Iwahori-Hecke algebras and the mod p Langlands program

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Mercredi 11, 18, 25 Mai et Mercredi 1er Juin à 13.30 heure

Salle B407, LAGA

One of the most important techniques in the smooth representation theory of p-adic reductive groups comes from examining the action of compact open subgroups. Of particular importance is the action of the Iwahori subgroup, and the residual action of the **Iwahori–Hecke algebra**. Namely, when the field of coefficients is the complex numbers, there is a close connection between modules over this algebra and a certain class of representations, due to Borel and Bernstein. This equivalence was utilized by Kazhdan–Lusztig to establish instances of the local Langlands correspondence.

When the coefficient field has characteristic p, the situation becomes more delicate. For the group $\mathrm{GL}_2(\mathbb{Q}_p)$, the equivalence between smooth representations and modules over the Hecke algebra continues to hold true, but is known to fail for certain other groups. This indicates that the link between the mod p Iwahori–Hecke algebra (the study of which was developed by Abe, Ollivier, and Vigneras) and mod p representations of p-adic groups is much more intricate than its complex counterpart. One proposed remedy to this problem is due to Schneider, who constructs a differential graded enhancement of the Iwahori–Hecke algebra in characteristic p. This allows us to recover a derived version of the classical equivalence between Iwahori–Hecke modules and smooth representations of p-adic groups.

The goal of this course is to define Iwahori–Hecke algebras and describe their properties, with a focus on the case of mod p coefficients. We will explore their use in the mod p representation theory of p-adic groups, and also define a differential graded enhancement due to Schneider. If time permits, we will additionally discuss the derived aspects of representation theory of p-adic groups.

Regular de Rham Galois representations in the completed cohomology of modular curves

Lue Pan
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Mercredi 18 et 25 Mai à 10.30 heure
Salle B407, LAGA

Let p be a prime. I plan to explain two results concerning regular de Rham Galois representations appearing in the completed cohomology of modular curves.

(1) Classicality. (This is a special case of the Fontaine–Mazur conjecture and was first proved by Emerton in this situation.)
(2) A geometric description of the $\mathrm{GL}_2(\mathbb{Q}_p)$-locally analytic vectors. (In the p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$, this is related to various results originally conjectured by Breuil and other people.)

One main ingredient is a construction of some differential operators on the modular curves with infinite level p. Using these operators, we can define certain de Rham complexes on the flag variety of GL_2 whose cohomology computes regular de Rham Galois representations in the subspace of locally analytic vectors of completed cohomology.

Cohomologie étale des grassmanniennes affines et géométrie des modèles locaux

João Lourenço
(Universitaet Bonn)

Mercredi 1 et 8 Juin à 10.30 heure
Salle B407, LAGA

Ce cours consiste de deux parties:

(1) Je vais expliquer la construction du foncteur central de Gaitsgory et du foncteur d'Arkhipov–Bezrukavnikov dans le cadre p-adique. Pour cela, je prends la grassmannienne de Beilinson–Drinfeld perfectoïde de Scholze–Weinstein et j'applique l'équivalence de Satake pour construire des faisceaux centraux sur la variété de drapeaux de Witt à niveau parahorique. Ceci s'appuie sur des travaux en commun avec Anschütz, Gleason, Richarz, Wu, et Yu.

(2) Je vais expliquer la théorie perfectoïde des modèles locaux p-adiques. Ceux-ci sont certains fermés de la grassmannienne de Beilinson–Drinfeld perfectoïde de Scholze–Weinstein et donnent la structure locale à quelque sens des champs de chtoucas. Je vais parler de leur morphisme de spécialisation, leur représentabilité et leur normalité. Ceci s'appuie sur des travaux en commun avec Anschütz, Gleason, et Richarz.