Assemblage de matrices de type différences finies sous MATLAB/Octave: algorithmes et performances

Laydine Youssouf

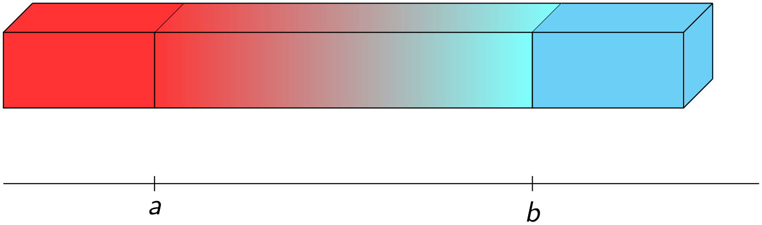
Mαthémαtiques Appliquées et Cαlcul Scieηtifiquε

08/12/2017

«Un ingénieur, des ordinateurs, des nombres»

Equation de Laplace 1D

Conditions limites mixtes



$$-\kappa u''(x) = f(x) \qquad \forall x \in]a, b[\tag{1a}$$

$$\alpha_1 u(a) + \beta_1 u'(a) = \gamma_1 \tag{1b}$$

$$\alpha_2 u(b) + \beta_2 u'(b) = \gamma_2 \tag{1c}$$

Schéma numérique

Discrétisation à l'ordre 2

$$u_{i+1} - 2u_i + u_{i-1} = -h^2 f_i \qquad \forall i \in [\![1, n[\![\tag{2a}$$

$$\nu_1 u_0 + 4\beta_1 u_1 - 2\beta_1 u_2 = 2h\gamma_1 \tag{2b}$$

$$\nu_2 u_n - 4\beta_2 u_{n-1} + 2\beta_2 u_{n-2} = 2h\gamma_2 \tag{2c}$$

Ecriture matricielle associée :

$$AU = \widehat{F} \tag{3}$$

Forme matricielle

Avec conditions limites

$$A = \begin{pmatrix} \nu_1 & 4\beta_1 & -2\beta_1 & 0 & 0 \\ 1 & -2 & 1 & 0 & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & 0 & 1 & -2 & 1 \\ 0 & 0 & 2\beta_2 & -4\beta_2 & \nu_2 \end{pmatrix}$$

$$U = \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_{n-1} \\ u_n \end{pmatrix}, \quad \widehat{F} = \begin{pmatrix} 2h\gamma_1 \\ -h^2 f_1 \\ \vdots \\ -h^2 f_{n-1} \\ 2h\gamma_2 \end{pmatrix}$$

Objectif : construire **rapidement** et **efficacement** A et $\widehat{F}$ sans les conditions limites.

$$A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}, \quad \widehat{F} = -\begin{pmatrix} h^2 f_0 \\ h^2 f_1 \\ \vdots \\ h^2 f_{n-1} \\ h^2 f_n \end{pmatrix}$$

Algorithme 1 Fonction initsyscore1d [alisc-nsnv]

Entrée(s) : $x, \ \kappa, \ f$
Sortie(s) : $A, \ \widehat{F}$

Fonction initsyscore1d(x, κ, f)
  $n \leftarrow$taille(x)
  $A \leftarrow$zeros(n, n), $\widehat{F} \leftarrow$zeros$(n, 1)$
  $c \leftarrow -(x(2) - x(1))^2/\kappa$
  $A(1,1) = -2, \ A(1,2) = 1$ ▷ arbitraire
  $A(n,n) = -2, \ A(n,n-1) = 1$ ▷ arbitraire
  Pour $i = 2$ **à** $n - 1$ **Faire**
    $A(i, i-1) \leftarrow 1$
    $A(i, i) \leftarrow -2$
    $A(i, i+1) \leftarrow 1$
    $\widehat{F}(i) \leftarrow c * f(x(i))$
  Fin Pour
Fin Fonction

Algorithme 2 Fonction initsyscore1d [alisc-snv]

Entrée(s) : $x, \ \kappa, \ f$

Sortie(s) : $A, \ \widehat{F}$

Fonction initsyscore1d(x, κ, f)

  $n \leftarrow$taille(x)

  $A \leftarrow$sparse(n, n), $\widehat{F} \leftarrow$zeros$(n, 1)$

  $c \leftarrow -(x(2) - x(1))^2/\kappa$

  $A(1, 1) = -2, \ A(1, 2) = 1$ ▷ arbitraire

  $A(n, n) = -2, \ A(n, n-1) = 1$ ▷ arbitraire

  Pour $i = 2$ **à** $n - 1$ **Faire**

    $A(i, i-1) \leftarrow 1$

    $A(i, i) \leftarrow -2$

    $A(i, i+1) \leftarrow 1$

    $\widehat{F}(i) \leftarrow c * f(x(i))$

  Fin Pour

Fin Fonction

Algorithme 3 Fonction initsyscore1d [alisc-sv]

Entrée(s) : $x, \ \kappa, \ f$
Sortie(s) : $A, \ \widehat{F}$

```
Fonction initsyscore1d(x, κ, f)
    n ←taille(x)
    c ← −(x(2) − x(1))^2/κ
    F̂ ← c ∗ f(x)
    i = [1 : n, 2 : n, 1 : n − 1]
    j = [1 : n, 1 : n − 1, 2 : n]
    nz = [−2∗ones(1, n), ones(1, n − 1), ones(1, n − 1)]
    A ←sparse(i, j, nz)
Fin Fonction
```

Algorithme 4 Fonction initsyscore1d [alisc-svs]

Entrée(s) : $x, \ \kappa, \ f$
Sortie(s) : $A, \ \widehat{F}$

Fonction initsyscore1d(x, κ, f)
  $n \leftarrow$taille(x)
  $c \leftarrow -(x(2) - x(1))^2/\kappa$
  $\widehat{F} \leftarrow c * f(x)$
  $i = [1 : n, \ 2 : n]$
  $j = [1 : n, \ 1 : n-1]$
  $nz = [-\text{ones}(1, n), \ \text{ones}(1, n-1)]$
  $A \leftarrow$sparse(i, j, nz)+sparse(j, i, nz)
Fin Fonction

Algorithme 5 Fonction initsyscore1d [alisc-svt]

Entrée(s) : $x,\ \kappa,\ f$

Sortie(s) : $A,\ \widehat{F}$

Fonction initsyscore1d(x, κ, f)

$n \leftarrow \text{taille}(x)$

$c \leftarrow -(x(2) - x(1))^2/\kappa$

$\widehat{F} \leftarrow c * f(x)$

$A \leftarrow \text{sparse}(1 : n,\ 1 : n,\ -2 * \text{ones}(1, n))$
$+\text{sparse}(2 : n,\ 1 : n-1,\ -\text{ones}(1, n-1), n, n)$
$+\text{sparse}(1 : n-1,\ 2 : n,\ -\text{ones}(1, n-1), n, n)$

Fin Fonction

Cas test

$$u_{ex}(x) = \sin(x\cos(x)),\ \ x \in [0, 2\pi] \tag{4a}$$

$$f(x) = -u''_{ex}(x),\ \ x \in]0, 2\pi[\tag{4b}$$

$$-u''(x) = f(x),\ \ x \in]0, 2\pi[\tag{5a}$$

$$\gamma_1 = u_{ex}(0) \tag{5b}$$

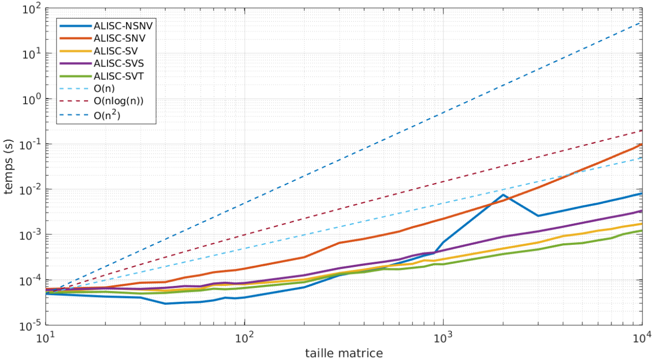
$$\gamma_2 = u'_{ex}(2\pi) \tag{5c}$$

Configurations matérielles :
Processeur : Intel Core i9-7940X CPU @ 3.10GHz
Mémoire : 62.6 Go
OS : Ubuntu 17.10

MATLAB R2017a

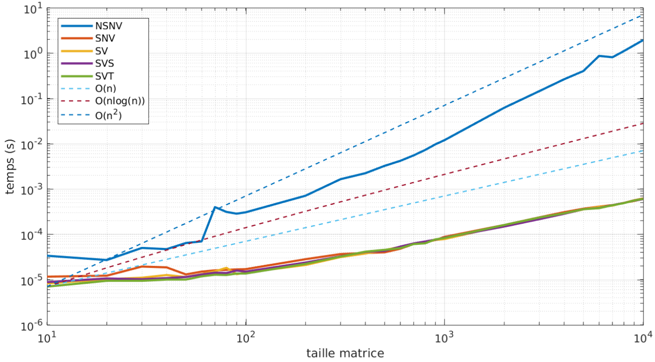
All modes

Assemblage



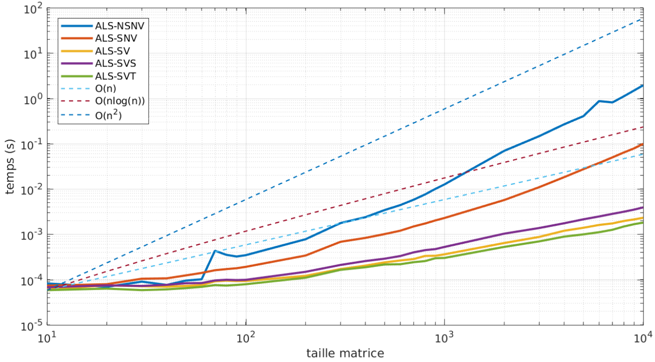
All modes

Résolution



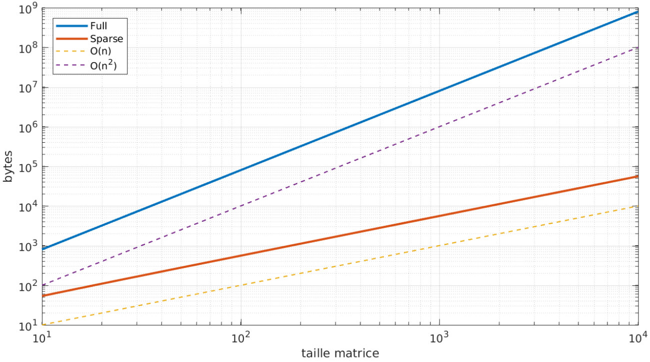
All modes

Assemblage + résolution



All modes

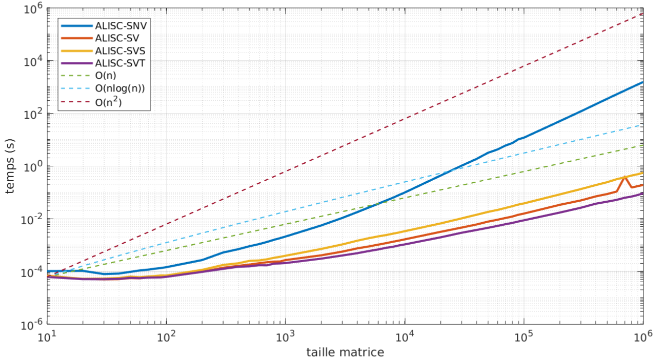
Taille mémoire matrice de discrétisation



MATLAB R2017a

Sparse modes

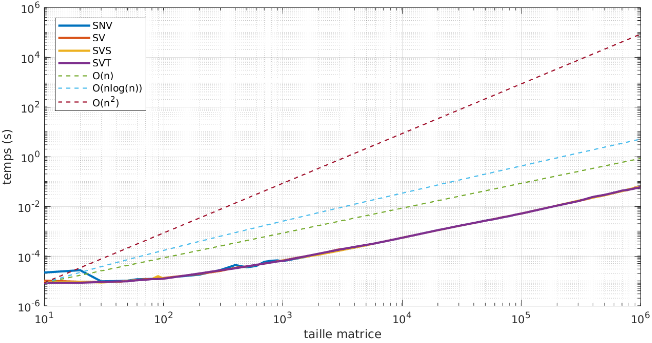
Assemblage



MATLAB R2017a

Sparse modes

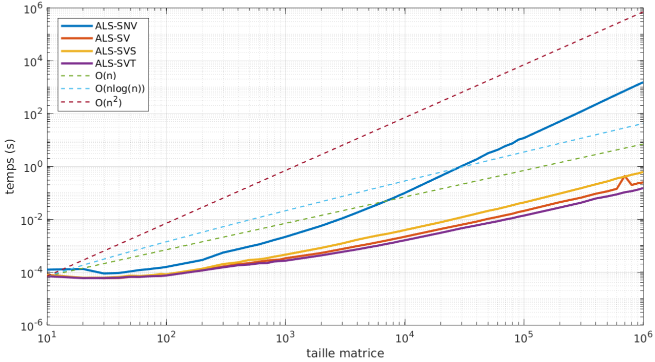
Résolution



MATLAB R2017a

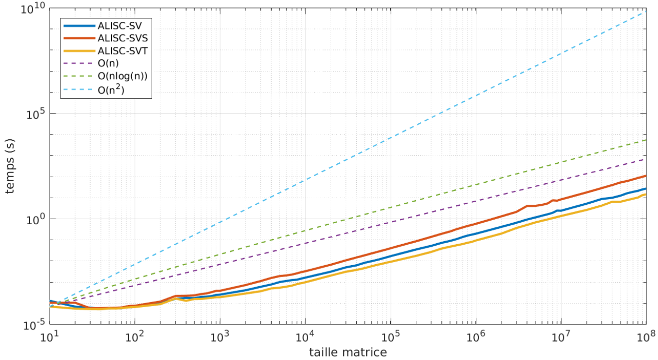
Sparse modes

Assemblage + résolution



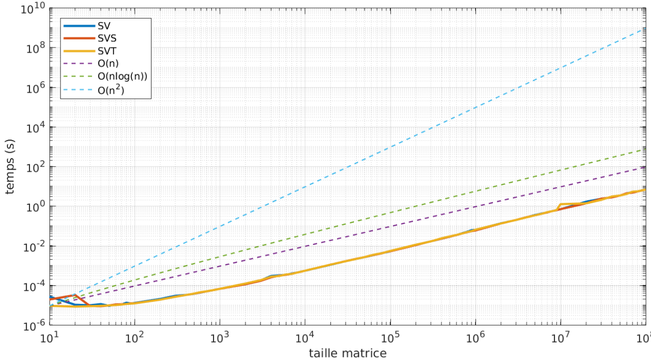
Vectorized sparse modes

Assemblage



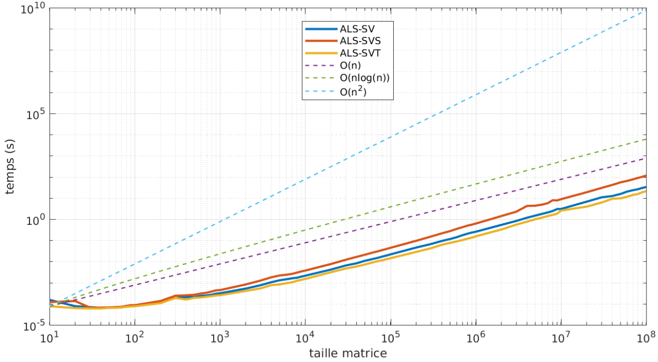
Vectorized sparse modes

Résolution



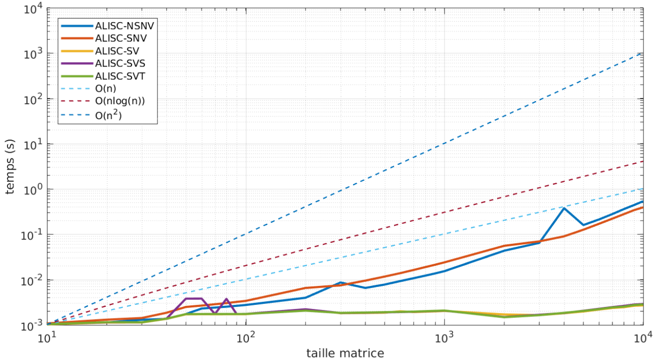
Vectorized sparse modes

Assemblage + résolution



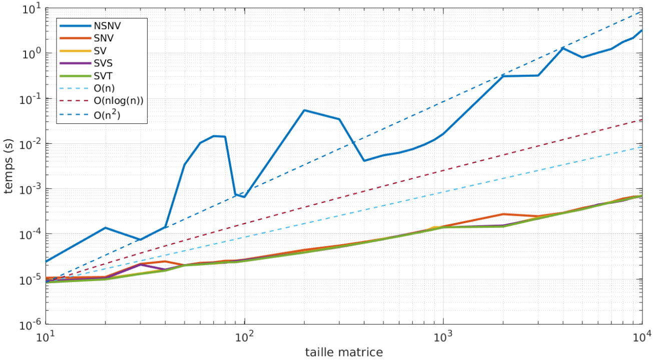
All modes

Assemblage



All modes

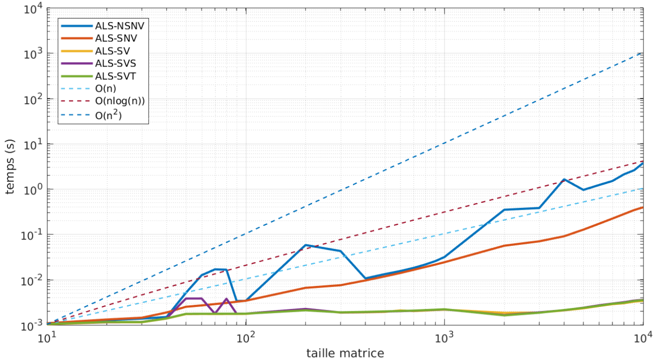
Résolution



Octave 4.2.1

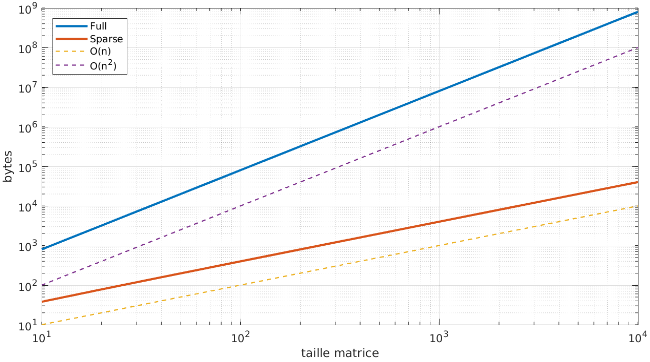
All modes

Assemblage + résolution



All modes

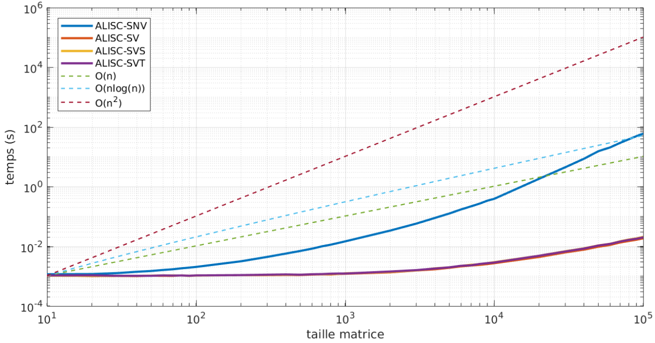
Taille mémoire matrice de discrétisation



Octave 4.2.1

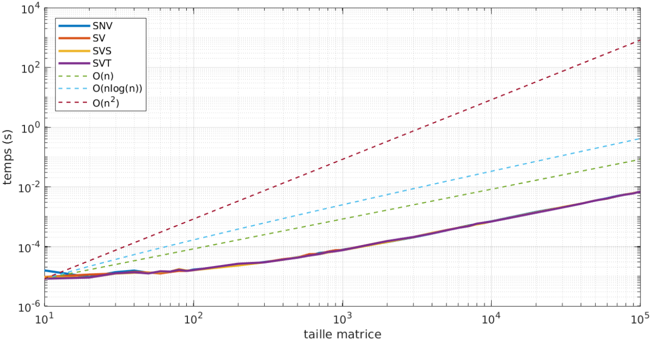
Sparse modes

Assemblage



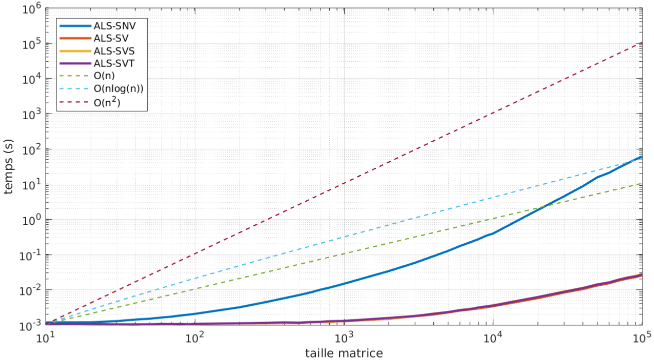
Sparse modes

Résolution



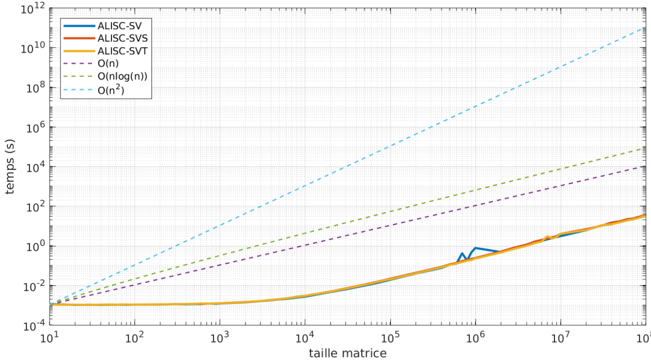
Sparse modes

Assemblage + résolution



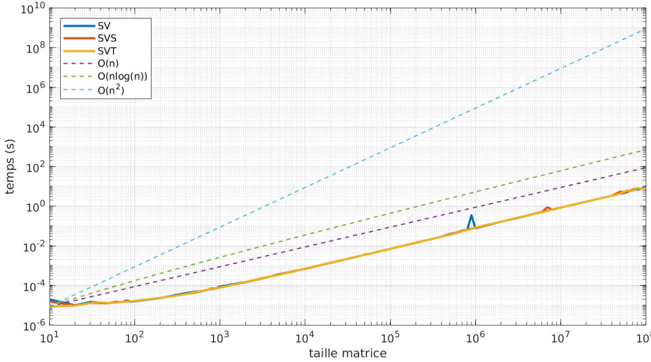
Vectorized sparse modes

Assemblage



Vectorized sparse modes

Résolution



Vectorized sparse modes

Assemblage + résolution