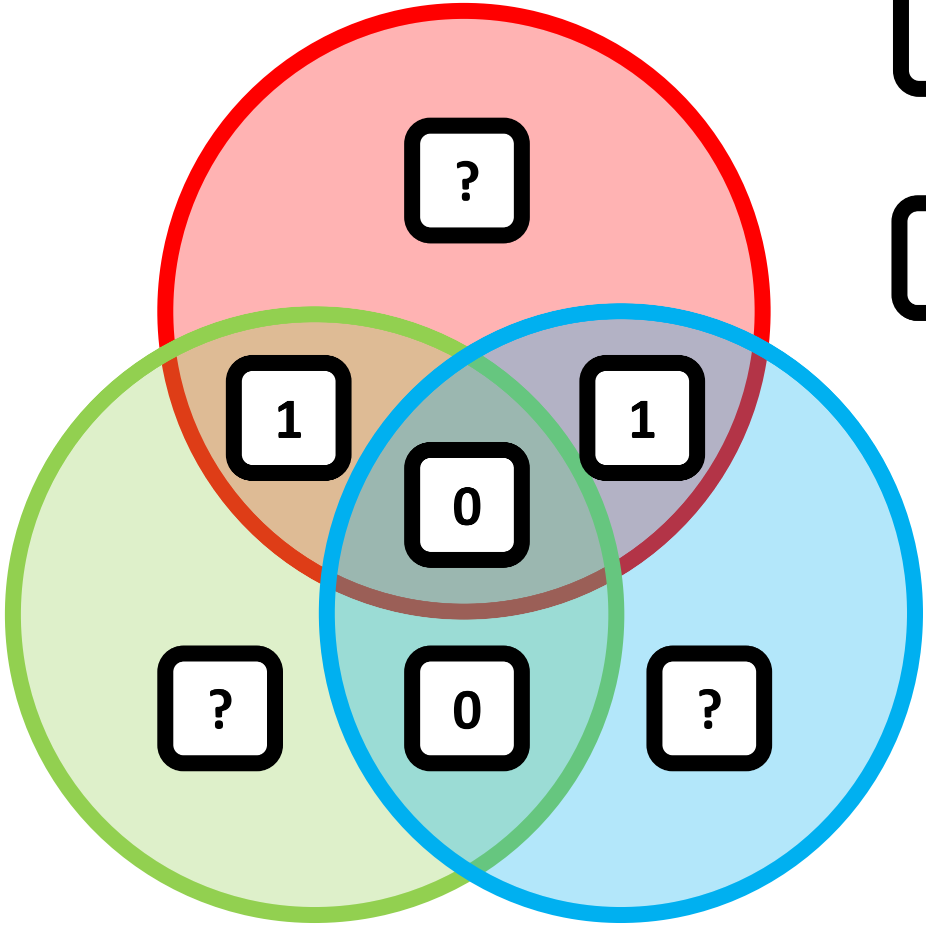
- $A = \{ a , b \}$
- $C \subseteq A^n$ capable de corriger 1 erreur

$$\#C \leq \frac{2^n}{n+1}$$

- $A = \{ a , b \}$
- $C \subseteq A^7$ capable de corriger 1 erreur

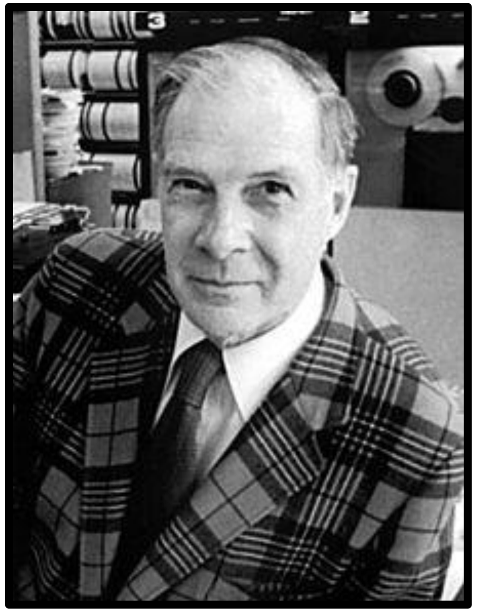
$$\#C \leq \frac{2^7}{7+1} = 16$$

- A = {0, 1}
- n = 7

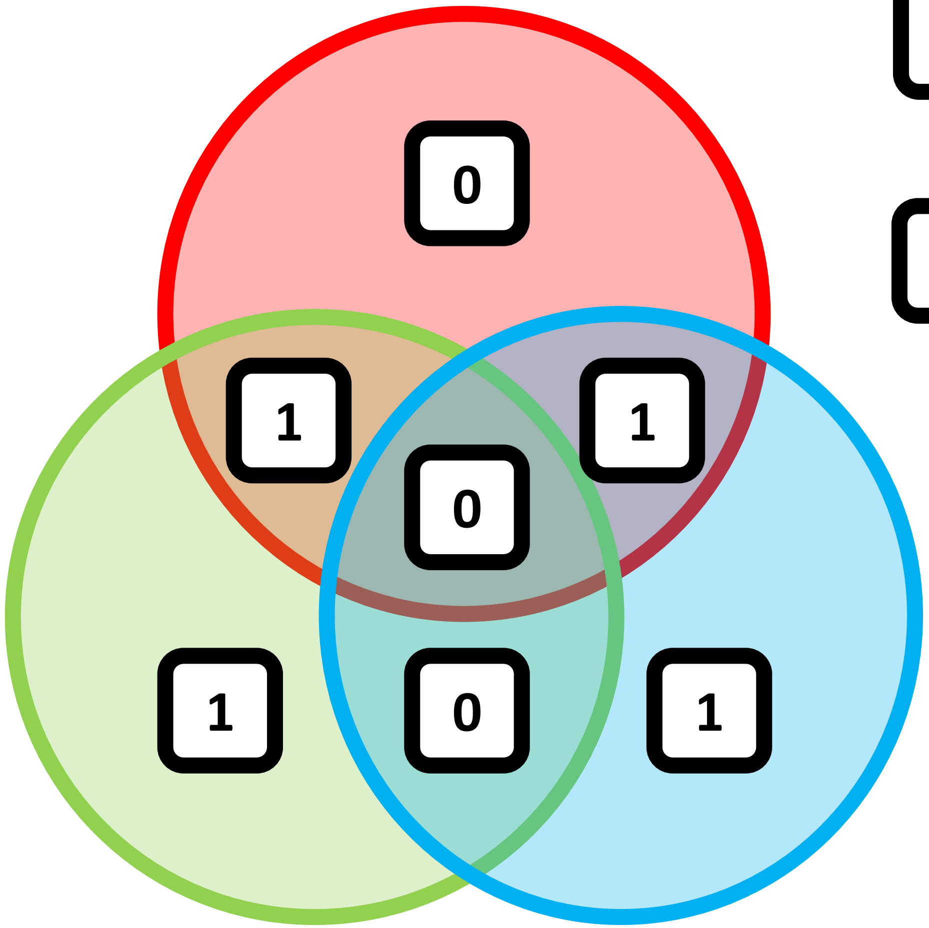


? = 0 si nombre pair de 1 dans le cercle

? = 1 autrement



- $A = \{$ 0 , 1 $\}$
- $n = 7$



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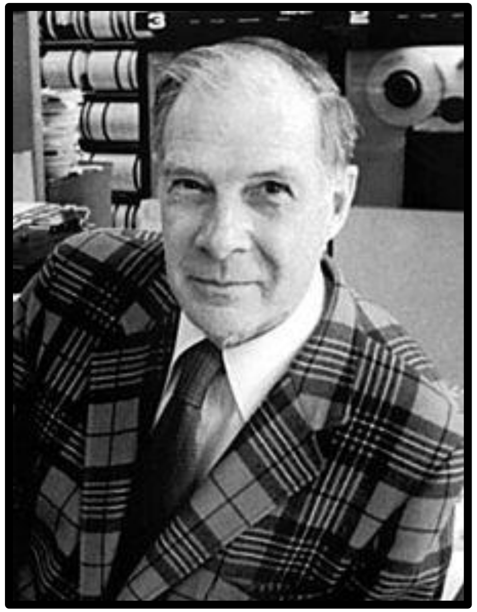

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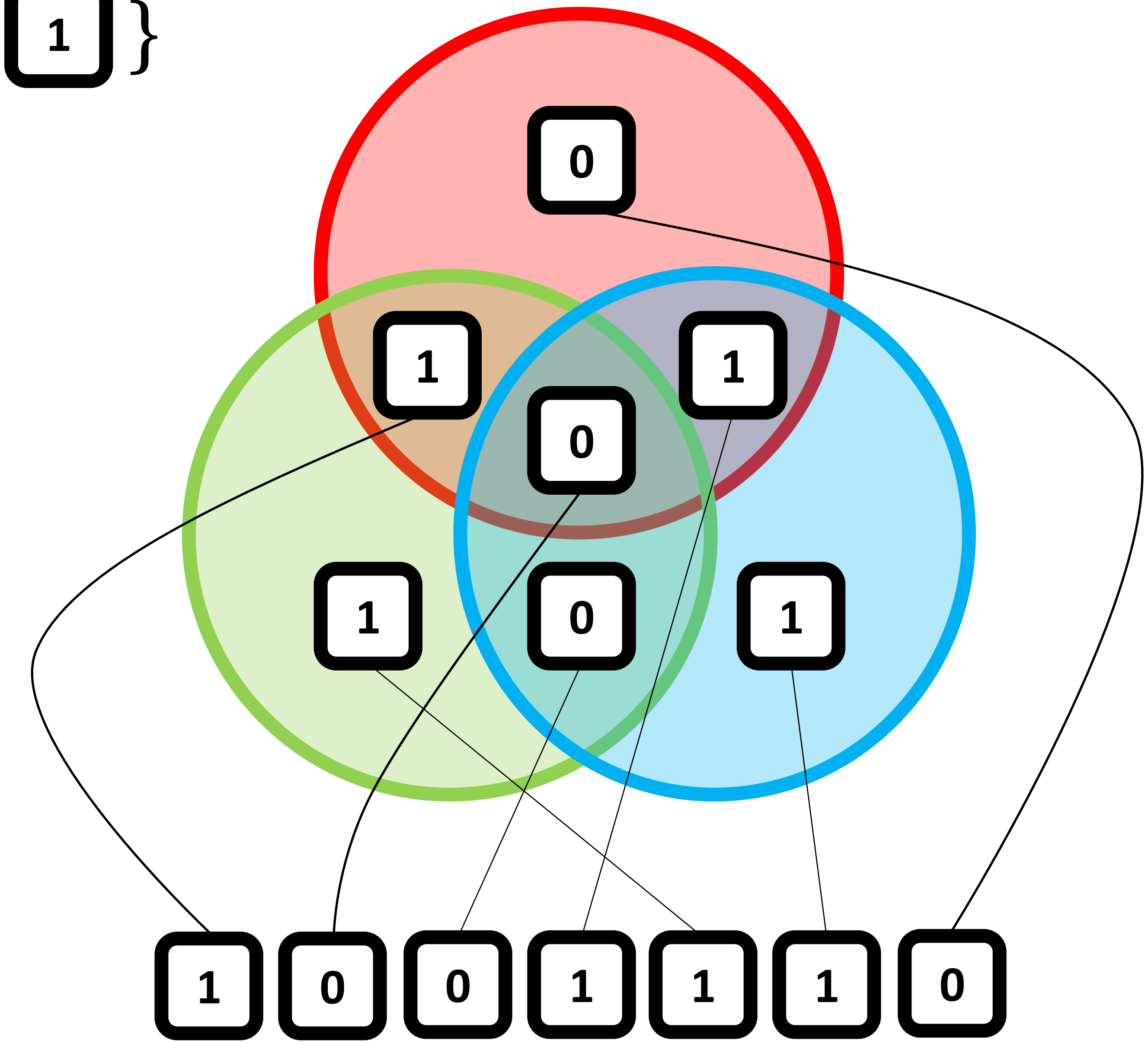

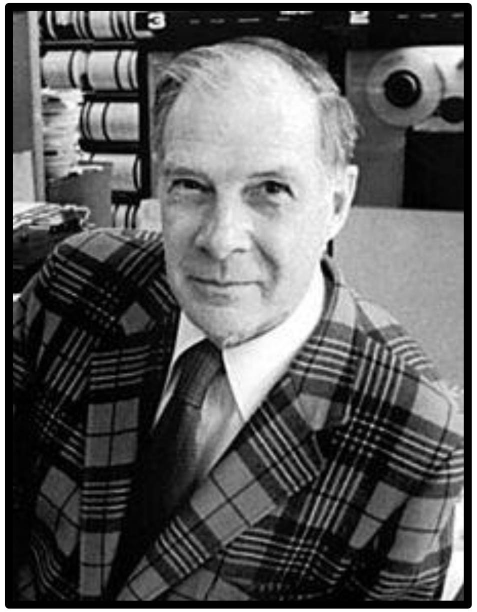

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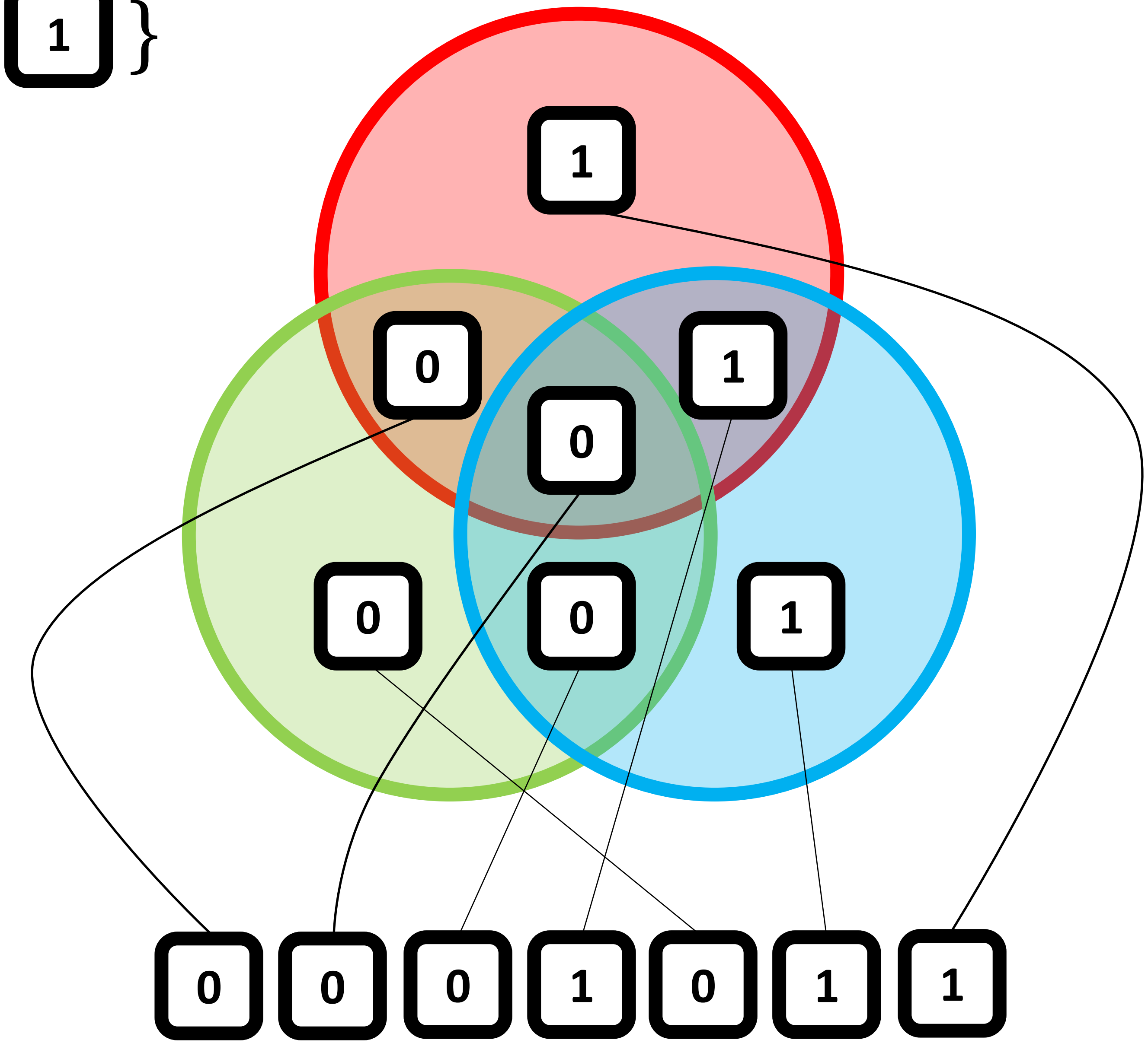

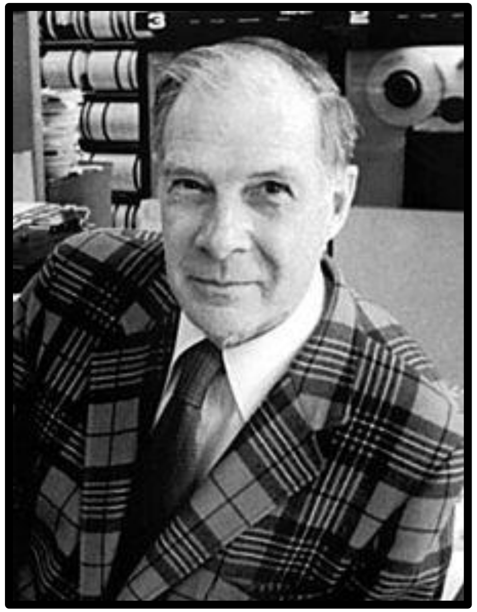

0	0	0	0	0	0	0		1	0	0	0	1	0	1
0	0	0	1	0	1	1		1	0	0	1	1	1	0
0	0	1	0	1	1	0		1	0	1	0	0	1	1
0	0	1	1	1	0	1		1	0	1	1	0	0	0
0	1	0	0	1	1	1		1	1	0	0	0	1	0
0	1	0	1	1	0	0		1	1	0	1	0	0	1
0	1	1	0	0	0	1		1	1	1	0	1	0	0
0	1	1	1	0	1	0		1	1	1	1	1	1	1

- $A = \{0, 1\}$
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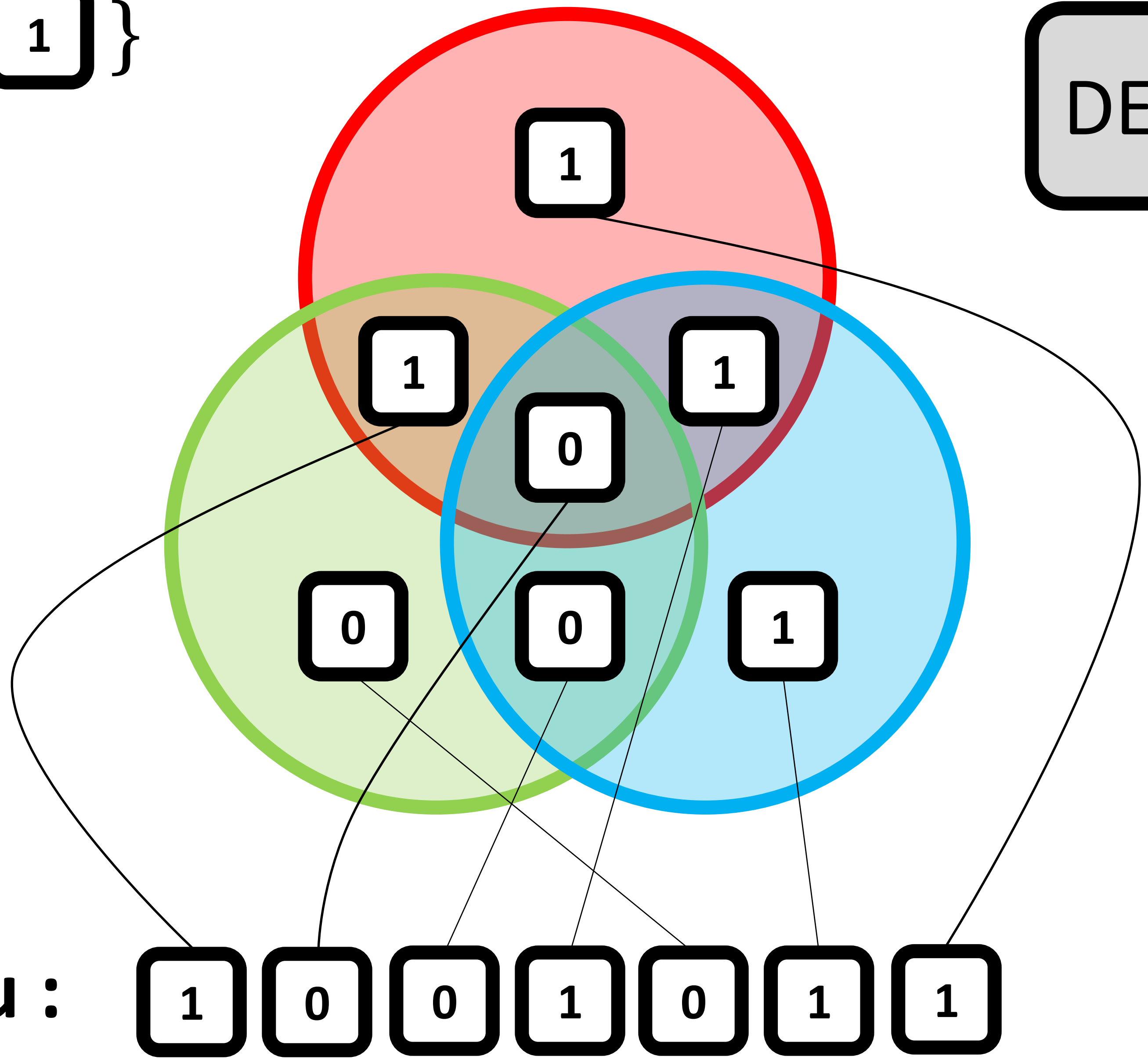
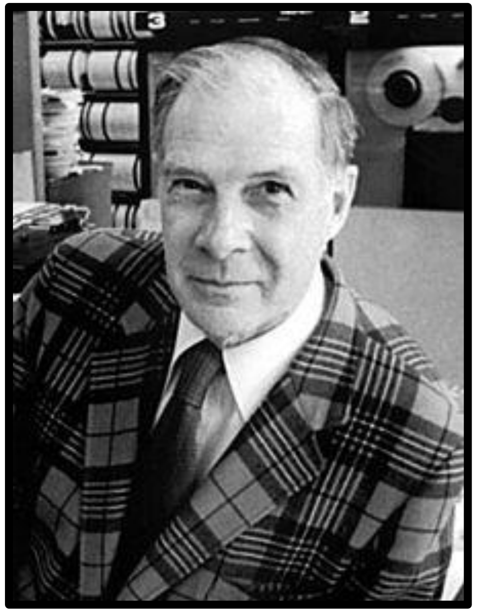
- $\# C = 16$

DECODEUR



Mot reçu :



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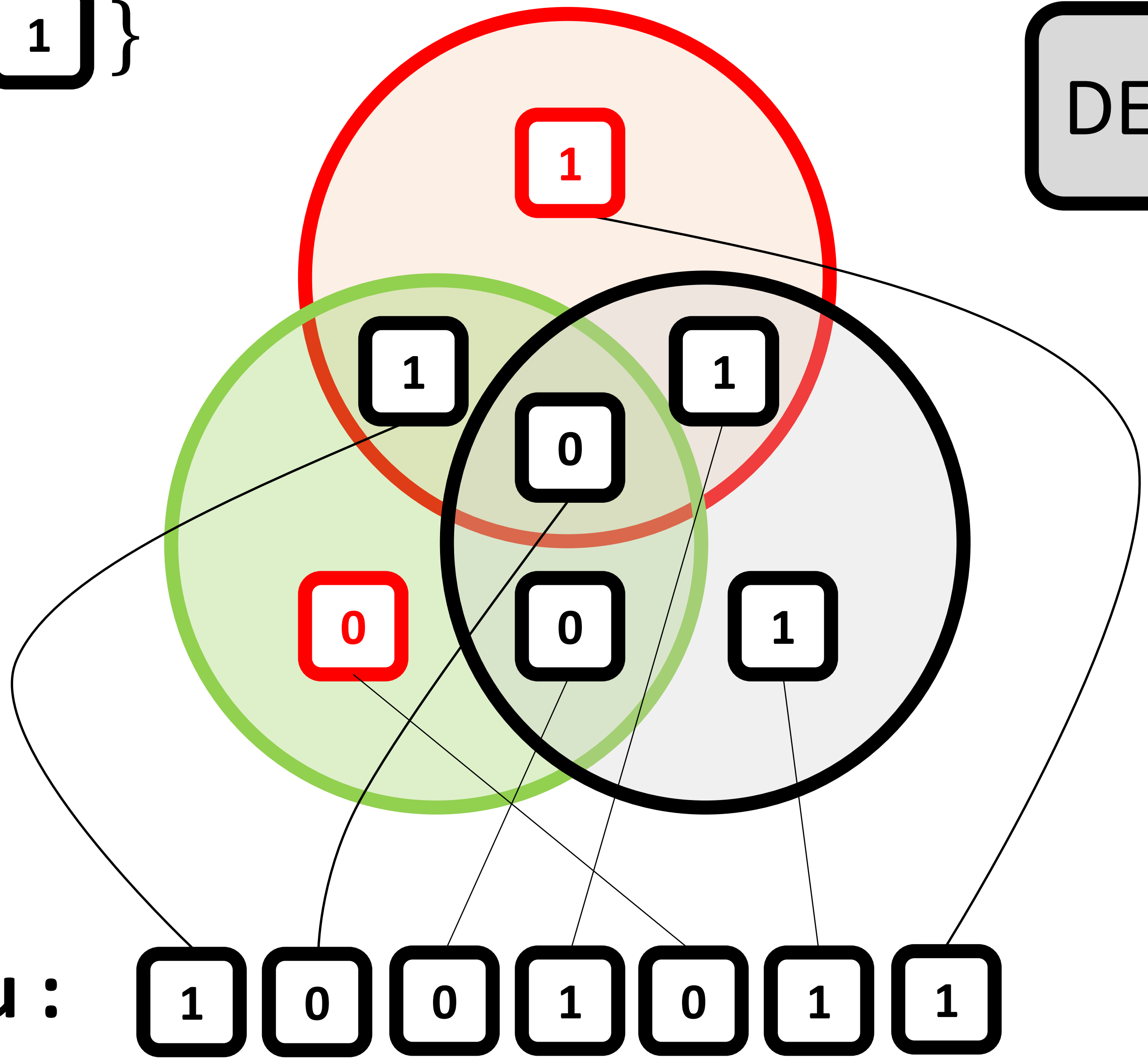
DECODEUR



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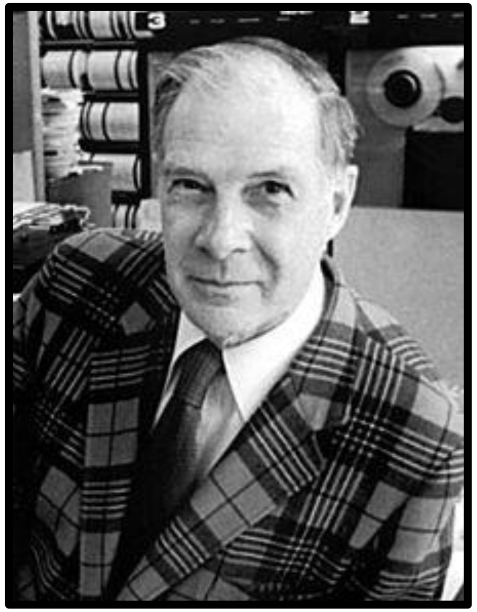

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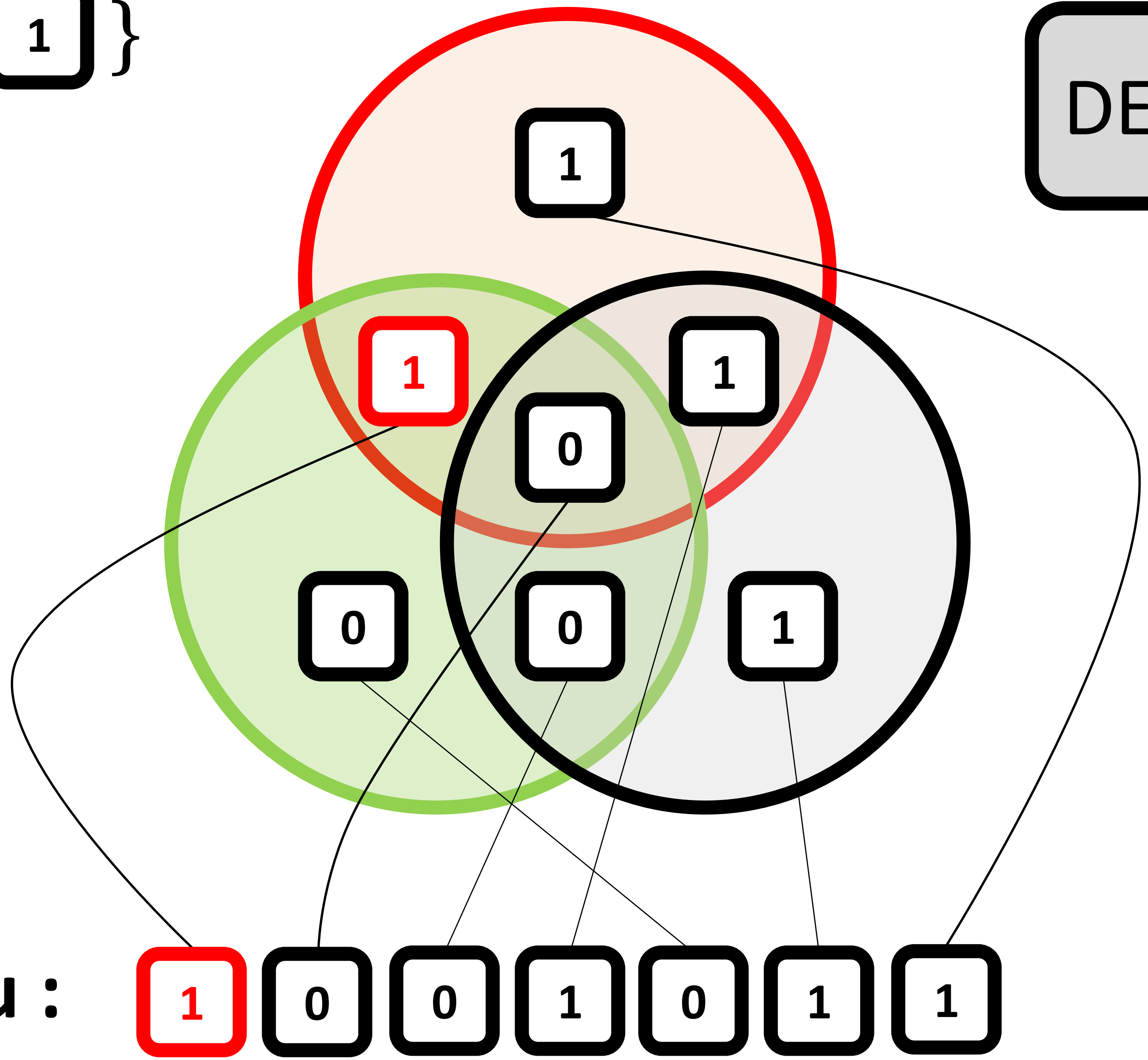

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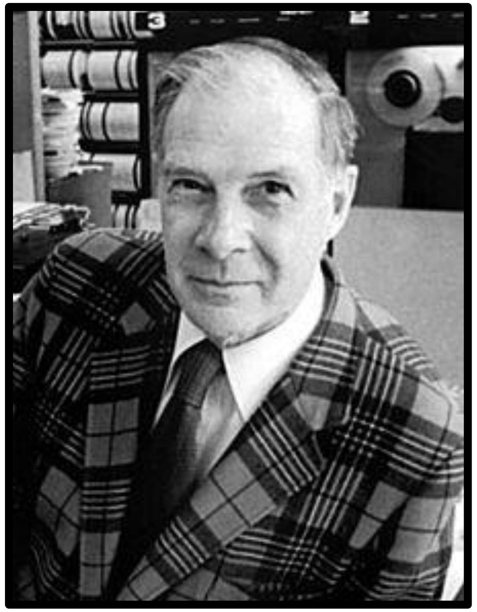

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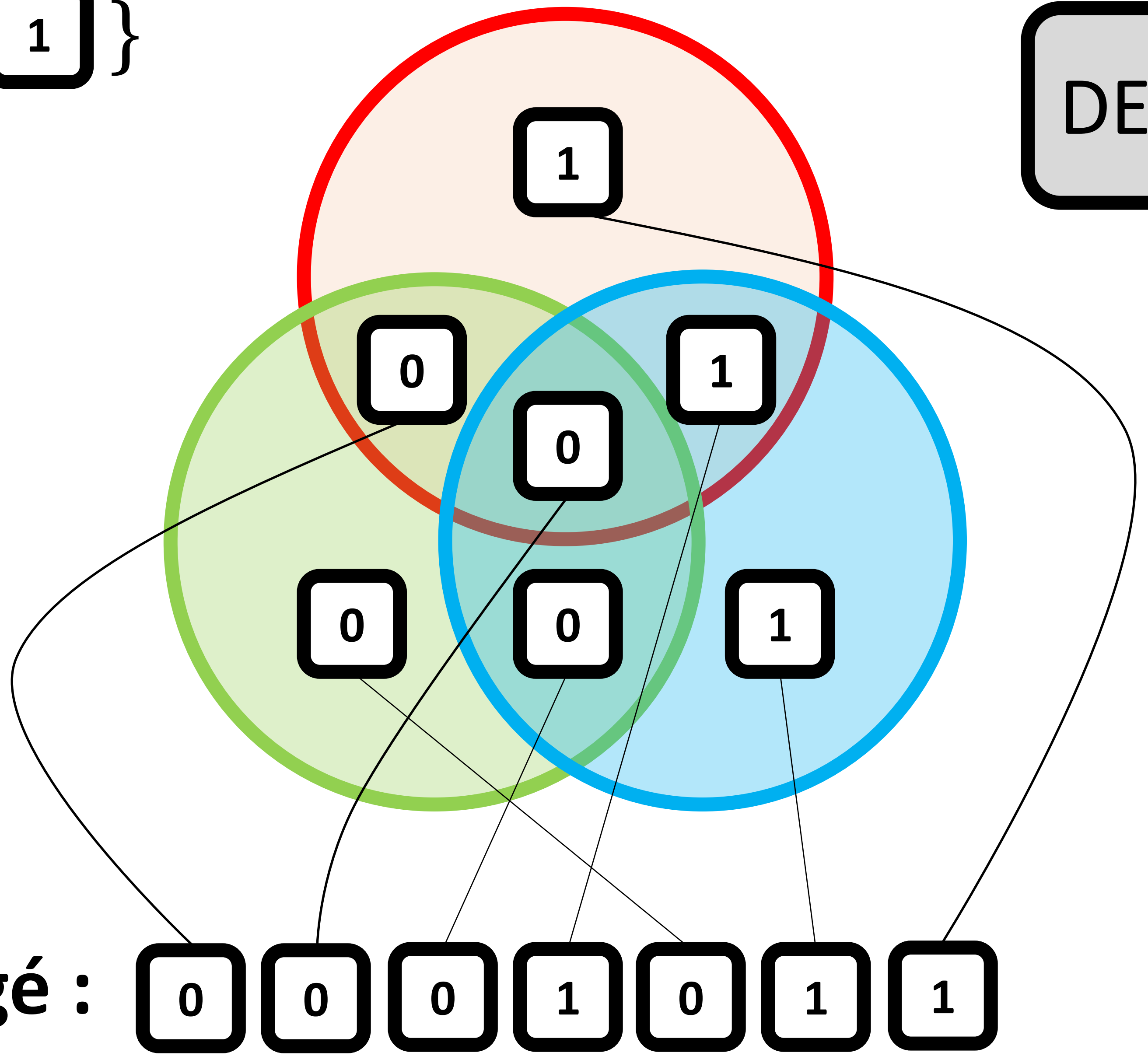
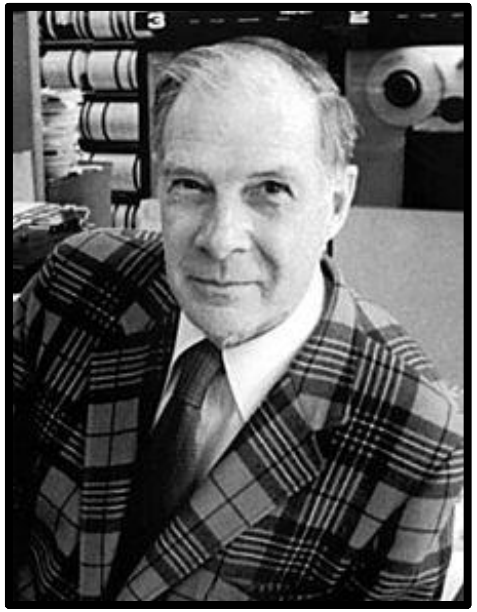


Mot corrigé : 0 0 0 1 0 1 1



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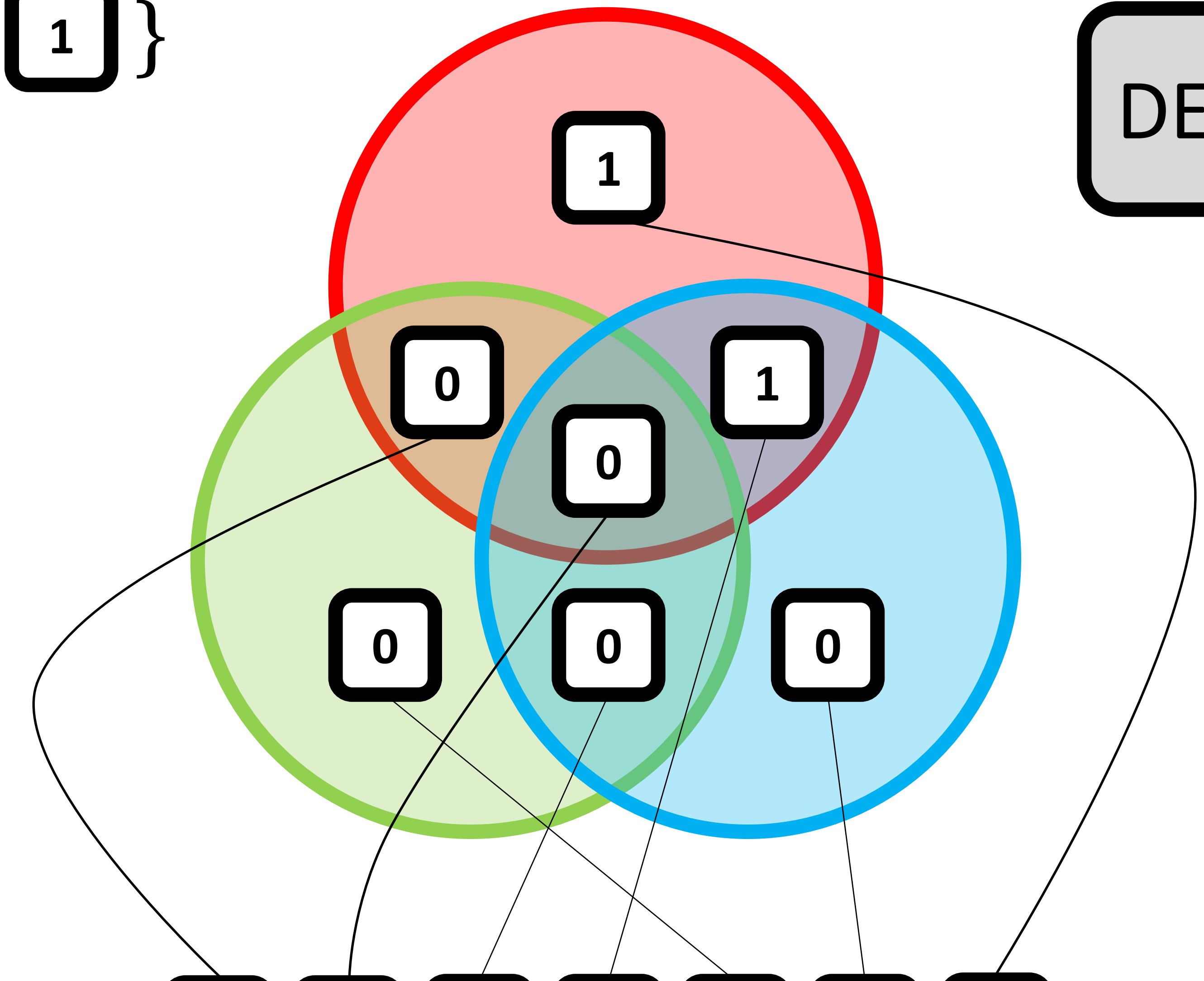
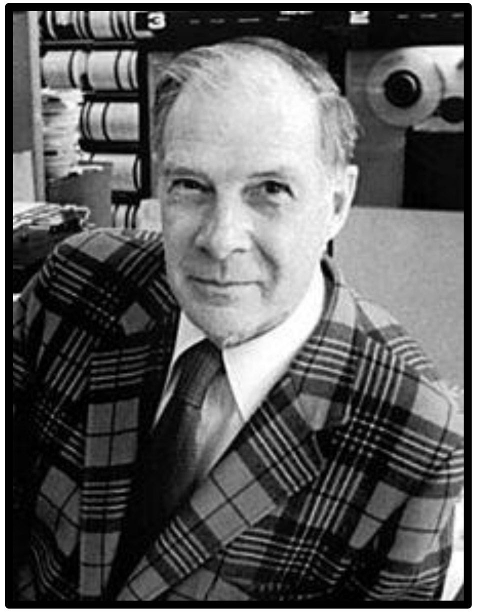
DECODEUR



Mot reçu : 0 0 0 1 0 0 1



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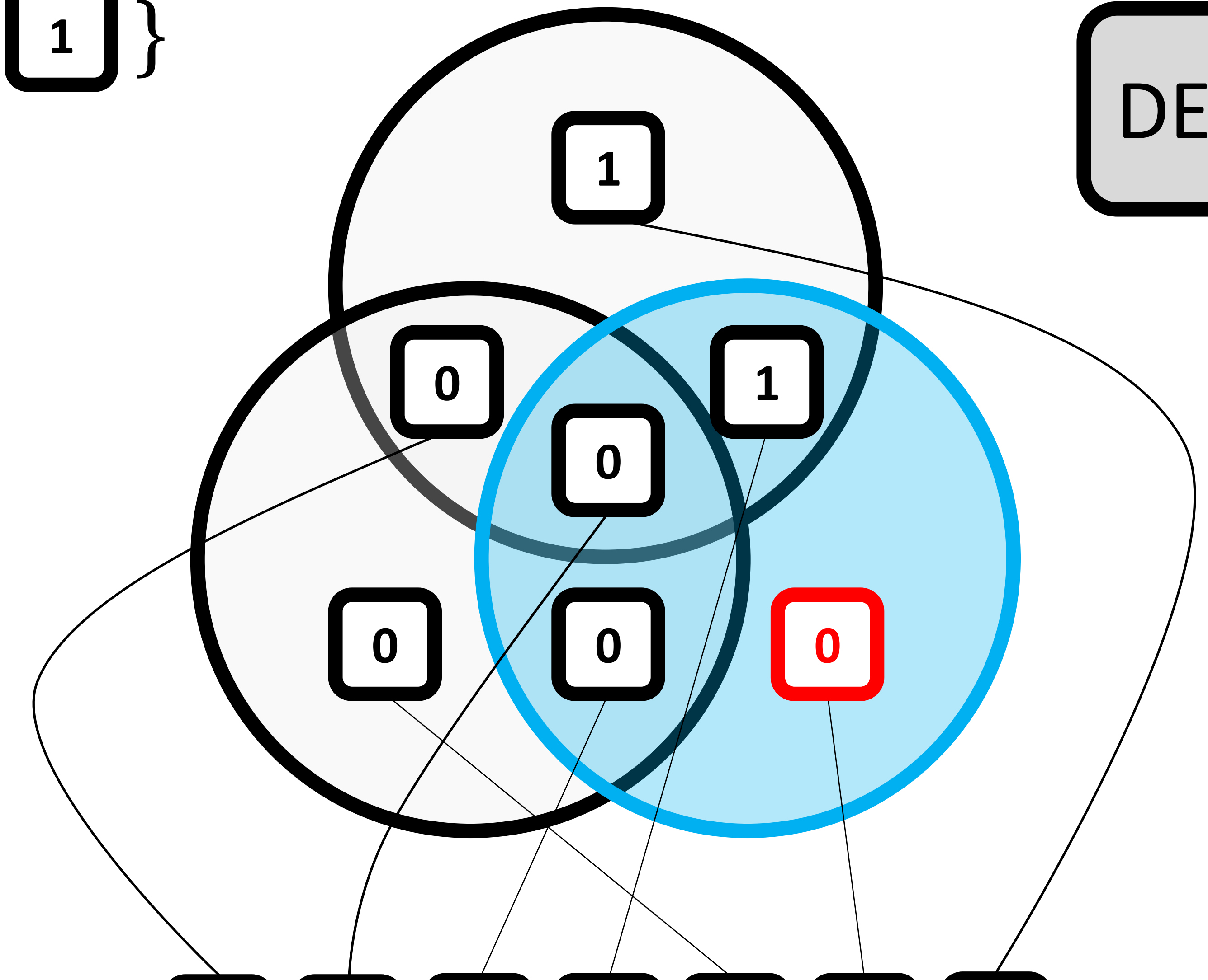
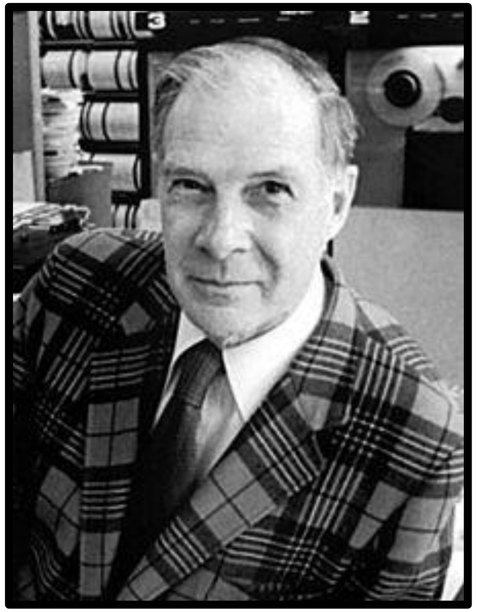


DECODEUR

Mot reçu : 0 0 0 1 0 0 1



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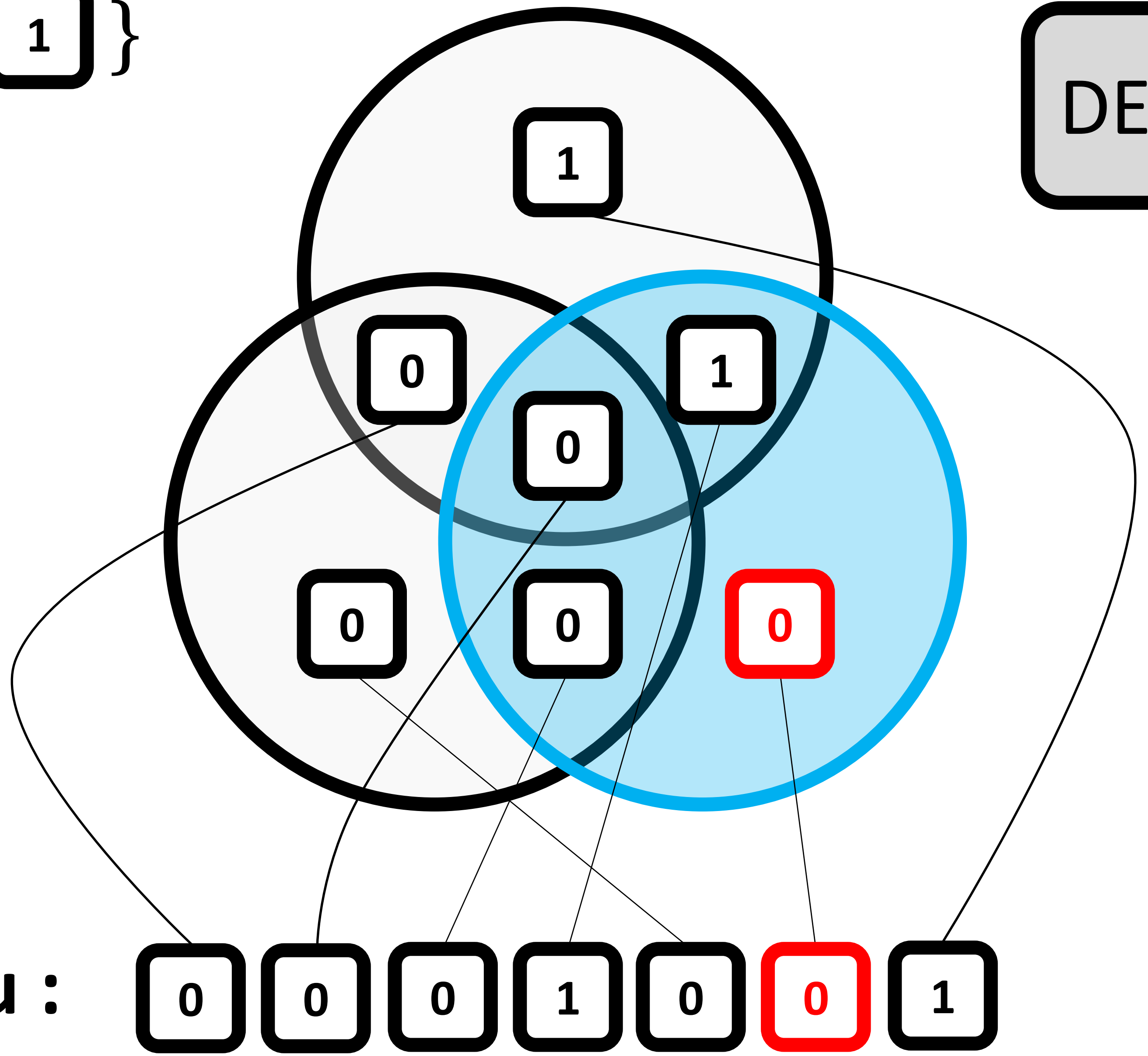

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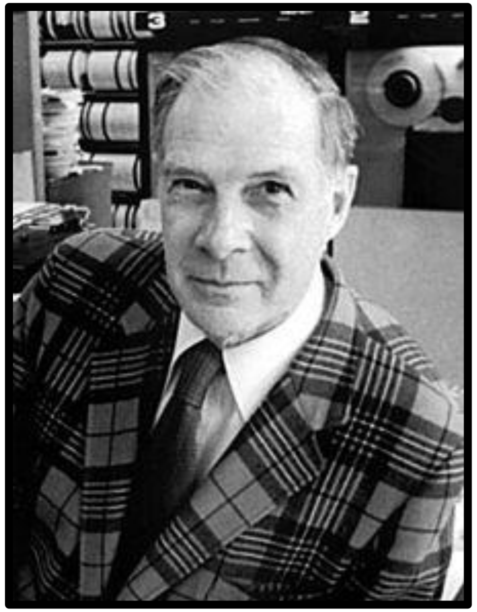

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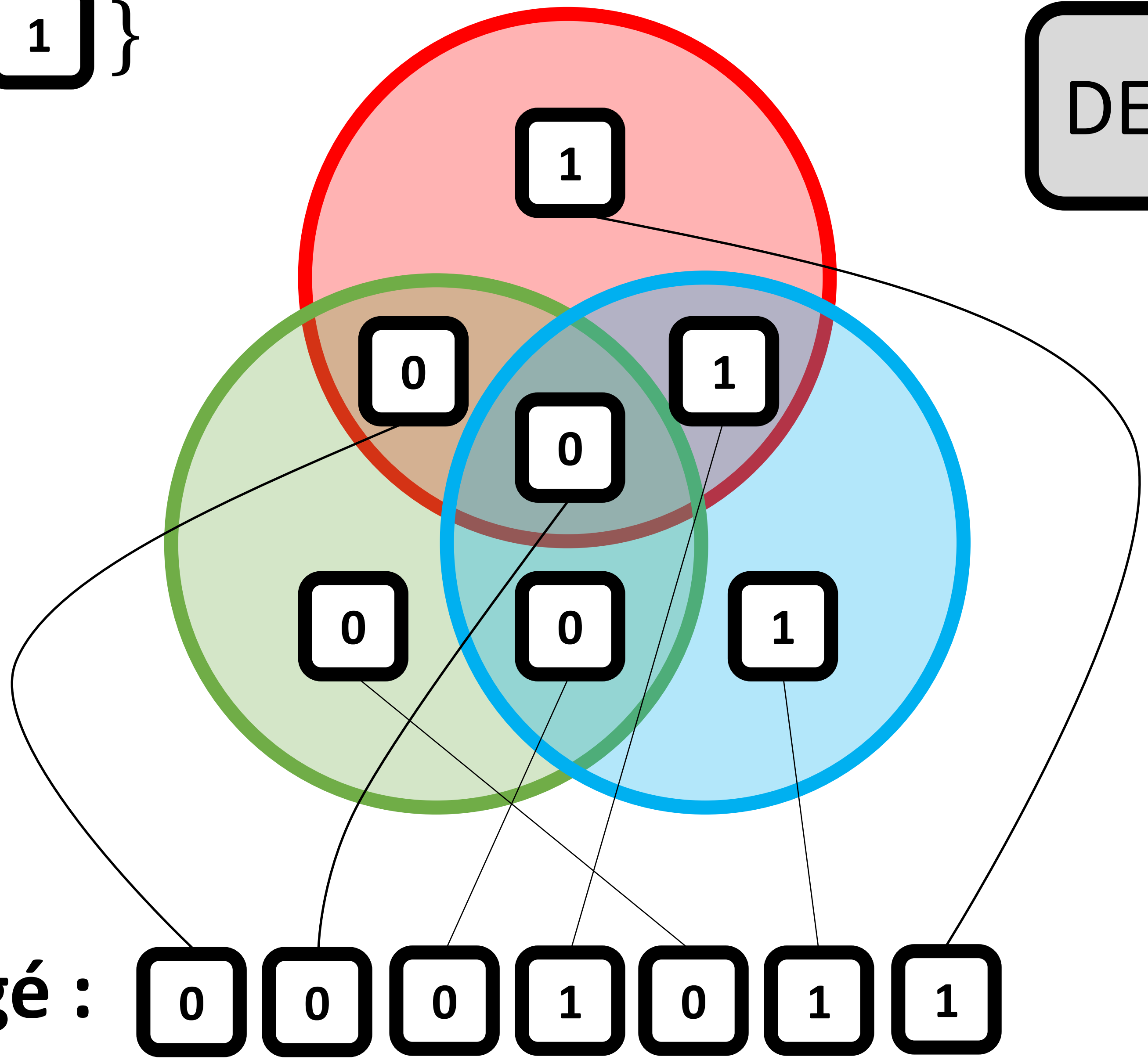

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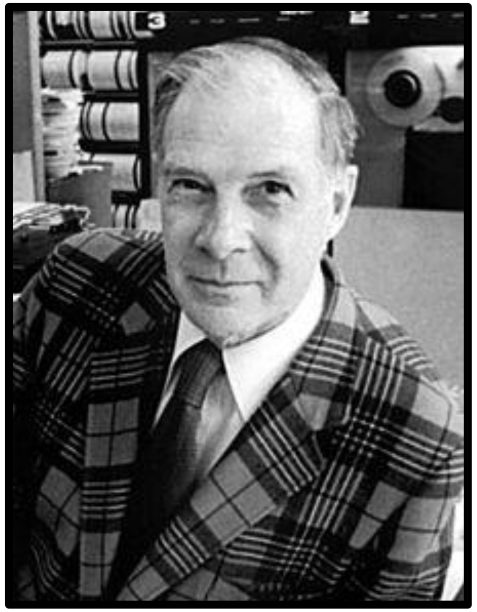
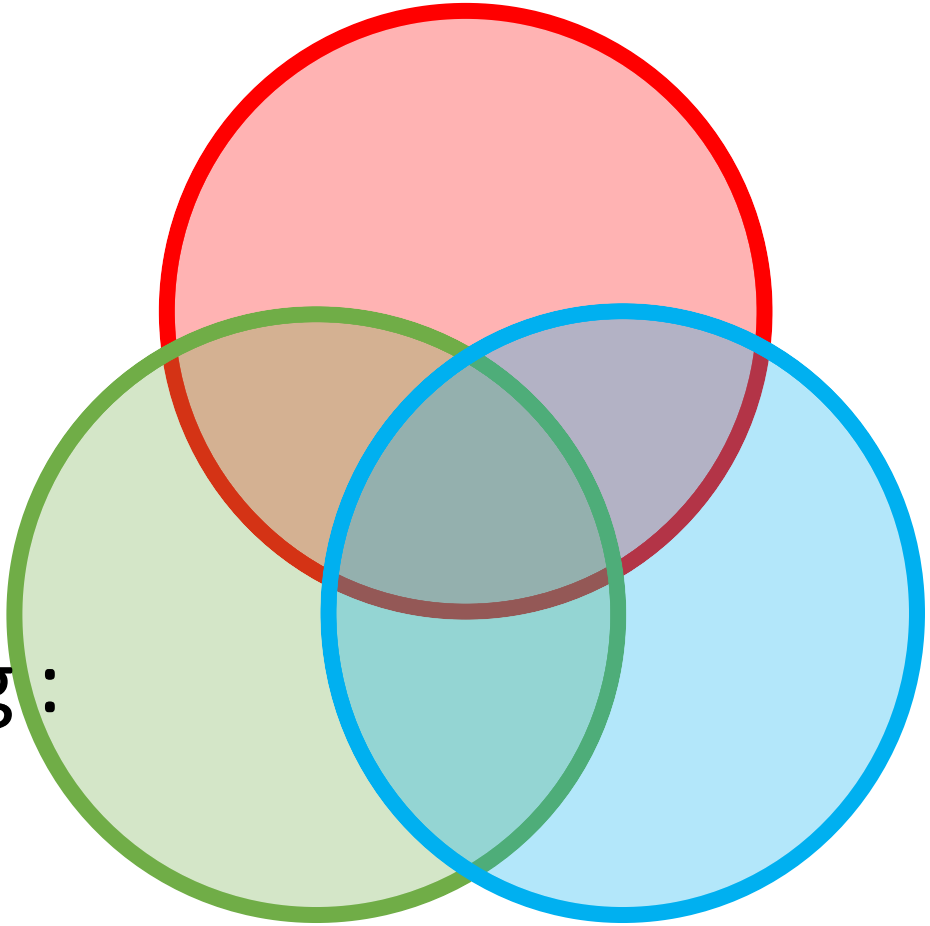
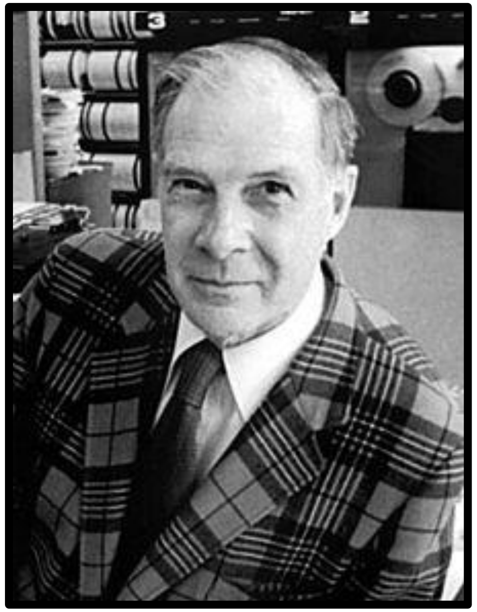
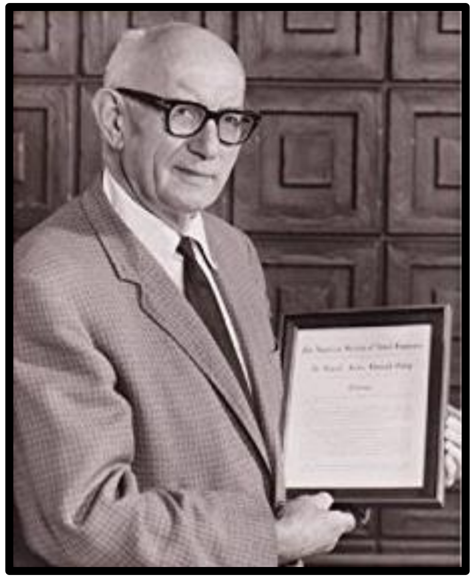


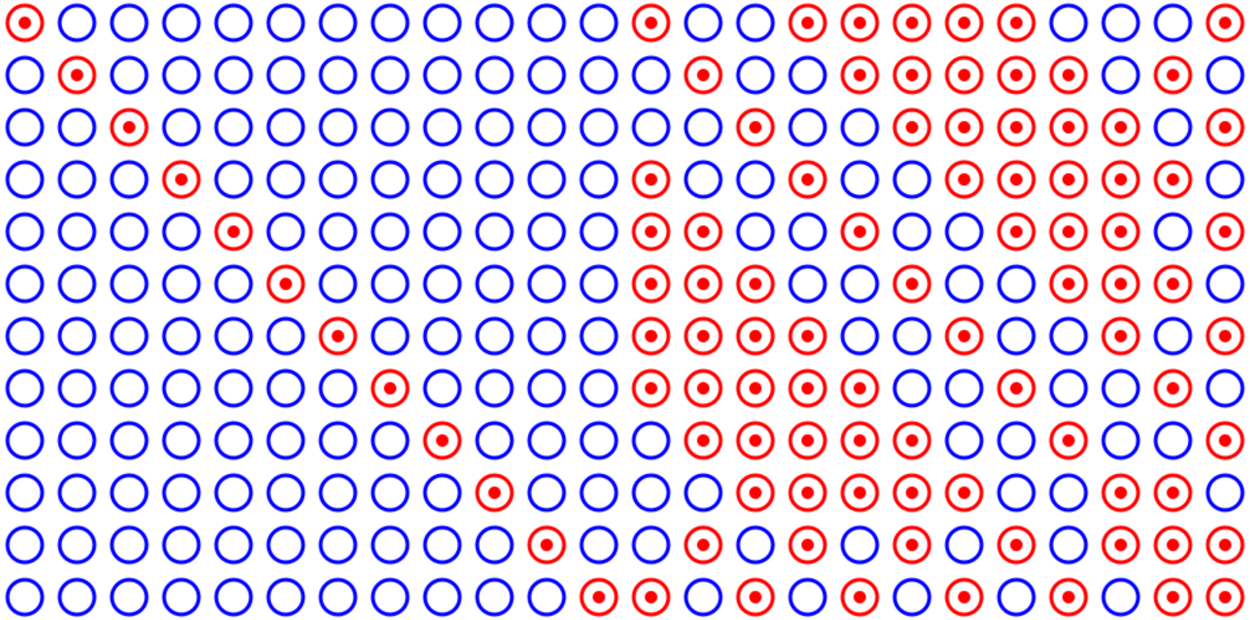


Code de Hamming :

- longueur 7;
- 16 mots;
- capable de corriger 1 erreurs.





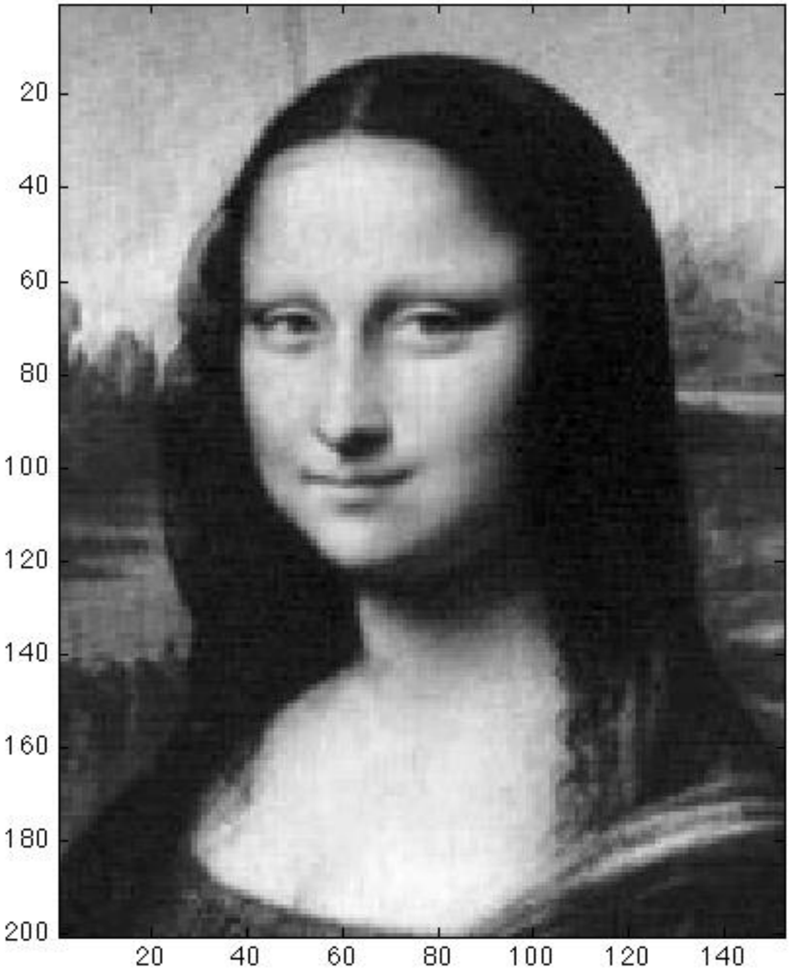


Code de Golay :

- longueur 24;
- 4096 mots;
- capable de corriger 3 erreurs.

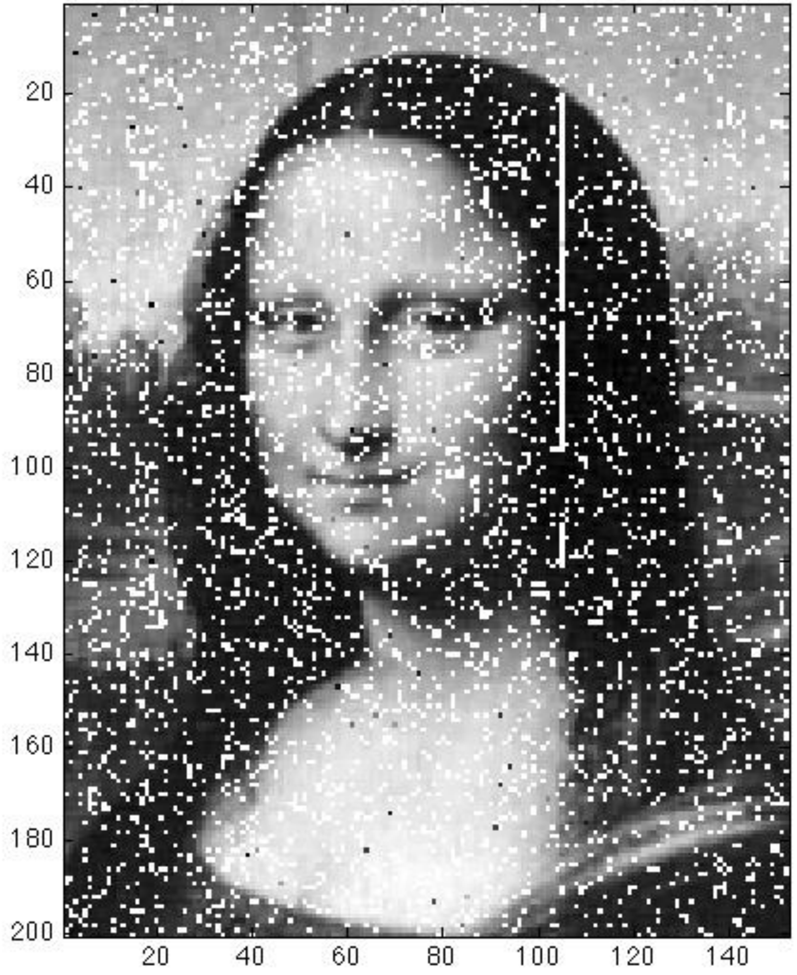
Sonde spatiale **Voyager 1**

Image transmise de la terre



NASA's Lunar Reconnaissance Orbiter (LRO)

Image reçue (sans code)



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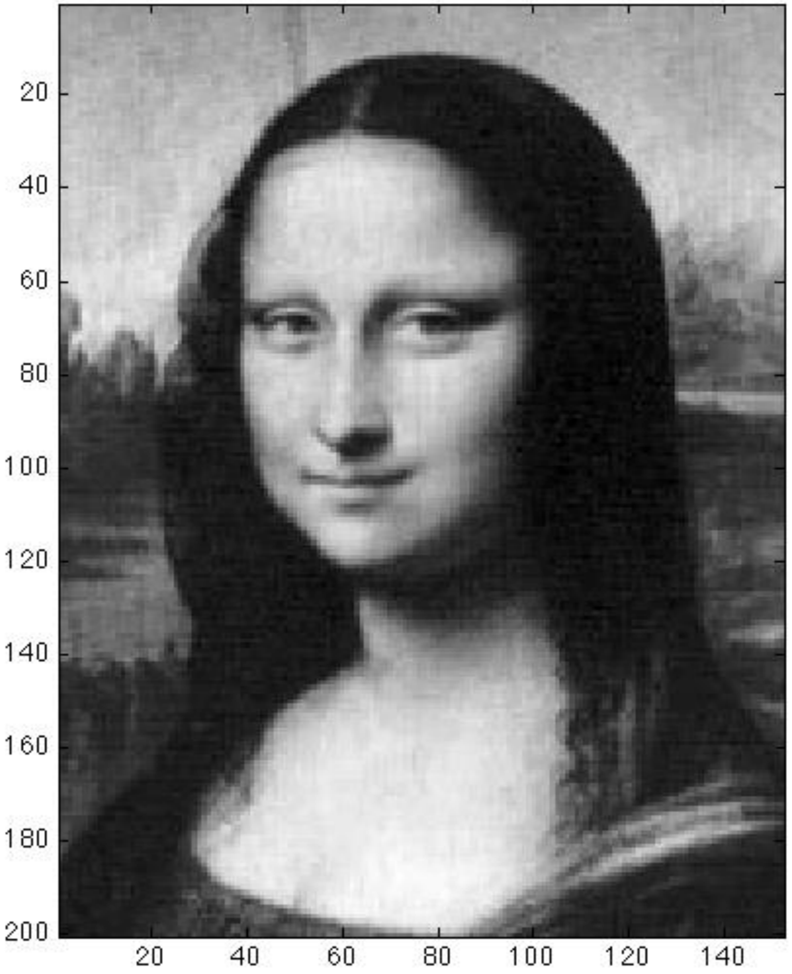
NASA's Lunar Reconnaissance Orbiter (LRO)

Image reçue avec code (de Reed-Solomon)



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NASA's Lunar Reconnaissance Orbiter (LRO)

Irving S. Reed et Gustave Solomon


MERCI POUR
L'ATTENNION !