amat Matlab toolbox, User's Guide*

version 0.1.1

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Abstract

This object-oriented Matlab toolbox allows to efficiently extend some linear algebra operations on array of matrices (with same size) as matrix product, determinant, factorization, solving, ...

0 Contents

*LaTeX manual, revision 0.1.1, compiled with Matlab 2019a, and toolboxes `fc-amat`[0.1.1], `fc-tools`[0.0.29], `fc-bench`[0.1.1]

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Initially the fc amat Matlab toolbox was created to be used with finite elements codes for computing volumes and gradiants of barycentric coordinates on each mesh elements. The volume of mesh element can be computed with the determinant of a matrix depending on the coordinates of the mesh element vertices. The gradients of the barycentric coordinates of a mesh element are solutions of linear systems. So we want to be able to do efficiently these operations on a very large number (few millions?) of very small matrices with same order (order less than 10?). In Matlab, all theses matrices can be stored as a N-by-m-by-m 3D-array. Currently, with Matlab from release R2017a (and Octave from version 4.0.3) only element-wise binary operators and functions can be used, as described in:

https://fr.mathworks.com/help/matlab/matlab_prog/compatible-array-sizes-for-basic-operations.html

For example, the sum of a m-by-n matrix with all the N matrices in a N-by-m-by-n 3D-array can be performed as follows:

```
A=rand(m,n);      % generate a m-by-n matrix (n>1)
B=randn(N,m,n); % generate a N-by-m-by-n 3D-array
C=reshape(A,[1,m,n])+B; % generate "A+B" 3D-array
```

Unfortunately, simple operation as matrix product between a m-by-n matrix and all the N matrices in a N-by-n-by-p 3D-array or between all the N matrices of two 3D-arrays with sizes N-by-m-by-n and N-by-n-by-p are not implemented yet.

The purpose of this toolbox is to give efficient operators and functions acting on amat object (array of matrices) to perform operations like sums, matrix product or more complex as determinants computation, factorization, solving, ... by only using Matlab language. One can referred to [1] for more details, tests and benchmarks.

In the first section, the fc amat toolbox is quickly presented. Thereafter, its installation process is described.

1 Presentation

The amat object provided in the fc amat toolbox represents an array of matrices of the same order. All the following functions return an amat object with N matrices whose order is n × m or d × d:

amat(N,m,n)	constructor with all matrices to zeros
fc_amat.zeros(N,m,n)	same as amat(N,m,n)
fc_amat.ones(N,m,n)	matrices of 1
fc_amat.eye(N,d)	identity matrices
fc_amat.random.randn(N,m,n)	normally distributed random elements
fc_amat.random.randnsym(N,d)	randomized symmetric matrices
fc_amat.random.randnher(N,d)	randomized hermitian matrices
fc_amat.random.randntril(N,d)	randomized lower triangular matrices
fc_amat.random.randntriu(N,d)	randomized upper triangular matrices

...

The complete list of constructor and generating functions is given in section 4.

Let A be an amat object with N matrices whose order are m × n. In a

more condensed way we say that `A` is a `N` × `m` × `m` `amat` object. One can easily manipulate and edit its content by using indexing. Here is a small part of the offered possibilities. These are detailed in section 5.

`A(k,i,j)`	return element (`i`, `j`) of the `k`-th matrix
`A(k)`	return the `k` -th matrix (order `m` × `n`)
`A(i,j)`	return elements (`i`, `j`) of all the matrices as an `N`-by-1-by-1 `amat`
`A(k,i,j)=c`	assign `c` scalar value to element (`i`, `j`) of the `k`-th matrix
`A(i,j)=c`	assign `c` value to elements (`i`, `j`) of all the matrices
`A(k)=B`	assign the `m` × `n` matrix `B` to the `k`-th matrix

...

It should be noted that resizing objects can happen when one of the indices is larger than the corresponding dimension. In Listing 1, some examples are provided.

```
A=fc_amat.random.randn(100,3,4);% A: 100-by-3-by-4 amat
B=randn(3,4);
A(10)=B; % B assign to the 10-th matrix
A(20:25)=B; % the matrices 20 to 25 are set to B
A(30:2:36)=0; % the matrices 30,32,34 and 36 are set to 0
A(120)=1; % now A is a 120-by-3-by-4 amat ...
A(1,2)=0; % elements (1,2) of all the matrices are set to 0
A(2:3,3)=1; % elements (2,3) and (3,3) of all the matrices are set to 1
A(4,5)=1;  % now A is a 120-by-4-by-5 amat ...
A(5,1,2)=pi;  % element (1,2) of the 5-th matrix is set to pi
A(10:15,1,2)=1; % element (1,2) of the matrices 10 to 15 are set to 1
A(130,6,7)=1; % now A is a 130-by-6-by-7 amat ...
```

Listing 1: Assigments with `amat` object

The `amat` class is provided with the usual elementary operations:

- `+` , `-` , `.*` , `./` , `.\` , , `.^` . (Arithmetic opertors)
- `==` , `>=` , `>` , `<=` , `<` , `~=` . (Relational opertors)
- `&` , `|` , `~` , `xor` , `all` , `any` . (Logical opertors)

These are detailed in section 6. In Listing 2, some examples are provided.

```
A=fc_amat.ones(100,3,4);% A: 100-by-3-by-4 amat
B=fc_amat.random.randn(100,3,4);% B: 100-by-3-by-4 amat
C=randn(3,4);
D1=-A+1;
D2=B.^2-A/2;
D3=-2*A.*C;
```

Listing 2: Element by elements operations with `amat` object

Matricial products can also be done between `amat` objects or between an `amat` object and a matrix if their dimensions are compatible. For this operation the operator `*` can be used. In Listing 3, some examples are provided.

Listing 3: : matricial products with amat object

```
A=fc_amat.ones(100,3,4);%   100-by-3-by-4
info(A)
B=fc_amat.random.randn(100,4,2);% 100-by-4-by-2
info(B)
C=randn(4,5);
D1=A*B; % 100-by-3-by-2
info(D1)
D2=A*C; % 100-by-3-by-5
info(D2)
```

Output

```
A is a 100x3x4 amat[double] object
B is a 100x4x2 amat[double] object
D1 is a 100x3x2 amat[double] object
D2 is a 100x3x5 amat[double] object
```

Some usual mathematical functions as `cos`, `sin`, `exp`, `sqrt`, `abs`, `max`, ... are available for `amat` objects. One can refered to section 7 for more details.

Other operations such as determinants computation (`det` method), LU factorization with partial pivot (`lu` method), Cholesky factorization (`chol` method), solving linear systems (`mldivide` method or `\` operator) are also implemented for `amat` objects and described in section 8. In Listing 4, some examples using these functions are given.

Thereafter in Listing 5, the benchmark function `fc_amat.benchs.mldivide` is used to obtain cputimes of the `X=mldivide(A,b)` command where `A` and `b` are respectively `N` $\times 3 \times 3$ and `N` $\times 3 \times 4$ `amat` objects. The provided error is computed by taking the maximum of the infinity norms of all the matrices in the error `amat` object `E=A*X-b` obtained by `max(norm(E))`.

Finally, in Table 1 benchmark functions `fc_amat.benchs.mtimes`, `fc_amat.benchs.lu`, `fc_amat.benchs.chol` and `fc_amat.benchs.mldivide` are respectively used to get cputimes of the `X=mtimes(A,B)`, `[L,U,P]=lu(A)`, `R=chol(A)` and `X=mldivide(A,b)` where `A` and `B` are `N` $\times 4 \times 4$ `amat` objects, and `b` is a `N` $\times 4 \times 1$ `amat` object.

Listing 4: : Linear algebra with amat object

```
% Generate 100-by-4-by-4 amat object symmetric positive definite ...
    matrices:
A=fc_amat.random.randnsympd(100,4);
% determinants computation:
D=det(A); % D: 100-by-1-by-1 amat object,  det(A(k))=D(k),     for all k
% LU factorizations:
[L,U,P]=lu(A);
E1=abs(L*U-P*A);
fprintf('max␣of␣E1␣elements:␣%.6e\n',max(E1(:)))
% Cholesky factorizations:
R=chol(A);
E2=abs(R'*R-A);
fprintf('max␣of␣E2␣elements:␣%.6e\n',max(E2(:)))
% Solving linear systems:
b=ones(4,1); % RHS
X=A\b;    % X: 100-by-4-by-1, X(k)=A(k)\b,     for all k
E3=abs(A*X-b);
fprintf('max␣of␣E3␣elements:␣%.6e\n',max(E3(:)))
B=fc_amat.random.randn(100,4,1); % RHS
Y=A\B;    % Y: 100-by-4-by-1, Y(k)=A(k)\B(k), for all k
E4=abs(A*Y-B);
fprintf('max␣of␣E4␣elements:␣%.6e\n',max(E4(:)))
whos
```

Output

```
max of E1 elements: 3.552714e-15
max of E2 elements: 7.105427e-15
max of E3 elements: 7.105427e-15
max of E4 elements: 2.153833e-14
  Name             Size           Bytes  Class    Attributes

  A              100x4x4           12832  amat
  B              100x4x1            3232  amat
  D              100x1x1             832  amat
  E1             100x4x4           12832  amat
  E2             100x4x4           12832  amat
  E3             100x4x1            3232  amat
  E4             100x4x1            3232  amat
  L              100x4x4           12832  amat
  P              100x4x4           12832  amat
  R              100x4x4           12832  amat
  SaveOptions      1x6               721  cell
  U              100x4x4           12832  amat
  X              100x4x1            3232  amat
  Y              100x4x1            3232  amat
  b                4x1                32  double
```

Listing 5: : Computational times of the `X=mldivide(A,b)` command where `A` and `b` are respectively `N` × 3 × 3 and `N` × 3 × 4 `amat` objects by using the benchmark function `fc_amat.benchs.mldivide`

```
LN=10^5*[2:2:10];
fc_amat.benchs.mldivide(LN,'d',3,'n',4,'nbruns',5)
```

Output

```
#------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[double] with (N,m,n)=(N,3,3)
#    containing strictly diagonally dominant matrices
# 2nd parameter is :
# -> amat[double] with (N,nr,nc)=(200000,3,4), size=[200000 3      4]
# Error function: @(X)max(norm(A*X-B))
#------------------------------------------------------------------------------
#date:2020/01/01 13:23:38
#nbruns:5
#numpy:       i4          f4          f4
#format:      %d         %.3f        %.3e
#labels:       N   mldivide(s)    Error[0]
          200000        0.308    2.801e-14
          400000        0.623    4.810e-14
          600000        0.951    1.024e-13
          800000        1.351    1.415e-13
         1000000        1.722    1.165e-13
```

N	mtimes (s)	chol (s)	lu (s)	mldivide (s)
200 000	0.115(s)	0.024(s)	0.275(s)	0.333(s)
400 000	0.235(s)	0.047(s)	0.536(s)	0.617(s)
600 000	0.417(s)	0.070(s)	0.809(s)	0.945(s)
800 000	0.526(s)	0.095(s)	1.216(s)	1.411(s)
1 000 000	0.662(s)	0.124(s)	1.481(s)	2.004(s)
5 000 000	6.172(s)	1.131(s)	10.875(s)	13.252(s)
10 000 000	12.175(s)	2.314(s)	21.584(s)	28.351(s)

Table 1: Computational times in seconds of `mtimes(A,B)` (i.e. `A*B`), `lu(A)` , `chol(A)` and `mldivide(A,b)` (i.e. `A\b`) with `A` and `B` `N` × 4 × 4 `amat` objects and `b` n `N` × 4 × 1 `amat` object.

2 Installation

This toolbox was tested on various OS and Matlab releases:

	Matlab
Linux	
CentOS 7.7.1908	**2015b** to **2019a**
Debian 9.11	**2015b** to **2019a**
Fedora 29	**2015b** to **2019a**
OpenSUSE Leap 15.0	**2015b** to **2019a**
Ubuntu 18.04.3 LTS	**2015b** to **2019a**
Apple Mac OS X	
MacOS High Sierra 10.13.6	**2015b** to **2019a**
MacOS Mojave 10.14.4	**2015b** to **2019a**
MacOS Catalina 10.15.2	**2015b** to **2019a**
Microsoft Windows	
Windows 10 (1909)	**2015b** to **2019a**

It is not compatible with Matlab release R2015a and previous.

2.0.1 Automatic installation, all in one (recommended)

For this method, one just has to get/download the install file

mfc_amat_install.m

or get it on the dedicated web page. Thereafter, one runs it under Matlab. This script downloads, extracts and configures the *fc-amat* and the required toolboxes (*fc-tools* and *fc-bench*) in the current directory.

For example, to install this toolbox in `~/Matlab/toolboxes` directory, one has to copy the file mfc_amat_install.m in the `~/Matlab/toolboxes` directory by using previous link. For example, in a Linux terminal, we can do:

```
cd ~/Matlab/toolboxes
HTTP=http://www.math.univ-paris13.fr/~cuvelier/software/codes/Matlab
wget $HTTP/fc-amat/0.1.1/mfc_amat_install.m
```

Then in a Matlab terminal run the following commands

```
>> cd ~/Matlab/toolboxes
>> mfc_amat_install()
```

The optional `'dir'` option can be used to specify installation directory:

```
mfc_amat_install('dir',dirname)
```

where `dirname` is the installation directory (string).
There is the output of the `mfc_amat_install()` command on a Linux computer:

```
Parts of the <fc-amat> Matlab toolbox.
Copyright (C) 2018-2019 F. Cuvelier

1- Downloading and extracting the toolboxes
2- Setting the <fc-amat> toolbox
Write in /home/cuvelier/tmp/fc-amat-full/fc_amat-0.1.1/configure_loc.m ...
    ...
3- Using toolboxes :
   ->             fc-tools : 0.0.29
   ->             fc-bench : 0.1.1
 with              fc-amat : 0.1.1
*** Using instructions
   To use the <fc-amat> toolbox:
   addpath('/home/cuvelier/tmp/fc-amat-full/fc_amat-0.1.1')
   fc_amat.init()

   See /home/cuvelier/tmp/mfc_amat_set.m
```

The complete toolbox (i.e. with all the other needed toolboxes) is stored in the directory `~/Matlab/toolboxes/fc-amat-full` and, for each Matlab session, one have to set the toolbox by:

```
>> addpath('~/Matlab/toolboxes/fc-amat-full/fc-amat-0.1.1')
>> fc_amat.init()
```

If it's the first time the `fc_amat.init()` function is used, then its output is

```
Try to use default parameters!
 Use fc_tools.configure to configure.
Write in ...
    /home/cuvelier/tmp/fc-amat-full/fc_tools-0.0.29/configure_loc.m ...
Try to use default parameters!
 Use fc_bench.configure to configure.
Write in ...
    /home/cuvelier/tmp/fc-amat-full/fc_bench-0.1.1/configure_loc.m ...
Using fc_amat[0.1.1] with fc_tools[0.0.29], fc_bench[0.1.1].
```

Otherwise, the output of the `fc_amat.init()` function is

```
Using fc_amat[0.1.1] with fc_tools[0.0.29], fc_bench[0.1.1].
```

For **uninstalling**, one just has to delete directory

`~/Matlab/toolboxes/fc-amat-full`

2.0.2 Manual installation

- Download one of the **full archives** (see web page) which contains all the needed toolboxes (*fc-amat*, *fc-tools* and *fc-bench*).
- Extract the archive in a folder.
- Set Matlab path by adding path of the needed toolboxes.

For example under Linux, to install this toolbox in `~/Matlab/toolboxes` directory, one can download fc-amat-0.1.1-full.tar.gz and extract it in the `~/Matlab/toolboxes` directory:

```
HTTP=http://www.math.univ-paris13.fr/~cuvelier/software/codes/Matlab
wget $HTTP/fc-amat/0.1.1/fc-amat-0.1.1-full.tar.gz
tar zxf fc-amat-0.1.1-full.tar.gz -C ~/Matlab/toolboxes
```

For each Matlab session, one has to set the toolbox by adding path of all toolboxes:

```
>> addpath('~/Matlab/toolboxes/fc-amat-0.1.1/fc_amat-0.1.1')
>> addpath('~/Matlab/toolboxes/fc-amat-0.1.1/fc_tools-0.0.29')
>> addpath('~/Matlab/toolboxes/fc-amat-0.1.1/fc_bench-0.1.1')
```

3 Notations

Some typographic conventions are used in the following:

- $\mathbb{Z}$, $\mathtt{N}$, $\mathbb{R}$, $\mathbb{C}$ are respectively the set of integers, positive integers, reals and complex numbers. $\mathbb{K}$ is either $\mathbb{R}$ or $\mathbb{C}$.
- All vectors or 1D-arrays are represented in bold: $\boldsymbol{v} \in \mathbb{R}^n$ or $\boldsymbol{X}$ a 1D-array.The first alphabetic characters are $\boldsymbol{aAbBcC}\ldots$.
- All matrices or 2D-arrays are represented with the blackboard font as: $\mathbb{M} \in \mathcal{M}_{m,n}(\mathbb{K})$ or $\mathbb{b}$ a m-by-n 2D-array. The first alphabetic characters are $\mathbb{aAbBcC}\ldots$.
- All arrays of matrices or 3D-arrays or `amat` objects are represented with the bold blackboard font as: $\mathbb{M} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N$ or $\mathbb{b}$ a N-by-m-by-n 3D-array. The first alphabetic characters are $\mathbb{aAbBcC}\ldots$.

We now introduce some notations. Let $\mathbb{A} = (\mathbb{A}_1, \ldots \mathbb{A}_N) \in (\mathcal{M}_{m,n}(\mathbb{K}))^N$ be a set of m-by-n matrices.We identify $\mathbb{A}$ as a N-by-m-by-n `amat` object and we said that the `amat` object `A` is in $(\mathcal{M}_{m,n}(\mathbb{K}))^N$. The `k`-th matrix of `A` is `A(k)` and the `(` i,j) entry of the `k`-th matrix of `A` is `A(k,i,j)` .

Thereafter, we said that an `amat` object `A` $\in (\mathcal{M}_{m,n}(\mathbb{K}))^N$ has a property of matrix if all its matrices have this property. For example, `A` is a symmetrical `amat` object if all its matrices are symmetrical.

4 Constructor and generators

We give properties of the `amat` class :

Properties of `amat` class

`nr`	:	number of rows
`nc`	:	number of columns
`N`	:	number of matrices (`nr`-by-`nc`)
`values`	:	`N`-by-`nr`-by-`nc` array which contains all the matrices

4.1 Constructor

Syntaxe

```
X=amat(N,nr,nc)
X=amat(T)
X=amat(N,A)
X=amat(...,classname)
```

Description

`X=amat(N,n,m)` returns a `N`-by-`n`-by-`m` `amat` object where all its elements are set to 0.

`X=amat(T)` when `T` is a `N`-by-`n`-by-`m` array, returns the `N`-by-`n`-by-`m` `amat` object set to `T`.
When `T` is a `N`-by-`n`-by-`m` `amat` object, returns a `N`-by-`n`-by-`m` zero `amat` object.

`X=amat(N,A)` with `A` a `n`-by-`m` matrix, return the `N`-by-`n`-by-`m` `amat` object where all its matrices are set to the matrix `A`.

`X=amat(...,classname)` returns an `amat` object with values of class `classname`.

In Listing 6, some examples are provided.

Listing 6: : `amat` constructors

```
X=amat(100,3,4);              % X: 100-by-3-by-4 amat
info(X)
W=amat(X);                    % W: 100-by-3-by-4 amat
info(W)
T=randn(200,2,3);             % T: 200-by-2-by-3 array
Y=amat(T);                    % Y: 200-by-2-by-3 amat
info(Y)
A=randi(10,[2,4],'int32');% A: 2-by-4 int32 matrix
Z=amat(30,A,'int64');         % Z: 30-by-2-by-4 int64 amat
disp('Print Z amat object :')
disp(Z)
```

Output

```
X is a 100x3x4 amat[double] object
W is a 100x3x4 amat[double] object
Y is a 200x2x3 amat[double] object
Print Z amat object :
Z is a 30x2x4 amat[int64] object
  Z(1)=
    9   10    1    8
    1    4    4    8

  Z(2)=
    9   10    1    8
    1    4    4    8

  ...

  Z(29)=
    9   10    1    8
    1    4    4    8

  Z(30)=
    9   10    1    8
    1    4    4    8
```

4.2 Particular generators

There is the list of functions which generate some particular `amat` objects:

- `fc_amat.zeros` , generates an zero `amat` object,
- `fc_amat.ones` , generates an `amat` object of one's,
- `fc_amat.eye` , generates an `amat` object of identity matrices.

4.2.1 `fc_amat.zeros` function

Syntaxe

```
X=fc_amat.zeros(N,m,n)
X=fc_amat.zeros([N,m,n])
X=fc_amat.zeros([N,d])
X=fc_amat.zeros(...,classname)
```

Description

`X=fc_amat.zeros(N,m,n)` return an `N`-by-`m`-by-`n` zero `amat` object.

`X=fc_amat.zeros([N,m,n])` same as `X=fc_amat.zeros(N,m,n)`

`X=fc_amat.zeros(N,d)` same as `X=fc_amat.zeros(N,d,d)`

`X=fc_amat.zeros(...,classname)` returns an `amat` object with values of class `classname`

In Listing 7, some examples are provided.

Listing 7: : examples of `fc_amat.zeros` function usage

```
X=fc_amat.zeros(100,2,4);                % X: 100-by-2-by-4 amat
Y=fc_amat.zeros(200,3);                  % Y: 100-by-3-by-3 amat
Z=fc_amat.zeros([50,2,3],'single');      % Y: 100-by-2-by-3 single amat
disp('List current variables:')
whos
disp('Print Z amat object:')
Z
```

Output

```
List current variables :
  Name             Size           Bytes  Class    Attributes

  SaveOptions      1x6              721  cell
  X              100x2x4           6432  amat
  Y              200x3x3          14432  amat
  Z               50x2x3           1232  amat

Print Z amat object :

Z =

Z is a 50x2x3 amat[single] object
  Z(1)=
     0     0     0
     0     0     0

  Z(2)=
     0     0     0
     0     0     0

  ...

  Z(49)=
     0     0     0
     0     0     0

  Z(50)=
     0     0     0
     0     0     0
```

4.2.2 `fc_amat.ones` function

Syntaxe

```
X=fc_amat.ones(N,m,n)
X=fc_amat.ones([N,m,n])
X=fc_amat.ones(N,d)
X=fc_amat.ones(...,classname)
```

Description

`X=fc_amat.ones(N,m,n)` return a `N`-by-`m`-by-`n` `amat` object of ones.

`X=fc_amat.ones([N,m,n])` same as `X=fc_amat.ones(N,m,n)`

`X=fc_amat.ones(N,d)` same as `X=fc_amat.ones(N,d,d)`

`X=fc_amat.ones(...,classname)` returns an `amat` object with values of class `classname`

In Listing 7, some examples are provided.

Listing 8: : examples of `fc_amat.ones` function usage

```
X=fc_amat.ones(100,2,4);                % X: 100-by-2-by-4 amat
Y=fc_amat.ones(200,3);                  % Y: 200-by-3-by-3 amat
Z=fc_amat.ones([50,2,3],'single');      % Y: 50-by-2-by-3 single amat
disp('List current variables:')
whos
disp('Print Z amat object:')
Z
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x2x4            6432  amat
  Y              200x3x3           14432  amat
  Z               50x2x3            1232  amat

Print Z amat object :

Z =

Z is a 50x2x3 amat[single] object
  Z(1)=
     1     1     1
     1     1     1

  Z(2)=
     1     1     1
     1     1     1

  ...

  Z(49)=
     1     1     1
     1     1     1

  Z(50)=
     1     1     1
     1     1     1
```

4.2.3 `fc_amat.eye` function

Syntaxe

```
X=fc_amat.eye(N,d)
X=fc_amat.eye(N,m,n)
X=fc_amat.eye([N,m,n])
X=fc_amat.eye(...,classname)
```

Description

`X=fc_amat.eye(N,d)` return a `N`-by-`d`-by-`d` `amat` object whose all its matrices are the `d`-by-`d` identity matrix.

`X=fc_amat.eye(N,m,n)` return a `N`-by-`m`-by-`n` `amat` object whose all its matrices are the `m`-by-`n` matrix with one's on the diagonal and zeros elsewhere.

`X=fc_amat.eye([N,m,n])` same as `X=fc_amat.eye(N,m,n)`

`X=fc_amat.eye(...,classname)` returns an `amat` object with values of class `classname`

In Listing 7, some examples are provided.

Listing 9: : examples of `fc_amat.eye` function usage

```
X=fc_amat.eye(100,2,4);          % X: 100-by-2-by-4 amat
Y=fc_amat.eye(200,3,'int32'); % Y: 200-by-3-by-3 int32 amat
Z=fc_amat.eye([50,2,3]);         % Z: 50-by-2-by-3 amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size            Bytes  Class   Attributes

  SaveOptions      1x6               721  cell
  X              100x2x4            6432  amat
  Y              200x3x3            7232  amat
  Z               50x2x3            2432  amat

Print Y amat object :

Y =

Y is a 200x3x3 amat[int32] object
  Y(1)=
   1   0   0
   0   1   0
   0   0   1

  Y(2)=
   1   0   0
   0   1   0
   0   0   1

  ...

  Y(199)=
   1   0   0
   0   1   0
   0   0   1

  Y(200)=
   1   0   0
   0   1   0
   0   0   1
```

4.3 Random generators

There is the list of functions which generate some `amat` objects with random elements. They all belong to the namespace `fc_amat.random` :

- `rand` , `randn` , `randi` random elements,
- `randsym` , `randnsym` , `randisym` random **symmetric** matrices,
- `randsym` , `randnsym` , `randisym` random **Hermitian** matrices,
- `randdiag` , `randndiag` , `randidiag` random **diagonal** matrices,
- `randtril` , `randntril` , `randitril` random **lower triangular** matrices,

- `randtriu` , `randntriu` , `randitriu` random **upper triangular** matrices,
- `randsdd` , `randnsdd` , `randisdd` random **stricly diagonally dominant** matrices,
- `randsympd` , `randnsympd` , `randisympd` random **symmetric positive definite** matrices,
- `randherpd` , `randnherpd` , `randiherpd` random **Hermitian positive definite** matrices.

4.3.1 `fc_amat.random.rand` function

The `fc_amat.random.rand` function return an `amat` object with random elements uniformly distributed on the interval $]0,1[$.

Syntaxe

```
X=fc_amat.random.rand(N,m,n)
X=fc_amat.random.rand([N,m,n])
X=fc_amat.random.rand(N,d)
X=fc_amat.random.rand(...,classname)
```

Description

`X=fc_amat.random.rand(N,m,n)` return a `N`-by-`m`-by-`n` `amat` object with random elements uniformly distributed on the interval $]0,1[$.

`X=fc_amat.random.rand([N,m,n])` same as `X=fc_amat.random.rand(N,m,n)`

`X=fc_amat.random.rand(N,d)` same as `X=fc_amat.random.rand(N,d,d)`

`X=fc_amat.random.rand(...,classname)` returns an `amat` object with values of class `classname` . `classname` could be `'single'` or `'double'` (default).

In Listing 10, some examples are provided.

Listing 10: : examples of `fc_amat.random.rand` function usage

```
X=fc_amat.random.rand(100,2,4);                % X: 100-by-2-by-4 amat
Y=fc_amat.random.rand(200,3);                  % Y: 200-by-3-by-3 amat
Z=fc_amat.random.rand([50,2,3],'single');      % Y: 50-by-2-by-3 single ...
    amat
disp('List current variables :')
whos
disp('Print Z amat object :')
Z
```

Output

```
List current variables :
  Name             Size           Bytes  Class   Attributes

  SaveOptions      1x6              721  cell
  X              100x2x4           6432  amat
  Y              200x3x3          14432  amat
  Z               50x2x3           1232  amat

Print Z amat object :

Z =

Z is a 50x2x3 amat[single] object
  Z(1)=
    0.0499   0.4716   0.4880
    0.6976   0.4794   0.9926

  Z(2)=
    0.5459   0.5430   0.4978
    0.2037   0.8985   0.4558

  ...

  Z(49)=
    0.1709   0.2399   0.9820
    0.0149   0.7151   0.8497

  Z(50)=
    0.3993   0.5977   0.5136
    0.1567   0.5048   0.2834
```

4.3.2 `fc_amat.random.randn` function

The `fc_amat.random.randn` function return an `amat` object with normally distributed random elements having zero mean and variance one.

Syntaxe

```
X=fc_amat.random.randn(N,m,n)
X=fc_amat.random.randn([N,m,n])
X=fc_amat.random.randn(N,d)
X=fc_amat.random.randn(...,classname)
```

Description

`X=fc_amat.random.randn(N,m,n)`

returns a `N`-by-`m`-by-`n` `amat` object with normally distributed random elements having zero mean and variance one.

`X=fc_amat.random.randn([N,m,n])`

same as `X=fc_amat.random.randn(N,m,n)`

`X=fc_amat.random.randn(N,d)`

same as `X=fc_amat.random.randn(N,d,d)`

`X=fc_amat.random.randn(...,classname)`

returns an `amat` object with values of class `classname`. `classname` could be `'single'` or `'double'` (default).

In Listing 10, some examples are provided.

Listing 11: : examples of `fc_amat.random.randn` function usage

```
X=fc_amat.random.randn(100,2,4);                % X: 100-by-2-by-4 amat
Y=fc_amat.random.randn(200,3);                  % Y: 200-by-3-by-3 amat
Z=fc_amat.random.randn([50,2,3],'single');      % Y: 50-by-2-by-3 single ...
    amat
disp('List current variables :')
whos
disp('Print Z amat object :')
Z
```

Output

```
List current variables :
  Name             Size           Bytes  Class   Attributes

  SaveOptions      1x6              721  cell
  X              100x2x4           6432  amat
  Y              200x3x3          14432  amat
  Z               50x2x3           1232  amat

Print Z amat object :

Z =

Z is a 50x2x3 amat[single] object
  Z(1)=
    1.1754   0.1834  -0.3981
    0.2753  -0.0281   0.9842

  Z(2)=
    0.3977  -0.2597   0.2564
    0.3533   0.1404   0.9931

  ...

  Z(49)=
    0.0987  -0.2084  -0.4723
   -0.2993   0.3512   0.0661

  Z(50)=
    1.6843   0.0247   0.2760
   -1.7210  -0.8723  -0.6379
```

4.3.3 `fc_amat.random.randi` function

The function `fc_amat.random.randi` return an `amat` object whose elements are random integers.

Syntaxe

```
X=fc_amat.random.randi(Imax,N,m,n)
X=fc_amat.random.randi(Imax,[N,m,n])
X=fc_amat.random.randi(Imax,N,d)
```

```
X=fc_amat.random.randi([Imin,Imax],...)
X=fc_amat.random.randi(...,classname)
```

Description

`X=fc_amat.random.randi(Imax,N,m,n)`

returns a `N`-by-`m`-by-`n` `amat` object containing pseudorandom integer values drawn from the discrete uniform distribution on `1:Imax`.

`X=fc_amat.random.randi(Imax,[N,m,n])`

same as `X=fc_amat.random.randi(Imax,N,m,n)`

`X=fc_amat.random.randi(Imax,N,d)`

same as `X=fc_amat.random.randi(Imax,N,d,d)`

`X=fc_amat.random.randi([Imin,Imax],...)`

returns an `amat` object containing integer values drawn from the discrete uniform distribution on `Imin:Imax`.

`X=fc_amat.random.randi(...,classname)`

returns an `amat` object with values of class `classname`. Accepted `classname` strings are those of the `randi` Matlab function. Default is `'double'`.

In Listing 10, some examples are provided.

Listing 12: : examples of `fc_amat.random.randi` function usage

```
X=fc_amat.random.randi(10,100,2,4);                % X: 100-by-2-by-4 amat
Y=fc_amat.random.randi(15,200,3);                  % Y: 200-by-3-by-3 amat
Z=fc_amat.random.randi([-5,5],[50,2,3],'int32'); % Z: 50-by-2-by-3 ...
    int32 amat
disp('List current variables:')
whos
disp('Print Z amat object:')
Z
```

Output

```
List current variables:
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x2x4            6432  amat
  Y              200x3x3           14432  amat
  Z               50x2x3            1232  amat

Print Z amat object:

Z =

Z is a 50x2x3 amat[int32] object
  Z(1)=
   -5    0    0
    2    0    5

  Z(2)=
    1    0    0
   -3    4    0

  ...

  Z(49)=
   -4   -3    5
   -5    2    4

  Z(50)=
   -1    1    0
   -4    0   -2
```

4.3.4 `fc_amat.random.randsym` function

The `fc_amat.random.randsym` function return an `amat` object whose matrices are symmetric with random elements uniformly distributed on the interval $]0,1[$.

Syntaxe

```
X=fc_amat.random.randsym(N,d)
X=fc_amat.random.randsym(N,d,'class',value)
```

Description

`X=fc_amat.random.randsym(N,d)`

return a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric with random elements uniformly distributed on the interval $]0,1[$.

`X=fc_amat.random.randsym(N,d,'class',classname)`

returns an `amat` object with values of class `classname`. `classname` could be `'single'` or `'double'` (default).

In Listing 13, some examples are provided.

Listing 13: : examples of `fc_amat.random.randsym` function usage

```
X=fc_amat.random.randsym(100,3);                    % X: 100-by-3-by-3 amat
Y=fc_amat.random.randsym(50,2,'class','single'); % Y: 50-by-2-by-2 ...
    single amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y               50x2x2             832  amat

Print Y amat object :

Y =

Y is a 50x2x2 amat[single] object
  Y(1)=
    0.8507   0.1465
    0.1465   0.5590

  Y(2)=
    0.5606   0.1891
    0.1891   0.8541

  ...

  Y(49)=
    0.1231   0.0252
    0.0252   0.0934

  Y(50)=
    0.2055   0.8422
    0.8422   0.3074
```

4.3.5 `fc_amat.random.randnsym` function

The `fc_amat.random.randnsym` function return an `amat` object whose matrices are symmetric with normally distributed random elements having zero mean and variance one.

Syntaxe

```
X=fc_amat.random.randnsym(N,d)
X=fc_amat.random.randnsym(N,d,'class',value)
```

Description

`X=fc_amat.random.randnsym(N,d)`

return a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric normally distributed random elements having zero mean and variance one.

`X=fc_amat.random.randnsym(N,d,'class',classname)`

returns an `amat` object with values of class `classname`. `classname`

could be `'single'` or `'double'` (default).

In Listing 14, some examples are provided.

Listing 14: : examples of `fc_amat.random.randnsym` function usage

```
X=fc_amat.random.randnsym(100,3);                    % X: 100-by-3-by-3 ...
    amat
Y=fc_amat.random.randnsym(50,2,'class','single'); % Y: 50-by-2-by-2 ...
    single amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y               50x2x2             832  amat

Print Y amat object :

Y =

Y is a 50x2x2 amat[single] object
  Y(1)=
    0.4759   -1.4827
   -1.4827    1.9085

  Y(2)=
    1.4122   -0.0438
   -0.0438    0.1222

  ...

  Y(49)=
    0.0931    0.5037
    0.5037   -0.7006

  Y(50)=
   -0.3782   -0.8927
   -0.8927   -1.6305
```

4.3.6 `fc_amat.random.randisym` function

The `fc_amat.random.randisym` function return an `amat` object whose matrices are symmetric with random integers values.

Syntaxe

```
X=fc_amat.random.randisym(Imax,N,d)
X=fc_amat.random.randisym([Imin,Imax],...)
X=fc_amat.random.randisym(...,'class',classname)
```

Description

`X=fc_amat.random.randisym(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric pseudo random integer values drawn from the discrete uniform distribution on `1:Imax`

`X=fc_amat.random.randisym([Imin,Imax], ...)`

pseudo random integer values are drawn from the discrete uniform distribution on `Imin:Imax`

`X=fc_amat.random.randisym(...,'class',classname)`

returns an `amat` object with values of class `classname` . Accepted `classname` strings are those of the `randi` Matlab function. Default is `'double'` .

In Listing 15, some examples are provided.

Listing 15: : examples of `fc_amat.random.randisym` function usage

```
X=fc_amat.random.randisym(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randisym([-5,5],100,2,'class','single');
% Y: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              100x2x2            1632  amat

Print Y amat object :

Y =

Y is a 100x2x2 amat[single] object
  Y(1)=
     4     1
     1     2

  Y(2)=
     1     4
     4    -4

  ...

  Y(99)=
    -5     3
     3    -4

  Y(100)=
     4     5
     5     1
```

4.3.7 `fc_amat.random.randher` function

The `fc_amat.random.randher` function return an `amat` object whose matrices are hermitian with random real part elements uniformly distributed on the interval $]0,1[$ and imaginary part elements uniformly distributed on the interval $]-1,1[$.

Syntaxe

```
X=fc_amat.random.randher(N,d)
X=fc_amat.random.randher(...,'class',value)
```

Description

`X=fc_amat.random.randher(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric with random elements uniformly distributed on the interval $]0,1[$.

`X=fc_amat.random.randher(...,'class',classname)`

returns an `amat` object with values of class `classname`. `classname` could be `'single'` or `'double'` (default).

In Listing 16, some examples are provided.

Listing 16: : examples of `fc_amat.random.randher` function usage

```
X=fc_amat.random.randher(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randher(50,2,'class','single');
% Y: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3           14432  amat
  Y               50x2x2            1632  amat

Print Y amat object :

Y =

Y is a 50x2x2 amat[complex single] object
  Y(1)=
   0.5038 - 0.4281i 0.4067 - 0.7659i
   0.4067 + 0.7659i 0.5038 - 0.9608i

  Y(2)=
   0.4896 + 0.0873i 0.6669 + 0.6294i
   0.6669 - 0.6294i 0.6128 - 0.1296i

  ...

  Y(49)=
   0.9227 + 0.3077i 0.4385 + 0.7957i
   0.4385 - 0.7957i 0.0563 - 0.2196i

  Y(50)=
   0.8004 - 0.8559i 0.4378 + 0.1867i
   0.4378 - 0.1867i 0.1525 - 0.2791i
```

4.3.8 `fc_amat.random.randnher` function

The `fc_amat.random.randnher` function return an `amat` object whose matrices are hermitian with normally distributed random real and imaginary part elements having zero mean and variance one.

Syntaxe

```
X=fc_amat.random.randnher(N,d)
X=fc_amat.random.randnher(...,'class',value)
```

Description

`X=fc_amat.random.randnher(N,d)`

returns a N-by-d-by-d amat object whose matrices are Hermitian normally distributed random elements having zero mean and variance one.

`X=fc_amat.random.randnher(...,'class',classname)`

returns an amat object with values of class classname. classname could be 'single' or 'double' (default).

In Listing 17, some examples are provided.

Listing 17: : examples of fc_amat.random.randnher function usage

```
X=fc_amat.random.randnher(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randnher(50,2,'class','single');
% Y: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Y amat object :')
Y
```

Output

```
List current variables :
  Name             Size           Bytes Class   Attributes

  SaveOptions      1x6              721 cell
  X              100x3x3          14432 amat
  Y               50x2x2           1632 amat

Print Y amat object :

Y =

Y is a 50x2x2 amat[complex single] object
  Y(1)=
   1.3419 + 0.3210i -0.4049 - 0.5741i
  -0.4049 + 0.5741i -1.5144 - 0.6035i

  Y(2)=
  -0.9884 + 1.6234i 0.5279 - 0.1952i
   0.5279 + 0.1952i 1.0262 - 0.7970i

  ...

  Y(49)=
   0.8751 - 1.0922i 0.3024 - 0.1006i
   0.3024 + 0.1006i 0.1736 + 0.4039i

  Y(50)=
   1.3954 - 0.2258i 0.0583 - 1.6250i
   0.0583 + 1.6250i 0.4536 + 1.1925i
```

4.3.9 fc_amat.random.randiher function

The fc_amat.random.randiher function return an amat object whose matrices are Hermitian with random integers values.

Syntaxe

```
X=fc_amat.random.randiher(Imax,N,d)
X=fc_amat.random.randiher([Imin,Imax],...)
X=fc_amat.random.randiher(...,'class',classname)
```

Description

`X=fc_amat.random.randiher(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are Hermitian where real and imaginay part values are respectively drawn from the discrete uniform distribution on `1:Imax` and the discrete uniform distribution on `1:Imax` times a random sign.

`X=fc_amat.random.randiher([Imin,Imax], ...)`

pseudorandom integer values are drawn from the discrete uniform distribution on `Imin:Imax`

`X=fc_amat.random.randiher(...,'class',classname)`

returns an `amat` object with values of class `classname`. Accepted `classname` strings are those of the `randi` Matlab function. Default is `'double'`.

In Listing 18, some examples are provided.

Listing 18: : examples of `fc_amat.random.randiher` function usage

```
X=fc_amat.random.randiher(10,100,3);  % X: 100-by-3-by-3 amat
info(X)
Y=fc_amat.random.randiher([-5,5],100,2,'class','single');
% Y: 50-by-2-by-2 single amat
disp('Print␣Y␣amat␣object␣:')
Y
```

Output

```
X is a 100x3x3 amat[complex double] object
Print Y amat object :

Y =

Y is a 100x2x2 amat[complex single] object
  Y(1)=
  -3.0000 - 3.0000i 5.0000 + 4.0000i
   5.0000 - 4.0000i 3.0000 - 5.0000i

  Y(2)=
  -3.0000 + 4.0000i 1.0000 + 4.0000i
   1.0000 - 4.0000i -3.0000 + 1.0000i

  ...

  Y(99)=
   1.0000 - 1.0000i 2.0000 + 0.0000i
   2.0000 + 0.0000i -1.0000 - 5.0000i

  Y(100)=
   4.0000 - 1.0000i 4.0000 - 3.0000i
   4.0000 + 3.0000i 1.0000 - 4.0000i
```

4.3.10 `fc_amat.random.randdiag` function

The `fc_amat.random.randdiag` function return an `amat` object whose matrices are diagonal with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

Syntaxe

```
X=fc_amat.random.randdiag(N,d)
X=fc_amat.random.randdiag(...,key,value)
```

Description

`X=fc_amat.random.randdiag(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are diagonal with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

`X=fc_amat.random.randdiag(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the diagonal matrices elements are also drawn from the uniform distribution on the interval $]a, b[$. (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'a'` , to set a (lower bound of the interval) value (0 by default).
- `'b'` , to set b (upper bound of the interval) value (1 by default).

In Listing 19, some examples are provided.

Listing 19: : examples of fc_amat.random.randdiag function usage

```
X=fc_amat.random.randdiag(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randdiag(200,3,'nc',4,'complex',true,'a',-1);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randdiag(50,3,'class','single','k',1,'b',5);
% Z: 50-by-3-by-3 single amat
disp('Print␣Z␣amat␣object␣:')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x3 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
        0   0.7013        0
        0        0   1.5901
        0        0        0

  Z(2)=
        0   1.3007        0
        0        0   3.0432
        0        0        0

  ...

  Z(49)=
        0   4.2819        0
        0        0   0.7392
        0        0        0

  Z(50)=
        0   2.0122        0
        0        0   0.9908
        0        0        0
```

4.3.11 fc_amat.random.randndiag function

The fc_amat.random.randndiag function return an amat object whose matrices are diagonal with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

Syntaxe

```
X=fc_amat.random.randndiag(N,d)
X=fc_amat.random.randndiag(...,key,value)
```

Description

`X=fc_amat.random.randndiag(N,d)`

returns a N-by-d-by-d amat object whose matrices are diagonal with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

`X=fc_amat.random.randndiag(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- 'complex' , if value is true the amat object is complex and the imaginary parts of the diagonal matrices elements are also drawn

from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)

- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'mean'` , to set mean of the normal distribution (0 by default).
- `'sigma'` , to set standard deviation of the normal distribution (1 by default).

In Listing 20, some examples are provided.

Listing 20: : examples of `fc_amat.random.randndiag` function usage

```
X=fc_amat.random.randndiag(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randndiag(200,3,'nc',4,'complex',true,'sigma',5);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randndiag(50,3,'class','single','k',-1,'mean',4);
% Z: 50-by-3-by-3 single amat
disp('Print␣Z␣amat␣object␣:')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x3 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
       0       0       0
  2.3152       0       0
       0  4.4274       0

  Z(2)=
       0       0       0
  4.4131       0       0
       0  4.1810       0

  ...

  Z(49)=
       0       0       0
  4.0053       0       0
       0  2.9046       0

  Z(50)=
       0       0       0
  5.1391       0       0
       0  4.5492       0
```

4.3.12 `fc_amat.random.randidiag` function

The `fc_amat.random.randidiag` function return an `amat` object whose matrices are diagonal and non zeros elements are random integers

Syntaxe

```
X=fc_amat.random.randidiag(Imax,N,d)
X=fc_amat.random.randidiag([Imin,Imax],...)
X=fc_amat.random.randidiag(...,key,value)
```

Description

`X=fc_amat.random.randidiag(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are diagonal and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `1:Imax` .

`X=fc_amat.random.randidiag([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are diagonal and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `Imin:Imax` .

`X=fc_amat.random.randidiag(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the diagonal matrices elements are also drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value are those of the `randi` Matlab function. Default is `'double'` .
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .

In Listing 21, some examples are provided.

Listing 21: : examples of `fc_amat.random.randidiag` function usage

```
X=fc_amat.random.randidiag(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randidiag(8,200,3,'nc',4,'complex',true);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randidiag([-5,5],50,3,'class','single','k',1);
% Z: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x3           28832  amat
  Z               50x3x3            1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
     0    -4     0
     0     0    -2
     0     0     0

  Z(2)=
     0    -3     0
     0     0     1
     0     0     0

  ...

  Z(49)=
     0     4     0
     0     0    -4
     0     0     0

  Z(50)=
     0    -1     0
     0     0    -3
     0     0     0
```

4.3.13 `fc_amat.random.randtril` function

The `fc_amat.random.randtril` function return an `amat` object whose matrices are lower triangular with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

Syntaxe

```
X=fc_amat.random.randtril(N,d)
X=fc_amat.random.randtril(...,key,value)
```

Description

`X=fc_amat.random.randtril(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are lower triangular with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

```
X=fc_amat.random.randtril(...,key,value)
```

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the lower triangular matrices elements are also drawn from the uniform distribution on the interval $]a, b[$. (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'a'` , to set a (lower bound of the interval) value (0 by default).
- `'b'` , to set b (upper bound of the interval) value (1 by default).

In Listing 22, some examples are provided.

Listing 22: : examples of `fc_amat.random.randtril` function usage

```
X=fc_amat.random.randtril(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randtril(200,3,'nc',4,'complex',true,'a',-1);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randtril(50,3,'class','single','k',1,'b',5);
% Z: 50-by-3-by-3 single amat
disp('Print␣Z␣amat␣object␣:')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x4 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
    1.8980   3.7119        0
    4.7610   1.7555   1.2448
    2.5337   2.7349   2.8706

  Z(2)=
    1.5953   4.6872        0
    2.7164   0.2715   1.9321
    1.9063   2.0148   2.4347

  ...

  Z(49)=
    0.1900   3.2218        0
    3.9782   2.3686   0.0738
    4.8427   1.0322   3.7255

  Z(50)=
    4.7712   0.8462        0
    3.9055   4.7560   3.5138
    0.4938   1.6930   3.6469
```

4.3.14 `fc_amat.random.randntril` function

The `fc_amat.random.randntril` function return an `amat` object whose matrices are lower triangular with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

Syntaxe

```
X=fc_amat.random.randntril(N,d)
X=fc_amat.random.randntril(...,key,value)
```

Description

`X=fc_amat.random.randntril(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are lower triangular with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

`X=fc_amat.random.randntril(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the lower triangular matrices elements are also drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'mean'` , to set mean of the normal distribution (0 by default).
- `'sigma'` , to set standard deviation of the normal distribution (1 by default).

In Listing 23, some examples are provided.

Listing 23: : examples of `fc_amat.random.randntril` function usage

```
X=fc_amat.random.randntril(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randntril(200,3,'nc',4,'complex',true,'sigma',5);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randntril(50,3,'class','single','k',-1,'mean',4);
% Z: 50-by-3-by-3 single amat
disp('Print Z amat object :')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x4 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
        0        0        0
   3.8961        0        0
   4.8726   4.6571        0

  Z(2)=
        0        0        0
   3.1771        0        0
   4.1179   3.2465        0

  ...

  Z(49)=
        0        0        0
   2.2368        0        0
   4.9195   3.3574        0

  Z(50)=
        0        0        0
   3.6767        0        0
   4.8522   3.9211        0
```

4.3.15 `fc_amat.random.randitril` function

The `fc_amat.random.randitril` function return an `amat` object whose matrices are lower triangular and non zeros elements are random integers

Syntaxe

```
X=fc_amat.random.randitril(Imax,N,d)
X=fc_amat.random.randitril([Imin,Imax],...)
X=fc_amat.random.randitril(...,key,value)
```

Description

`X=fc_amat.random.randitril(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are lower triangular and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `1:Imax` .

`X=fc_amat.random.randitril([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are lower triangular and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `Imin:Imax` .

`X=fc_amat.random.randitril(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the lower triangular matrices elements are also drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value are those of the `randi` Matlab function. Default is `'double'` .
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .

In Listing 24, some examples are provided.

Listing 24: : examples of `fc_amat.random.randitril` function usage

```
X=fc_amat.random.randitril(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randitril(8,200,3,'nc',4,'complex',true);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randitril([-5,5],50,3,'class','single','k',1);
% Z: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size           Bytes Class   Attributes

  SaveOptions      1x6              721 cell
  X              100x3x3           7232 amat
  Y              200x3x4          38432 amat
  Z               50x3x3           1832 amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
    -1    3    0
     5   -2   -3
     0    1    1

  Z(2)=
    -2    5    0
     0   -5   -1
    -1   -1    0

  ...

  Z(49)=
    -5    2    0
     3    0   -5
     5   -3    3

  Z(50)=
     5   -4    0
     3    5    2
    -4   -2    3
```

4.3.16 `fc_amat.random.randtriu` function

The `fc_amat.random.randtriu` function return an `amat` object whose matrices are upper triangular with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

Syntaxe

```
X=fc_amat.random.randtriu(N,d)
X=fc_amat.random.randtriu(...,key,value)
```

Description

`X=fc_amat.random.randtriu(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are diagonal with non zeros elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

`X=fc_amat.random.randtriu(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the upper triangular matrices elements are also drawn from the uniform distribution on the interval $]a, b[$. (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'a'` , to set a (lower bound of the interval) value (0 by default).
- `'b'` , to set b (upper bound of the interval) value (1 by default).

In Listing 25, some examples are provided.

Listing 25: : examples of fc_amat.random.randtriu function usage

```
X=fc_amat.random.randtriu(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randtriu(200,3,'nc',4,'complex',true,'a',-1);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randtriu(50,3,'class','single','k',-1,'b',5);
% Z: 50-by-3-by-3 single amat
disp('Print␣Z␣amat␣object␣:')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x4 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
    0.6266   1.2508   2.6523
    4.2032   3.7222   4.7936
         0   4.9171   3.2082

  Z(2)=
    1.8224   3.1229   0.3704
    2.3441   4.1967   3.5592
         0   4.4815   3.9319

  ...

  Z(49)=
    4.9196   0.7504   4.9152
    2.3943   3.4633   4.6067
         0   0.2439   1.2681

  Z(50)=
    4.8989   4.7924   2.9260
    1.5406   1.0191   4.9201
         0   1.5692   4.2159
```

4.3.17 fc_amat.random.randntriu function

The fc_amat.random.randntriu function return an amat object whose matrices are upper triangular with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

Syntaxe

```
X=fc_amat.random.randntriu(N,d)
X=fc_amat.random.randntriu(...,key,value)
```

Description

`X=fc_amat.random.randntriu(N,d)`

returns a N-by-d-by-d amat object whose matrices are upper triangular with non zeros elements drawn from the normal distribution having zero mean and unit standard deviation.

`X=fc_amat.random.randntriu(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- 'complex' , if value is true the amat object is complex and the imaginary parts of the upper triangular matrices elements are also

drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)

- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .
- `'mean'` , to set mean of the normal distribution (0 by default).
- `'sigma'` , to set standard deviation of the normal distribution (1 by default).

In Listing 26, some examples are provided.

Listing 26: : examples of `fc_amat.random.randntriu` function usage

```
X=fc_amat.random.randntriu(100,3);
info(X) % X: 100-by-3-by-3 amat
Y=fc_amat.random.randntriu(200,3,'nc',4,'complex',true,'sigma',5);
info(Y) % Y: 200-by-3-by-4 amat
Z=fc_amat.random.randntriu(50,3,'class','single','k',-1,'mean',4);
% Z: 50-by-3-by-3 single amat
disp('Print␣Z␣amat␣object␣:')
disp(Z)
```

Output

```
X is a 100x3x3 amat[double] object
Y is a 200x3x4 amat[complex double] object
Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
    1.9633   3.9498   3.9330
    2.8153   3.9781   5.8515
         0   3.9147   2.2565

  Z(2)=
    4.0779   3.6113   3.2827
    4.3404   5.0548   3.9939
         0   2.7159   5.4148

  ...

  Z(49)=
    3.2304   4.5675   3.1868
    4.7068   4.8699   3.9731
         0   3.0846   4.8731

  Z(50)=
    2.7639   4.2766   4.1320
    5.5403   3.8129   3.2579
         0   4.9401   3.4746
```

4.3.18 `fc_amat.random.randitriu` function

The `fc_amat.random.randitriu` function return an `amat` object whose matrices are upper triangular and non zeros elements are random integers

Syntaxe

```
X=fc_amat.random.randitriu(Imax,N,d)
X=fc_amat.random.randitriu([Imin,Imax],...)
X=fc_amat.random.randitriu(...,key,value)
```

Description

`X=fc_amat.random.randitriu(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are upper triangular and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `1:Imax` .

`X=fc_amat.random.randitriu([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are upper triangular and non zeros elements are pseudorandom integer drawn from the discrete uniform distribution on `Imin:Imax` .

`X=fc_amat.random.randitriu(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the upper triangular matrices elements are also drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value are those of the `randi` Matlab function. Default is `'double'` .
- `'nc'` , number of columns of the matrices (default: `d`)
- `'k'` , offset of `k` diagonals above or below the main diagonal; above for positive `k` and below for negative `k` .

In Listing 27, some examples are provided.

Listing 27: : examples of `fc_amat.random.randitriu` function usage

```
X=fc_amat.random.randitriu(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randitriu(8,200,3,'nc',4,'complex',true);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randitriu([-5,5],50,3,'class','single','k',1);
% Z: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x4           38432  amat
  Z               50x3x3            1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
     0    -4    -3
     0     0     0
     0     0     0

  Z(2)=
     0    -1     1
     0     0    -5
     0     0     0

  ...

  Z(49)=
     0     5    -4
     0     0     5
     0     0     0

  Z(50)=
     0     5     5
     0     0     1
     0     0     0
```

4.3.19 `fc_amat.random.randsdd` function

The `fc_amat.random.randsdd` function return an `amat` object whose matrices are strictly diagonally dominant with non-diagonal elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

Syntaxe

```
X=fc_amat.random.randsdd(N,d)
X=fc_amat.random.randsdd(...,key,value)
```

Description

`X=fc_amat.random.randsdd(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant with non-diagonal elements drawn from the uniform distribution on the interval $]a, b[=]0, 1[$.

```
X=fc_amat.random.randsdd(...,key,value)
```

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts elements are also drawn from the uniform distribution on the interval $]a, b[=]0, 1[$. (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'a'` , to set a (lower bound of the interval) value (0 by default).
- `'b'` , to set b (upper bound of the interval) value (1 by default).

In Listing 28, some examples are provided.

Listing 28: : examples of `fc_amat.random.randsdd` function usage

```
X=fc_amat.random.randsdd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randsdd(200,3,'a',-2,'b',2);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randsdd(50,3,'complex',true,'a',-1,'class','single');
% Z: 50-by-3-by-3 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x3           14432  amat
  Z               50x3x3            3632  amat

Print Z amat object :
Z is a 50x3x3 amat[complex single] object
  Z(1)=
   2.8843 - 0.3015i -0.6963 - 0.7494i 0.7247 + 0.6813i
   0.5270 + 0.7701i -1.4158 - 1.7475i 0.5716 + 0.4889i
   0.6897 - 0.2498i 0.8189 + 0.0609i -0.6773 + 2.3895i

  Z(2)=
   1.1409 - 0.8701i -0.2387 - 0.2710i 0.7928 - 0.0624i
  -0.9023 + 0.6796i 0.7288 - 3.1322i 0.8451 + 0.6787i
   0.6331 + 0.6578i -0.8843 - 0.8518i 3.1005 + 0.5283i

  ...

  Z(49)=
   1.8708 + 0.5921i 0.0946 + 0.9678i 0.3094 - 0.0423i
   0.5501 - 0.4129i -0.0539 + 2.3680i -0.6998 + 0.3853i
   0.6844 + 0.5281i -0.4048 + 0.9661i 0.0471 + 3.0565i

  Z(50)=
  -1.1309 + 2.6668i 0.1070 + 0.9596i -0.9477 - 0.3838i
  -0.5264 - 0.7696i 1.6560 - 2.4976i 0.9826 - 0.5924i
   0.0333 + 0.9759i 0.0501 + 0.1704i -2.1163 + 0.7265i
```

4.3.20 `fc_amat.random.randnsdd` function

The `fc_amat.random.randnsdd` function return an `amat` object whose matrices are strictly diagonally dominant with non-diagonal elements drawn from the normal distribution having zero mean and unit standard deviation.

Syntaxe

```
X=fc_amat.random.randnsdd(N,d)
X=fc_amat.random.randnsdd(...,key,value)
```

Description

`X=fc_amat.random.randnsdd(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant with non-diagonal elements drawn from the normal distribution having zero mean and unit standard deviation.

`X=fc_amat.random.randnsdd(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the upper triangular matrices elements are also drawn from the normal distribution having zero mean and unit standard deviation (default `false` i.e real `amat` object)
- `'class'` , to set `amat` object data type; value could be `'single'` or `'double'` (default).
- `'mean'` , to set mean of the normal distribution (0 by default).
- `'sigma'` , to set standard deviation of the normal distribution (1 by default).

In Listing 29, some examples are provided.

Listing 29: : examples of `fc_amat.random.randnsdd` function usage

```
X=fc_amat.random.randnsdd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randnsdd(200,3,'complex',true,'sigma',5);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randnsdd(50,3,'class','single','mean',5);
% Z: 50-by-3-by-3 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x3           28832  amat
  Z               50x3x3            1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
   14.9935   3.8108   4.8541
    6.5726  16.0219   4.0699
    5.1120   5.3285 -15.7116

  Z(2)=
  -11.7322   3.0651   3.8616
    6.0557 -18.1866   5.3382
    6.6613   4.0512  16.0673

  ...

  Z(49)=
   14.4837   5.0342   5.1701
    5.1146 -15.5134   7.1708
    5.3230   6.1006 -17.4436

  Z(50)=
  -16.8319   4.6791   6.3048
    4.2776 -13.9625   4.6583
    4.0640   5.3629  14.7697
```

4.3.21 `fc_amat.random.randisdd` function

The `fc_amat.random.randisdd` function return an `amat` object whose matrices are strictly diagonally dominant with random integers

Syntaxe

```
X=fc_amat.random.randisdd(Imax,N,d)
X=fc_amat.random.randisdd([Imin,Imax],...)
X=fc_amat.random.randisdd(...,key,value)
```

Description

`X=fc_amat.random.randisdd(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudo random integer drawn from the discrete uniform distribution on `1:Imax` .

`X=fc_amat.random.randisdd([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudo random integer drawn from the discrete uniform distribution on `Imin:Imax` .

`X=fc_amat.random.randisdd(...,key,value)`

Some optional key/value pairs arguments are available with keys:

- `'complex'` , if value is `true` the `amat` object is complex and the imaginary parts of the non-diagonal elements are also drawn from the discrete uniform distribution (default `false` i.e real `amat` object).
- `'class'` , to set `amat` object data type; value are those of the `randi` Matlab function. Default is `'double'` .

In Listing 30, some examples are provided.

Listing 30: : examples of `fc_amat.random.randisdd` function usage

```
X=fc_amat.random.randisdd(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randisdd(8,200,3,'class','single');
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randisdd([-5,5],50,3,'class','single','complex',true);
% Z: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size           Bytes  Class    Attributes

  SaveOptions      1x6              721  cell
  X              100x3x3           7232  amat
  Y              200x3x3           7232  amat
  Z               50x3x3           3632  amat

Print Z amat object :
Z is a 50x3x3 amat[complex single] object
  Z(1)=
   9.0000 +17.0000i 4.0000 + 4.0000i 2.0000 + 5.0000i
   1.0000 - 3.0000i 4.0000 -14.0000i 4.0000 + 2.0000i
   5.0000 + 0.0000i -1.0000 + 5.0000i 10.0000 +10.0000i

  Z(2)=
   8.0000 - 9.0000i 4.0000 + 0.0000i 1.0000 + 4.0000i
   5.0000 + 1.0000i -13.0000 +13.0000i 4.0000 + 3.0000i
  -5.0000 - 5.0000i 5.0000 + 2.0000i 21.0000 + 3.0000i

  ...

  Z(49)=
  -7.0000 -14.0000i 2.0000 + 0.0000i -3.0000 - 5.0000i
   3.0000 - 4.0000i 7.0000 -14.0000i -2.0000 - 3.0000i
  -2.0000 + 5.0000i -4.0000 + 5.0000i 3.0000 -18.0000i

  Z(50)=
   5.0000 -17.0000i -5.0000 - 2.0000i -5.0000 - 2.0000i
   0.0000 + 5.0000i -12.0000 -14.0000i -4.0000 + 4.0000i
   0.0000 + 1.0000i 3.0000 + 5.0000i 2.0000 +14.0000i
```

4.3.22 `fc_amat.random.randsympd` function

The `fc_amat.random.randsympd` function return an `amat` object whose matrices are symmetric positive definite. This object is generated by using `randsdd` function from `fc_amat.random` namespace.

Syntaxe

```
X=fc_amat.random.randsympd(N,d)
X=fc_amat.random.randsympd(...,key,value)
```

Description

`X=fc_amat.random.randsympd(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric positive definite.

`X=fc_amat.random.randsympd(...,key,value)`

Optional key/value pairs arguments are those of the `fc_amat.random.randnsdd` function except for `'complex'` key which is forced to `false`. Keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).
- `'a'`, to set a (lower bound of the interval) value (0 by default).
- `'b'`, to set b (upper bound of the interval) value (1 by default).

In Listing 31, some examples are provided.

Listing 31: : examples of fc_amat.random.randsympd function usage

```
X=fc_amat.random.randsympd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randsympd(200,3,'a',-2,'b',2);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randsympd(50,3,'a',-1,'class','single');
% Z: 50-by-3-by-3 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x3           14432  amat
  Z               50x3x3            1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
    5.8555   -0.2914    0.7971
   -0.2914    3.0042   -1.0823
    0.7971   -1.0823    3.5891

  Z(2)=
    2.7439   -3.3075    1.7550
   -3.3075    8.2281   -0.0391
    1.7550   -0.0391    7.2384

  ...

  Z(49)=
    1.1942   -1.3214    0.6896
   -1.3214    3.3648    0.5249
    0.6896    0.5249    4.0609

  Z(50)=
    1.4089    0.6835    0.5095
    0.6835    2.2980   -1.5578
    0.5095   -1.5578    2.1885
```

4.3.23 fc_amat.random.randnsympd function

The fc_amat.random.randnsympd function return an amat object whose matrices are symmetric positive definite. This object is generated by using fc_amat.random.randnsdd function.

Syntaxe

```
X=fc_amat.random.randnsympd(N,d)
X=fc_amat.random.randnsympd(...,key,value)
```

Description

`X=fc_amat.random.randnsympd(N,d)`

returns a N-by-d-by-d amat object whose matrices are symmetric positive definite.

```
X=fc_amat.random.randnsympd(...,key,value)
```

Optional key/value pairs arguments are those of the `fc_amat.random.randnsdd` function except for `'complex'` key which is forced to `false`. Keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).
- `'mean'`, to set mean of the normal distribution (0 by default).
- `'sigma'`, to set standard deviation of the normal distribution (1 by default).

In Listing 32, some examples are provided.

Listing 32: : examples of `fc_amat.random.randnsympd` function usage

```
X=fc_amat.random.randnsympd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randnsympd(200,3,'sigma',5);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randnsympd(50,3,'class','single','mean',5);
% Z: 50-by-3-by-3 single amat
disp('List␣current␣variables␣:')
whos
disp('Print␣Z␣amat␣object␣:')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size           Bytes  Class    Attributes

  SaveOptions      1x6              721  cell
  X              100x3x3           7232  amat
  Y              200x3x3          14432  amat
  Z               50x3x3           1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
  259.0745 -150.2612 -99.9897
 -150.2612 365.9420 -141.3501
  -99.9897 -141.3501 224.8158

  Z(2)=
  290.4875 -168.3739 36.5276
 -168.3739 358.9001  54.7849
   36.5276  54.7849 297.8479

  ...

  Z(49)=
  308.4285   1.9906  54.0225
    1.9906 224.9108 151.7461
   54.0225 151.7461 301.8139

  Z(50)=
  259.6917 137.2443   8.2705
  137.2443 253.3166  67.6369
    8.2705  67.6369 269.5292
```

4.3.24 `fc_amat.random.randisympd` function

The `fc_amat.random.randisympd` function return an `amat` object whose matrices are symmetric positive definite with random integers. This object is generated by using `randisympd` function from `fc_amat.random` namespace.

Syntaxe

```
X=fc_amat.random.randisympd(Imax,N,d)
X=fc_amat.random.randisympd([Imin,Imax],...)
X=fc_amat.random.randisympd(...,key,value)
```

Description

`X=fc_amat.random.randisympd(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudo random integer drawn from the discrete uniform distribution on `1:Imax`.

`X=fc_amat.random.randisympd([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudo random integer drawn from the discrete uniform distribution on `Imin:Imax`.

`X=fc_amat.random.randisympd(...,key,value)`

Optional key/value pairs arguments are those of the `randisdd` function except for `'complex'` key which is forced to `false` and `'class'` key which can only be `'single'` or `'double'`. Keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).

In Listing 33, some examples are provided.

Listing 33: : examples of `fc_amat.random.randisympd` function usage

```
X=fc_amat.random.randisympd(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randisympd(8,200,3,'class','single');
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randisympd([-5,5],50,3,'class','single');
% Z: 50-by-2-by-2 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3            7232  amat
  Y              200x3x3            7232  amat
  Z               50x3x3            1832  amat

Print Z amat object :
Z is a 50x3x3 amat[single] object
  Z(1)=
   147    48    66
    48    98    36
    66    36    84

  Z(2)=
    99   -72   -48
   -72   266    19
   -48    19   242

  ...

  Z(49)=
    62    24    -5
    24   120   -66
    -5   -66   113

  Z(50)=
   144   -60   -60
   -60   134    92
   -60    92   122
```

4.3.25 `fc_amat.random.randherpd` function

The `fc_amat.random.randherpd` function return an `amat` object whose matrices are hermitian positive definite. This object is generated by using `randsdd` function from `fc_amat.random` namespace.

Syntaxe

```
X=fc_amat.random.randherpd(N,d)
X=fc_amat.random.randherpd(...,key,value)
```

Description

`X=fc_amat.random.randherpd(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are symmetric positive definite.

`X=fc_amat.random.randherpd(...,key,value)`

Optional key/value pairs arguments are those of the `fc_amat.random.randnsdd` function except for `'complex'` key which is forced to `true`. keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).
- `'a'`, to set a (lower bound of the interval) value (0 by default).
- `'b'`, to set b (upper bound of the interval) value (1 by default).

In Listing 34, some examples are provided.

Listing 34: : examples of `fc_amat.random.randherpd` function usage

```
X=fc_amat.random.randherpd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randherpd(200,3,'a',-2,'b',2);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randherpd(50,3,'a',-1,'class','single');
% Z: 50-by-3-by-3 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size           Bytes  Class    Attributes

  SaveOptions      1x6              721  cell
  X              100x3x3          14432  amat
  Y              200x3x3          28832  amat
  Z               50x3x3           3632  amat

Print Z amat object :
Z is a 50x3x3 amat[complex single] object
  Z(1)=
   2.6363 + 0.0000i -0.3336 - 0.9376i 2.2724 - 0.2571i
  -0.3336 + 0.9376i 6.6370 + 0.0000i -1.8343 + 3.4740i
   2.2724 + 0.2571i -1.8343 - 3.4740i 7.1614 + 0.0000i

  Z(2)=
   6.7001 + 0.0000i 1.0940 - 4.1729i 1.7635 - 0.3262i
   1.0940 + 4.1729i 8.1097 + 0.0000i 1.0601 - 0.0978i
   1.7635 + 0.3262i 1.0601 + 0.0978i 2.9646 + 0.0000i

  ...

  Z(49)=
   9.0330 + 0.0000i 1.7173 - 1.4282i 3.4638 - 0.9305i
   1.7173 + 1.4282i 4.7652 + 0.0000i 0.0708 + 2.4355i
   3.4638 + 0.9305i 0.0708 - 2.4355i 5.5916 + 0.0000i

  Z(50)=
  13.2686 + 0.0000i -0.2314 + 0.8565i -6.4115 - 2.9403i
  -0.2314 - 0.8565i 11.8201 + 0.0000i -3.7681 + 0.8334i
  -6.4115 + 2.9403i -3.7681 - 0.8334i 9.5450 + 0.0000i
```

4.3.26 `fc_amat.random.randnherpd` function

The `fc_amat.random.randnherpd` function return an `amat` object whose matrices are hermitian positive definite. This object is generated by using `randnsdd` function from `fc_amat.random` namespace.

Syntaxe

```
X=fc_amat.random.randnherpd(N,d)
X=fc_amat.random.randnherpd(...,key,value)
```

Description

`X=fc_amat.random.randnherpd(N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are Hermitian positive definite.

`X=fc_amat.random.randnherpd(...,key,value)`

Optional key/value pairs arguments are those of the `randnsdd` function except for `'complex'` key which is forced to `true`. Keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).
- `'mean'`, to set mean of the normal distribution (0 by default).
- `'sigma'`, to set standard deviation of the normal distribution (1 by default).

In Listing 35, some examples are provided.

Listing 35: : examples of `fc_amat.random.randnherpd` function usage

```
X=fc_amat.random.randnherpd(100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randnherpd(200,3,'sigma',5);
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randnherpd(50,3,'class','single','mean',5);
% Z: 50-by-3-by-3 single amat
disp('List current variables :')
whos
disp('Print Z amat object :')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3           14432  amat
  Y              200x3x3           28832  amat
  Z               50x3x3            3632  amat

Print Z amat object :
Z is a 50x3x3 amat[complex single] object
  Z(1)=
   1.0e+02 *

   5.5952 + 0.0000i  1.4791 + 3.2441i -0.6622 + 2.4489i
   1.4791 - 3.2441i  6.2931 + 0.0000i -0.4961 + 0.3382i
  -0.6622 - 2.4489i -0.4961 - 0.3382i  6.8153 + 0.0000i

  Z(2)=
   1.0e+02 *

   5.6793 + 0.0000i  2.9387 - 1.8670i  2.7489 - 1.5817i
   2.9387 + 1.8670i  5.6718 + 0.0000i  2.2443 + 0.2267i
   2.7489 + 1.5817i  2.2443 - 0.2267i  5.2918 + 0.0000i

  ...

  Z(49)=
   1.0e+02 *

   5.0513 + 0.0000i -2.1143 + 0.7183i -1.3965 - 0.3477i
  -2.1143 - 0.7183i  5.3161 + 0.0000i -1.7460 + 0.2400i
  -1.3965 + 0.3477i -1.7460 - 0.2400i  4.7632 + 0.0000i

  Z(50)=
   1.0e+02 *

   5.2906 + 0.0000i  1.6293 - 0.2617i  0.6918 - 2.6042i
   1.6293 + 0.2617i  4.9432 + 0.0000i  0.6616 - 2.8367i
   0.6918 + 2.6042i  0.6616 + 2.8367i  5.2710 + 0.0000i
```

4.3.27 `fc_amat.random.randiherpd` function

The `fc_amat.random.randiherpd` function return an `amat` object whose matrices are Hermitian positive definite with random integers. This object is generated by using `randiherpd` function from `fc_amat.random` namespace.

Syntaxe

```
X=fc_amat.random.randiherpd(Imax,N,d)
X=fc_amat.random.randiherpd([Imin,Imax],...)
X=fc_amat.random.randiherpd(...,key,value)
```

Description

`X=fc_amat.random.randiherpd(Imax,N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudorandom integer drawn from the discrete uniform distribution on `1:Imax`.

`X=fc_amat.random.randiherpd([Imin,Imax],N,d)`

returns a `N`-by-`d`-by-`d` `amat` object whose matrices are strictly diagonally dominant and non-diagonal elements are pseudorandom integer drawn from the discrete uniform distribution on `Imin:Imax`.

`X=fc_amat.random.randiherpd(...,key,value)`

Optional key/value pairs arguments are those of the `randisdd` function except for `'complex'` key which is forced to `true` and `'class'` key which can only be `'single'` or `'double'`. Keys can be:

- `'class'`, to set `amat` object data type; value can be `'single'` or `'double'` (default).

In Listing 36, some examples are provided.

Listing 36: : examples of `fc_amat.random.randiherpd` function usage

```
X=fc_amat.random.randiherpd(10,100,3);
% X: 100-by-3-by-3 amat
Y=fc_amat.random.randiherpd(8,200,3,'class','single');
% Y: 200-by-3-by-4 amat
Z=fc_amat.random.randiherpd([-5,5],50,3,'class','single');
% Z: 50-by-2-by-2 single amat
disp('List current variables:')
whos
disp('Print Z amat object:')
disp(Z,'n',2)
```

Output

```
List current variables :
  Name             Size            Bytes  Class    Attributes

  SaveOptions      1x6               721  cell
  X              100x3x3           14432  amat
  Y              200x3x3           14432  amat
  Z               50x3x3            3632  amat

Print Z amat object :
Z is a 50x3x3 amat[complex single] object
  Z(1)=
   1.0e+02 *

    2.8000 + 0.0000i -0.3200 - 0.0700i -0.5800 - 1.0000i
   -0.3200 + 0.0700i  1.6700 + 0.0000i -0.6300 + 0.4800i
   -0.5800 + 1.0000i -0.6300 - 0.4800i  2.1500 + 0.0000i

  Z(2)=
   1.0e+02 *

    1.9900 + 0.0000i -1.3900 + 0.0300i  0.7100 + 0.5000i
   -1.3900 - 0.0300i  3.2900 + 0.0000i -0.8000 + 0.2600i
    0.7100 - 0.5000i -0.8000 - 0.2600i  2.8300 + 0.0000i

   ...

  Z(49)=
   1.0e+02 *

    3.2600 + 0.0000i  0.8600 - 0.4800i -0.6400 - 1.0400i
    0.8600 + 0.4800i  2.2800 + 0.0000i -0.6100 - 0.6500i
   -0.6400 + 1.0400i -0.6100 + 0.6500i  2.9600 + 0.0000i

  Z(50)=
   1.0e+02 *

    3.1500 + 0.0000i  0.6600 - 0.5200i  0.7600 - 0.5300i
    0.6600 + 0.5200i  2.6000 + 0.0000i  0.0500 - 0.4200i
    0.7600 + 0.5300i  0.0500 + 0.4200i  2.1300 + 0.0000i
```

5 Indexing

5.1 Subscripted reference

Let `A` be a `N`-by-`m`-by-`n` `amat` object.

5.1.1 `A(K,I,J)`

- With `K`, `I`, `J` three 1D-arrays of indices, a `length(K)`-by-`length(I)`-by-`length(J)` `amat` object is returned where $\forall$`i` $\in$ `1:length(I)`, $\forall$`j` $\in$ `1:length(J)`, $\forall$ `k` $\in$ `1:length(K)` the element (`i`, `j`) of its `k`-th matrix is the element (`I(i)`, `J(j)`)) of `K(k)`-th matrix of `A`, i.e. with `B` denoting

the output `amat` object:

$$\texttt{B(k,i,j)} \leftarrow \texttt{A(k, I(i), J(j))}.$$

If `length(K)==1` , then the returned object is a `length(I)`-by-`length(J)` matrix such that

$$\texttt{B(i,j)} \leftarrow \texttt{A(k, I(i), J(j))}.$$

- *(experimental)* With `K` , `I` , `J` three `M`-by-`p`-by-`q` `amat` object a `M`-by-`p`-by-`q` `amat` object is returned where $\forall$`i` $\in$ `1:p`, $\forall$`j` $\in$ `1:q`, $\forall$ `k` $\in$ `1:M` the element (`i`, `j`) of its `k`-th matrix is the element (`I(k,i,j)`, `J(k,i,j)`) of `K(k,i,j)`-th matrix of `A`, i.e. with `B` denoting the output `amat` object:

$$\texttt{B(k,i,j)} \leftarrow \texttt{A(K(k,i,j), I(k,i,j), J(k,i,j))}.$$

The commands `A(K,I,:)` and `A(K,I,1:end)` are equivalent to `A(K,I,1:n)` .
The commands `A(K,:,J)` and `A(K,:,J)` are equivalent to `A(K,:,1:n)` .
The commands `A(:,I,J)` and `A(1:end,I,J)` are equivalent to `A(1:N,I,J)` .
The commands `A(K,:,:)` and `A(K,1:end,1:end)` are equivalent to `A(K,1:m,1:m)` .
...

5.1.2 `A(K)`

Identically to `A(K,:,:)` .

5.1.3 `A(I,J)`

Identically to `A(:,I,J)` .
In Listing 37, some examples are provided.

Listing 37: : examples of `subsref` method usage

```
N=100;m=2;n=3;
X=fc_amat.random.randi(9,[N,m,n]);
A=X(1,2,2); % A is a scalar
B=X([2,end-1],1:2,[1,3]);
info(B)
C=X(1); % C is a m-by-n matrix
D=X(1:10);
info(D)
E=X(1,2);
info(E)
F=X(1,[1,3]);
info(F)
p=2;q=2;
K=fc_amat.ones(N,p,q).*[1:N]';
I=fc_amat.random.randi(m,[N,p,q]);
J=fc_amat.random.randi(n,[N,p,q]);
sK=1:2:N;
G=X(K(sK),I(sK),J(sK));
info(G)
H=X(I,J);
info(H)
disp('List␣of␣some␣variables␣:')
whos A C sK
```

Output

```
B is a 2x2x2 amat[double] object
D is a 10x2x3 amat[double] object
E is a 100x1x1 amat[double] object
F is a 100x1x2 amat[double] object
G is a 50x2x2 amat[double] object
H is a 100x2x2 amat[double] object
List of some variables :
  Name      Size            Bytes  Class    Attributes

  A         1x1                 8  double
  C         2x3                48  double
  sK        1x50              400  double
```

5.2 Subscripted assignment

Let `A` be a `N`-by-`m`-by-`n` `amat` object.

5.2.1 `A(K,I,J)=B`

- `I`, `J` and `K` are scalars indices, `B` must be a scalar and it is assigned to element (`I`, `J`) of the `K`-th matrix of `A`.
- `I`, `J` and `K` are 1D-arrays of indices. Then three cases are possible
  - `B` is a scalar, then

    $$\texttt{A(k,i,j)=B},\quad \forall \texttt{i} \in \texttt{I}, \forall \texttt{j} \in \texttt{J}, \forall \texttt{k} \in \texttt{K}.$$

  - `B` is a `length(I)` $\times$ `length(J)` matrix, then $\forall$`k` $\in$ `1:length(K)` the `K(k)`-th matrix of `A` is set to `B`, i.e. $\forall$`i` $\in$ `1:length(I)`, $\forall$`j` $\in$ `1:length(J)`,

    $$\texttt{A(K(k),I(i),J(j))=B(i,j)}.$$

– `B` is a `length(K)`-by-`length(I)`-by-`length(J)` `amat` object then $\forall$ `k` $\in$ `1:length(K)` the `K(k)`-th matrix of `A` is set to `k`-th matrix of `B`, i.e. $\forall$`i` $\in$ `1:length(I)`, $\forall$`j` $\in$ `1:length(J)`,

`A(K(k),I(i),J(j))=B(k,i,j)`.

- `I`, `J` and `K` are `M`-by-`p`-by-`q` `amat` objects of indices

  Then three cases are possible

  – `B` is a scalar, then $\forall$`i` $\in$ `1:p`, $\forall$`j` $\in$ `1:q`, $\forall$`k` $\in$ `1:M`

  `A(K(k,i,j),I(k,i,j),J(k,i,j))=B`

  – *(experimental)* `B` is a `M`-by-`p`-by-`q` `amat` object then $\forall$`i` $\in$ `1:p`, $\forall$`j` $\in$ `1:q`, $\forall$`k` $\in$ `1:M`

  `A(K(k,i,j),I(k,i,j),J(k,i,j))=B(k,i,j)`

If `max(I)>m`, `max(J)>n` or `max(K)>N` then before assignment `A` is reshaped to fit the new size by setting 0 for missing elements.

5.2.2 `A(K)=B`

Identically to the equivalent commands `A(K,1:m,1:n)=B` or `A(K,:,:)=B` or `A(K,1:end,1:end)=B`

5.2.3 `A(I,J)=B`

If `B` is a scalar or a matrix or an `amat` object, this command is equivalent to one of these commands `A(1:N,I,J)=B` or `A(:,I,J)=B` or `A(1:end,I,J)=B`. If `B` is a `N`-by-1 array then $\forall$`k` $\in$ `1:N`, $\forall$`i` $\in$ `1:length(I)`, $\forall$`j` $\in$ `1:length(J)`,

`A(k,I(i),J(j))=B(k)`.

In Listing 38, some examples are provided.

Listing 38: : examples of `subsasgn` method usage

```
N=100;m=3;n=2;
X=fc_amat.ones(N,m,n,'int32');
X(2,1,2)=3;
X([2,N],1:2,[1,3])=2;
X(1)=-1;
X([2,N])=0;
X(3,3)=1:N;
disp('Print␣X␣amat␣object␣:')
X
```

Output

```
Print X amat object :

X =

X is a 100x3x3 amat[int32] object
  X(1)=
   -1   -1   -1
   -1   -1   -1
   -1   -1    1

  X(2)=
    0    0    0
    0    0    0
    0    0    2

  ...

  X(99)=
     1    1    0
     1    1    0
     1    1   99

  X(100)=
     0    0    0
     0    0    0
     0    0  100
```

6 Elementary operations

6.1 Arithmetic operations

The implemented element by element arithmetic operators/methods for `amat` objects are:

- `+` / `plus` , addition
- `+` / `uplus` , unary plus
- `-` / `minus` , subtraction
- `-` / `uminus` , unary minus
- `.*` / `times` , element-wise multiplication
- `./ rdivide` , element-by-element right division
- `.\` / `ldivide` , element-by-element left division

Let $\mathbb{A} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N$, (i.e. a N-by-m-by-n `amat` object) we now explain how a generic binary operator, denoted by $\bigotimes$, act between $\mathbb{A}$ and another input

data. We define four kinds of element by element arithmetic binary operations when $\mathbb{A}$ is the left operand.

1. Let $\mathbb{B} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N$, we have

$$\mathbb{A} \bigotimes \mathbb{B} \stackrel{\text{def}}{=} \mathbb{C} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N \tag{1}$$

   where $\forall k \in [\![1, N]\!]$

$$\mathbb{C}_k(i,j) = \mathbb{A}_k(i,j) \bigotimes \mathbb{B}_k(i,j), \quad \forall i \in [\![1,m]\!],\ \forall j \in [\![1,n]\!].$$

2. Let $\mathbb{B} \in \mathcal{M}_{m,n}(\mathbb{K})$, we have

$$\mathbb{A} \bigotimes \mathbb{B} \stackrel{\text{def}}{=} \mathbb{C} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N \tag{2}$$

   where $\forall k \in [\![1, N]\!]$

$$\mathbb{C}_k(i,j) = \mathbb{A}_k(i,j) \bigotimes \mathbb{B}(i,j), \quad \forall i \in [\![1,m]\!],\ \forall j \in [\![1,n]\!].$$

3. Let $\boldsymbol{B} \in \mathbb{K}^N$, (i.e. a N-by-1 array) we have

$$\mathbb{A} \bigotimes \boldsymbol{B} \stackrel{\text{def}}{=} \mathbb{C} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N \tag{3}$$

   where $\forall k \in [\![1, N]\!]$

$$\mathbb{C}_k(i,j) = \mathbb{A}_k(i,j) \bigotimes \boldsymbol{B}(k), \quad \forall i \in [\![1,m]\!],\ \forall j \in [\![1,n]\!].$$

4. Let $B \in \mathbb{K}$, we have

$$\mathbb{A} \bigotimes B \stackrel{\text{def}}{=} \mathbb{C} \in (\mathcal{M}_{m,n}(\mathbb{K}))^N \tag{4}$$

   where $\forall k \in [\![1, N]\!]$

$$\mathbb{C}_k(i,j) = \mathbb{A}_k(i,j) \bigotimes B, \quad \forall i \in [\![1,m]\!],\ \forall j \in [\![1,n]\!].$$

When $\mathbb{A}$ is the right operand element by element binary operations can be easily deduced.

In Listing 39, some examples are provided.

Listing 39: : examples of element by element operations

```
N=100; m=2;n=3;
X=fc_amat.ones(N,m,n);
A=X+2;
B=[1:N]'.*X;
M=rand(m,n);
C=M-X;
D=C./(2.*X);
disp('List␣current␣variables␣:')
whos
disp('Print␣D␣amat␣object␣:')
disp(D,'n',2)
```

Output

```
List current variables :
  Name             Size           Bytes  Class    Attributes

  A              100x2x3            4832  amat
  B              100x2x3            4832  amat
  C              100x2x3            4832  amat
  D              100x2x3            4832  amat
  M                2x3                48  double
  N                1x1                 8  double
  SaveOptions      1x6               721  cell
  X              100x2x3            4832  amat
  m                1x1                 8  double
  n                1x1                 8  double

Print D amat object :
D is a 100x2x3 amat[double] object
  D(1)=
   -0.0926  -0.4365  -0.1838
   -0.0471  -0.0433  -0.4512

  D(2)=
   -0.0926  -0.4365  -0.1838
   -0.0471  -0.0433  -0.4512

  ...

  D(99)=
   -0.0926  -0.4365  -0.1838
   -0.0471  -0.0433  -0.4512

  D(100)=
   -0.0926  -0.4365  -0.1838
   -0.0471  -0.0433  -0.4512
```

6.2 Relational operators

The implemented element by element relational operators/methods for `amat` objects are:

- `==` / `eq` , equality
- `>=` / `ge` , greater than or equal
- `>` / `gt` , greater than
- `<=` / `le` , less than or equal
- `<` / `lt` , less than
- `~=` / `ne` , inequality

With these binary operators, four kind element by element operations occur. They are the same as those described for the *element by element arithmetic*

operations, Section 6.1, and given by (1) to (4) except that the output differs: it is a **logical** `amat` object.

In Listing 40, some examples are provided.

Listing 40: : examples of relational operators

```
N=100; m=2;n=3;
X=fc_amat.random.randn(N,m,n);
Y=randn(m,n);
Z=randn(N,1);
W=fc_amat.random.randn(N,m,n);
A= X>=0;
info(A)
B= X<Y;
info(B)
C= X==Z;
info(C)
D= X~=W;
disp(D)
```

Output

```
A is a 100x2x3 amat[logical] object
B is a 100x2x3 amat[logical] object
C is a 100x2x3 amat[logical] object
D is a 100x2x3 amat[logical] object
  D(1)=
   1   1   1
   1   1   1

  D(2)=
   1   1   1
   1   1   1

  ...

  D(99)=
   1   1   1
   1   1   1

  D(100)=
   1   1   1
   1   1   1
```

6.3 Logical operations

The implemented logical operators/methods for `amat` objects are:

- `&` / `and` , logical and
- `|` / `or` , logical or
- `~` / `not` , logical not
- `xor` , logical xor
- `all` , ...
- `any` , ...

With the binary operators `and` , `or` , and `xor` four kind element by element operations occur. They are the same as those described for the *element by element arithmetic operations*, section 6.1, and given by (1) to (4) except that the output differs: it is a **logical** `amat` object.

In Listing 41, some examples are provided.

Listing 41: : examples of relational operators

```
N=100; m=2;n=3;
X=( fc_amat.random.randi([-2,2],N,m,n) >=0 );
y=( randi([-2,2],m,n) <0 );
w=( randi([-2,2],N,1) <=1 );
A= X & y;
info(A)
B= X | w;
info(B)
C= ~B;
info(C)
D= xor(X,C);
disp(D)
```

Output

```
A is a 100x2x3 amat[logical] object
B is a 100x2x3 amat[logical] object
C is a 100x2x3 amat[logical] object
D is a 100x2x3 amat[logical] object
  D(1)=
   1   1   1
   1   1   1

  D(2)=
   1   1   1
   1   1   1

  ...

  D(99)=
   0   1   0
   1   0   0

  D(100)=
   0   1   1
   1   0   0
```

6.3.1 `all` method

Let `X` be a `N`-by-`m`-by-`n` `amat` object. The `all` method of `X` return a `N`-by-`1`-by-`1` logical `amat` object such that its k-th element (`1`-by-`1` matrix) is `true` (logical 1) if all elements of the k-th matrix of `X` are all nonzero.

Syntaxe

```
B=all(X)
B=all(X,dim)
```

Description

`B=all(X)`

return a `N`-by-`1`-by-`1` logical `amat` object such that `B(k,1,1)` is one (logical `true`) if $\forall$`i` $\in$ `[1:m]`, $\forall$`j` $\in$ `[1:n]`, `A(k,i,j)` is nonzero. Otherwise `B(k,1,1)` is zero (logical `false`).

`B=all(X,dim)`

- `dim=1` , along rows of matrices of `X`. Returns a `N`-by-`1`-by-`n` logical `amat` object such that `B(k,1,j)` is one (logical `true`) if $\forall$`i` $\in$ `[1:m]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,1,j)` is zero (logical `false`).
- `dim=2` , along columns of matrices of `X`. Returns a `N`-by-`m`-by-`1` logical `amat` object such that `B(k,i,1)` is one (logical `true`) if $\forall$`j` $\in$ `[1:n]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,i,1)` is zero (logical `false`).
- `dim=3` , (default value) , along rows and columns of matrices of `X`. Returns a `N`-by-`1`-by-`1` logical `amat` object such that `B(k,1,1)` is one (logical `true`) if $\forall$`i` $\in$ `[1:m]`, $\forall$`j` $\in$ `[1:n]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,1,1)` is zero (logical `false`).
- `dim=0` , along matrices index of `X`. Returns return a `m`-by-`n` logical matrix such that `B(i,j)` is one (logical `true`) if $\forall$`k` $\in$ `[1:N]`, `A(k,i,j)` is nonzero. Otherwise, `B(i,j)` is zero (logical `false`).

In Listing 42, some examples are provided.

Listing 42: : examples of `all` function usage

```
X=fc_amat.random.rand(100,2,3);
info(X)
A=all(X>0); info(A)
B=all(X>0,1); info(B)
C=all(X>0,2); info(C)
D=all(X>0,0);
fprintf('␣D␣is␣\n␣');disp(D)
E=all(all(X>0),0);
fprintf('␣E␣is␣\n␣');disp(E)
```

Output

```
X is a 100x2x3 amat[double] object
A is a 100x1x1 amat[logical] object
B is a 100x1x3 amat[logical] object
C is a 100x2x1 amat[logical] object
 D is
   1   1   1
   1   1   1

 E is
   1
```

6.3.2 `any` method

Let `X` be a `N`-by-`m`-by-`n` `amat` object. The `any` method of `X` return a `N`-by-`1`-by-`1` logical `amat` object such that its k-th element (`1`-by-`1` matrix) is `true` (logical 1) if any of the elements of the k-th matrix of `X` is nonzero.

Syntaxe

```
B=any(X)
B=any(X,dim)
```

Description

`B=any(X)`

return a `N`-by-`1`-by-`1` logical `amat` object such that `B(k,1,1)` is one (logical `true`) if ∃`i` ∈ `[1:m]`, ∃`j` ∈ `[1:n]`, `A(k,i,j)` is nonzero.

`B=any(X,dim)`

- `dim=1` , along rows of matrices of `X`. Returns a `N`-by-`1`-by-`n` logical `amat` object such that `B(k,1,j)` is one (logical `true`) if ∃`i` ∈ `[1:m]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,1,j)` is zero (logical `false`).
- `dim=2` , along columns of matrices of `X`. Returns a `N`-by-`m`-by-`1` logical `amat` object such that `B(k,i,1)` is one (logical `true`) if ∃`j` ∈ `[1:n]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,i,1)` is zero (logical `false`).
- `dim=3` , (default value) , along rows and columns of matrices of `X`. Returns a `N`-by-`1`-by-`1` logical `amat` object such that `B(k,1,1)` is one (logical `true`) if ∃`i` ∈ `[1:m]`, ∃`j` ∈ `[1:n]`, `A(k,i,j)` is nonzero. Otherwise, `B(k,1,1)` is zero (logical `false`).
- `dim=0` , along matrices index of `X`. Returns return a `m`-by-`n` logical matrix such that `B(i,j)` is one (logical `true`) if ∃`k` ∈ `[1:N]`, `A(k,i,j)` is nonzero. Otherwise, `B(i,j)` is zero (logical `false`).

In Listing 43, some examples are provided.

Listing 43: : examples of `fc_amat.random.randher` function usage

```
X=fc_amat.random.rand(100,2,3);
info(X)
A=any(X>0); info(A)
B=any(X>0,1); info(B)
C=any(X>0,2); info(C)
D=any(X>0,0);
fprintf('␣D␣is␣\n␣');disp(D)
E=any(any(X>0),0);
fprintf('␣E␣is␣\n␣');disp(E)
```

Output

```
X is a 100x2x3 amat[double] object
A is a 100x1x1 amat[logical] object
B is a 100x1x3 amat[logical] object
C is a 100x2x1 amat[logical] object
 D is
   1   1   1
   1   1   1

 E is
   1
```

7 Elementary mathematical functions

A lot of elementary mathematical functions can be used with `amat` objects. In Listing 44, some examples are provided and complete lists are given thereafter.

Listing 44: : examples of elementary mathematical functions

```
A=fc_amat.random.randiher(10,100,3);
info(A);
X=cos(A);
info(X);
Y=sin(A);
info(Y) ;
Z=X.^2+Y.^2;
disp('Print␣Z␣amat␣object␣:')
Z
```

Output

```
A is a 100x3x3 amat[complex double] object
X is a 100x3x3 amat[complex double] object
Y is a 100x3x3 amat[complex double] object
Print Z amat object :

Z =

Z is a 100x3x3 amat[complex double] object
  Z(1)=
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 - 0.0000i 1.0000 - 0.0000i 1.0000 - 0.0000i
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i

  Z(2)=
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 - 0.0000i 1.0000 + 0.0000i 1.0000 - 0.0000i
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 - 0.0000i

  ...

  Z(99)=
   1.0000 - 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 - 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i

  Z(100)=
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 - 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
   1.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i
```

7.1 trigonometric functions

- `sin` , `asin` , `sind` , `asind` , `sinh` , `asinh` for sine functions
- `cos` , `acos` , `cosd` , `acosd` , `cosh` , `acosh` for cosine functions
- `tan` , `atan` , `tand` , `atand` , `tanh` , `atanh` , `atan2` , `atan2d` for tangent functions
- `csc` , `acsc` , `cscd` , `acscd` , `csch` , `acsch` for cosecant functions
- `sec` , `asec` , `secd` , `asecd` , `sech` , `asech` for secant functions
- `cot` , `acot` , `cotd` , `acotd` , `coth` , `acoth` for cotangent functions
- `hypot` , square root of the sum of the squares

- `deg2rad` , `rad2deg` for convert functions

7.2 Exponents and Logarithms

- `exp` , exponential function
- `expm1` , exponential function minus one
- `log` , natural logarithm
- `reallog` , real-valued natural logarithm
- `log1p` , compute `log(1+x)`
- `log10` , base-10 logarithm
- `log2` , base-2 logarithm
- `pow2` , base-2 power
- `nextpow2` , exponent of next higher power of 2
- `realpow` , real-valued power
- `sqrt` , square root
- `realsqrt` , real-valued square root
- `cbrt` , cube root
- `cbrtsqrt` , real-valued cube root
- `nthroot` , real (non-complex) n-th root

7.3 Complex Arithmetic

- `abs` , magnitude
- `arg` , `angle` , argument
- `conj` , complex conjugate
- `imag` , imaginary part
- `real` , real part

- `ceil` , round toward positive infinity
- `fix` , round toward zero
- `floor` , round toward negative infinity
- `round` , Round to the nearest integer

7.4.1 `max` method

Let `X` be a `N`-by-`m`-by-`n` `amat` object. The `max` method of `X` return its maximum values.

Syntaxe

```
W = max (X)
W = max (X, [], DIM)
W = max (X, Y)
[W, I] = max (X)
[W, I] = max (X, [], DIM)
[W, I, J] = max (X, [], 3)
```

Description

`W=max(X)`

return a `m`-by-`n` matrix such that `W(i,j)` is the maximum value of `X(:,i,j)`

`W = max (X, [], dim)`

- `dim=0` , along the number of matrices of `X`. Same as `W = max (X)`.
- `dim=1` , along rows of matrices of `X`. Returns a `N`-by-`1`-by-`n` `amat` object such that `W(k,1,j)` is the maximum value of `X(k,:,j)` .
- `dim=2` , along columns of matrices of `X`. Returns a `N`-by-`m`-by-`1` `amat` object such that `W(k,i,1)` is the maximum value of `X(k,i,:)` .
- `dim=3` , along rows and columns of matrices of `X`. Returns a `N`-by-`1`-by-`1` `amat` object such that `W(k,1,1)` is the maximum value of `X(k,:,:)` .

`W = max (X, Y)`

Returns a `N`-by-`m`-by-`n` `amat` object such that

- `W(k,i,j)=max(X(k,i,j),Y(k,i,j))` if `Y` is a `N`-by-`m`-by-`n` `amat` object,
- `W(k,i,j)=max(X(k,i,j),Y(i,j))` if `Y` is a `m`-by-`n` matrix,

- `W(k,i,j)=max(X(k,i,j),Y(k))` if `Y` is a `N`-by-`1` or `1`-by-`N` array,
- `W(k,i,j)=max(X(k,i,j),Y)` if `Y` is a scalar.

`[W, K] = max (X)`

Returns two `m`-by-`n` matrices such that

`W(i,j)=max(X(:,i,j))` and `W(i,j)=X(K(i,j),i,j)`

`[W, Idx] = max (X, [], DIM)`

- if `DIM=0` , command is equivalent to `[W, Idx] = max (X)`,
- if `DIM=1` , returns two `N`-by-`1`-by-`n` `amat` objects such that

  `W(k,1,j)=max(X(k,:,j))`and`W(k,1,j)=X(K,Idx(k,1,j),j)`,
- if `DIM=2` , returns two `N`-by-`m`-by-`1` `amat` objects such that

  `W(k,i,1)=max(X(k,i,:))` and `W(k,i,1)=X(K,i,Idx(k,i,1))`.

`[W, I, J] = max (X, [], 3)`

returns three `N`-by-`1`-by-`1` `amat` objects such that

`W(k,1,1)=max(X(k,:,:))` and `W(k,1,1)=X(K,I(k,1,1),J(k,1,1))`.

In Listing 45, some examples are provided.

Listing 45: : examples of `fc_amat.random.randher` function usage

```
N=3;m=2;n=3;
X=fc_amat.random.randi(9,[N,m,n]);
Y=fc_amat.random.randi(9,[N,m,n]);
disp(X)
W=max(X);
fprintf('W=max(X)␣->\n')
disp(W)
W1=max(X,[],1);
fprintf('W1=max(X,[],1)␣->␣\n')
```

Output

```
X is a 3x2x3 amat[double] object
  X(1)=
     8     3     9
     9     9     2

  X(2)=
     9     5     5
     6     2     4

  X(3)=
     2     9     8
     1     9     9

W=max(X) ->
     9     9     9
     9     9     9

W1=max(X,[],1) ->
```

7.4.2 `min` method

Let `X` be a `N`-by-`m`-by-`n` `amat` object. The `min` method of `X` return its minimum values.

Syntaxe

```
W = min (X)
W = min (X, [], DIM)
W = min (X, Y)
[W, I] = min (X)
[W, I] = min (X, [], DIM)
[W, I, J] = min (X, [], 3)
```

Description

`W=min(X)`

return a `m`-by-`n` matrix such that `W(i,j)` is the minimum value of `X(:,i,j)`

`W = min (X, [], dim)`

- `dim=0` , along the number of matrices of `X`. Same as `W = min (X)`.
- `dim=1` , along rows of matrices of `X`. Returns a `N`-by-`1`-by-`n` `amat` object such that `W(k,1,j)` is the minimum value of `X(k,:,j)` .
- `dim=2` , along columns of matrices of `X`. Returns a `N`-by-`m`-by-`1` `amat` object such that `W(k,i,1)` is the minimum value of `X(k,i,:)` .
- `dim=3` , along rows and columns of matrices of `X`. Returns a `N`-by-`1`-by-`1` `amat` object such that `W(k,1,1)` is the minimum value of `X(k,:,:)` .

`W = min (X, Y)`

Returns a `N`-by-`m`-by-`n` `amat` object such that

- `W(k,i,j)=min(X(k,i,j),Y(k,i,j))` if `Y` is a `N`-by-`m`-by-`n` `amat` object,
- `W(k,i,j)=min(X(k,i,j),Y(i,j))` if `Y` is a `m`-by-`n` matrix,
- `W(k,i,j)=min(X(k,i,j),Y(k))` if `Y` is a `N`-by-`1` or `1`-by-`N` array,
- `W(k,i,j)=min(X(k,i,j),Y)` if `Y` is a scalar.

`[W, K] = min (X)`

Returns two `m`-by-`n` matrices such that

`W(i,j)=min(X(:,i,j))` and `W(i,j)=X(K(i,j),i,j)`

`[W, Idx] = min (X, [], DIM)`

- if `DIM=0` , command is equivalent to `[W, Idx] = min (X)`,
- if `DIM=1` , returns two `N`-by-`1`-by-`n amat` objects such that

  `W(k,1,j)=min(X(k,:,j))`and`W(k,1,j)=X(K,Idx(k,1,j),j)`,

- if `DIM=2` , returns two `N`-by-`m`-by-`1 amat` objects such that

  `W(k,i,1)=min(X(k,i,:))` and `W(k,i,1)=X(K,i,Idx(k,i,1))`.

`[W, I, J] = min (X, [], 3)`

returns three `N`-by-`1`-by-`1 amat` objects such that

`W(k,1,1)=min(X(k,:,:))` and `W(k,1,1)=X(K,I(k,1,1),J(k,1,1))`.

In Listing 46, some examples are provided.

Listing 46: : examples of `fc_amat.random.randher` function usage

```
N=10;m=2;n=3;
X=fc_amat.random.randi(9,[N,m,n]);
disp(X)
W=min(X);
fprintf('W=min(X)␣->\n')
disp(W)
W1=min(X,[],1);
fprintf('W1=min(X,[],1)␣->␣\n')
disp(W1)
```

Output

```
X is a 10x2x3 amat[double] object
  X(1)=
     8     6     4
     2     7     3

  X(2)=
     9     1     4
     9     1     7

  ...

  X(9)=
     9     6     7
     8     9     6

  X(10)=
     9     2     7
     9     1     3

W=min(X) ->
     1     1     2
     2     1     2

W1=min(X,[],1) ->
W1 is a 10x1x3 amat[double] object
  W1(1)=
     2
     6
     3

  W1(2)=
     9
     1
     4

  ...

  W1(9)=
     8
     6
     6

  W1(10)=
     9
     1
     3
```

8 Linear algebra

8.1 Linear combination

. Let `X` be a `N`-by-`m`-by-`n` `amat` object, `alpha` and `beta` two scalars. We define four kinds of linear combinations for the Matlab instruction:

```
Z= alpha*X + beta*Y
```
(5)

where Z be also a N-by-m-by-n amat object, and we have $\forall$k $\in$ 1:N, $\forall$i $\in$ 1:m, $\forall$j $\in$ 1:n,

$$\texttt{Z(k,i,j)= alpha*X(k,i,j)} + \begin{cases} \texttt{beta*Y(k,i,j)} & \text{if Y is a N-by-m-by-n amat object} \\ \texttt{beta*Y(i,j)} & \text{if Y is a m-by-n matrix} \\ \texttt{beta*Y(i,j)} & \text{if Y is a scalar} \\ \texttt{beta*Y(k)} & \text{if Y is a N-by-1 array} \end{cases}$$

In Listing 47, some examples are provided.

Listing 47: : examples of linear combinations

```
N=100;m=2;n=3;
X=fc_amat.random.randi(9,[N,m,n]);
info(X)
Y=fc_amat.random.randi(9,[N,m,n]);
info(Y)
A=3*X-2*Y;
info(A)
Y2=randi(9,[m,n]);
B=2*Y2-4*X;
info(B)
C=3*X-1;
info(C)
Y3=randi(9,[N,1]);
D=3*Y3-X;
info(D)
```

Output

```
X is a 100x2x3 amat[double] object
Y is a 100x2x3 amat[double] object
A is a 100x2x3 amat[double] object
B is a 100x2x3 amat[double] object
C is a 100x2x3 amat[double] object
D is a 100x2x3 amat[double] object
```

8.2 Matrix product

We define (and extend) matricial products for amat objects by using operator * (i.e. mtimes method)

$$\texttt{Z= X*Y} \tag{6}$$

where X and/or Y are amat objects. Explanations on programming techniques can be found in [1].

We choose to only described this operator when the left operand X is a N-by-m-by-n amat object. We can easily deduced results when X is not an amat object and Y is an amat object.

- With Y a N-by-n-by-p amat object (compatible dimensions), instruction (6) defines Z as a N-by-m-by-p amat object and is equivalent to the N matricial products

  $$\texttt{Z(k)= X(k)*Y(k)} \ , \quad \forall \texttt{k} \in \texttt{1:N}$$

  i.e. $\forall$i $\in$ 1:m, $\forall$j $\in$ 1:p,

  $$\texttt{Z(k,i,j)=} \sum_{\texttt{r}=1}^{\texttt{n}} \texttt{X(k,i,r)*Y(k,r,j)}, \quad \forall \texttt{k} \in \texttt{1:N}.$$

- With `Y` a `n`-by-`p` matrix (compatible dimensions), instruction (6) defines `Z` as a `N`-by-`m`-by-`p` `amat` object and is equivalent to the `N` matricial products

$$\texttt{Z(k)= X(k)*Y} \ , \quad \forall \texttt{k} \in \texttt{1:N}$$

  i.e. $\forall \texttt{i} \in \texttt{1:m}$, $\forall \texttt{j} \in \texttt{1:p}$,

$$\texttt{Z(k,i,j)=} \sum_{\texttt{r}=1}^{\texttt{n}} \texttt{X(k,i,r)*Y(r,j)}, \quad \forall \texttt{k} \in \texttt{1:N}.$$

- With `Y` a `N`-by-`1` 1D-array, instruction (6) defines `Z` as a `N`-by-`m`-by-`n` `amat` object and we have

$$\texttt{Z(k)= X(k)*Y(k)} \ , \quad \forall \texttt{k} \in \texttt{1:N}$$

  i.e. $\forall \texttt{i} \in \texttt{1:m}$, $\forall \texttt{j} \in \texttt{1:n}$,

$$\texttt{Z(k,i,j)= X(k,i,j)*Y(k)}, \quad \forall \texttt{k} \in \texttt{1:N}.$$

- With `Y` a scalar, instruction (6) defines `Z` as a `N`-by-`m`-by-`n` `amat` object and we have

$$\texttt{Z(k)= X(k)*Y} \ , \quad \forall \texttt{k} \in \texttt{1:N}$$

  i.e. $\forall \texttt{i} \in \texttt{1:m}$, $\forall \texttt{j} \in \texttt{1:n}$,

$$\texttt{Z(k,i,j)= X(k,i,j)*Y} \ , \quad \forall \texttt{k} \in \texttt{1:N}.$$

In Listing 47, some examples are provided.

Listing 48: : examples of matricial products

```
N=100;m=2;n=4;p=3;
X=fc_amat.random.randi(9,[N,m,n]);
info(X)
Y=fc_amat.random.randi(9,[N,n,p]);
info(Y)
A=X*Y; % <- matricial products
info(A)
X2=randi(9,[m,n]);
B=X2*Y;% <- matricial products
info(B)
Y2=randi(9,[n,p]);
C=X*Y2;% <- matricial products
info(C)
T=C(1)-X(1)*Y2;
fprintf('T is\n')
disp(T)
```

Output

```
X is a 100x2x4 amat[double] object
Y is a 100x4x3 amat[double] object
A is a 100x2x3 amat[double] object
B is a 100x2x3 amat[double] object
C is a 100x2x3 amat[double] object
T is
    0    0    0
    0    0    0
```

8.2.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.mtimes` can be used and is described in Section 8.2.2. This function uses the FC-BENCH Matlab toolbox described in [2] and performs all computational times of this section.

Let `X` and `Y` be `N`-by-`d`-by-`d` `amat` objects, in Table 2 computational times in seconds of `mtimes(X,Y)` (`X*Y` matricial products) are given. In Figure 1, computational times in seconds for a given `N` are represented in function of very small values of `d` .

N	mtimes
200 000	0.120(s)
400 000	0.245(s)
600 000	0.410(s)
800 000	0.565(s)
1 000 000	0.715(s)
5 000 000	6.183(s)
10 000 000	11.373(s)

Table 2: Computational times in seconds of `mtimes(X,Y)` (`X*Y` matrix product) where `X` and `Y` are `N`-by-`d`-by-`d` `amat` objects.

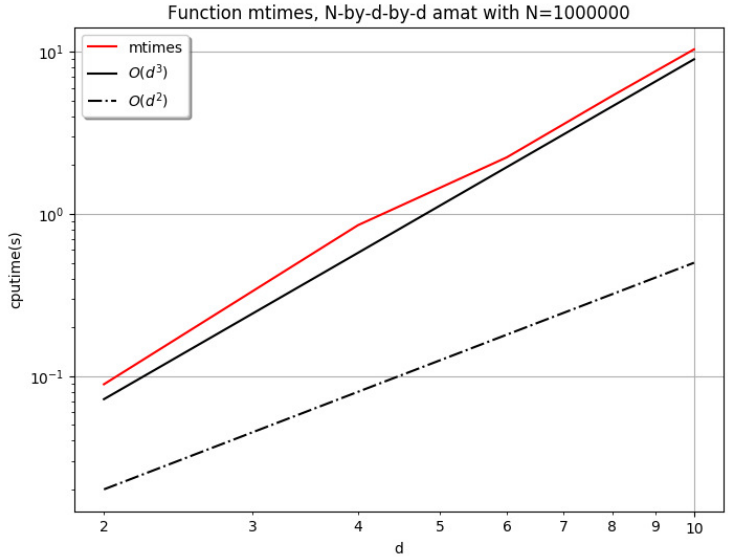

Figure 1: Computational times in seconds of `mtimes(X,Y)` or `X*Y` (matrix product) where `X` and `Y` are `N`-by-`d`-by-`d` `amat` objects.

8.2.2 Benchmark function

The function `fc_amat.benchs.mtimes` measures performance of matricial products of `amat` objects done by `mtimes(X,Y)` or `X*Y` command. At least one of

the inputs must be an `amat` object. When running this function the matrices orders are fixed and only the number `N` of matrices contained in `amat` objects varies and it is given by a list of values `LN` .

Syntaxe

```
fc_amat.benchs.mtimes(LN)
fc_amat.benchs.mtimes(LN,key,value,...)
```

Description

`fc_amat.benchs.mtimes(LN)`

runs a benchmark of the `mtimes` method of the `amat` class between two `N`-by-`2`-by-`2` `amat` objects for all `N` in `LN` .

`fc_amat.benchs.mtimes(LN,key,value,...)`

Optional key/value pairs arguments are available. `key` can be one of the following strings

- `'d'` , left and right matrices dimension (default `value` is `[2,2]`)
- `'type'` , to set type of left and right operands. `value` is either `'amat'` (`amat` object), `'mat'` (matrix), `'array1d'` (`N`-by-`1` 1D-array) or `'scalar'` (default `value` is `'amat'`).
- `'class'` , to set classname of left and right operands. Value can be `'double'` (default), `'single'` , `'int32'` ,...
- `'complex'` , if `true` left and right operands are complex (default `value` is `false`).
- `'ld'` , same as `'d'` but only for left operand.
- `'rd'` , same as `'d'` but only for right operand.
- `'ltype'` same as `'type'` but only for left operand.
- `'rtype'` same as `'type'` but only for right operand.
- `'lclass'` same as `'class'` but only for left operand.
- `'rclass'` same as `'class'` but only for right operand.
- `'lcomplex'` same as `'complex'` but only for left operand.
- `'rcomplex'` same as `'complex'` but only for right operand.

In Listings 49 and 50 two examples with outputs are provided.

Listing 49: : Benchmarking `mtimes(X,Y)` with `X` a 3-by-4 matrix and `Y` a N-by-4-by-5 `amat` object

```
LN=10^5*[2:2:10];
fc_amat.benchs.mtimes(LN,'ltype','mat','ld',[3,4],'rd',[4,5]);
```

Output

```
#-------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#-------------------------------------------------------------------------------
# 1st parameter is :
# -> matrix[double] with (m,n)=(3,4), size=[3 4]
# 2nd parameter is :
# -> amat[double] with (N,nr,nc)=(200000,4,5), size=[200000 4      5]
#-------------------------------------------------------------------------------
#date:2020/01/01 15:22:04
#nbruns:5
#numpy:       i4         f4
#format:      %d       %.3f
#labels:       N   mtimes(s)
          200000      0.080
          400000      0.149
          600000      0.248
          800000      0.347
         1000000      0.442
```

Listing 50: : Benchmarking `mtimes(X,Y)` where `X` and `Y` are N-by-4-by-4 `amat` object with `complex single` values.

```
LN=10^5*[2:2:10];
fc_amat.benchs.mtimes(LN,'d',[4,4],'complex',true,'class','single', ...
    'info',false);
```

Output

```
#-------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[complex single] with (N,nr,nc)=(200000,4,4), size=[200000 4      4]
# 2nd parameter is :
# -> amat[complex single] with (N,nr,nc)=(200000,4,4), size=[200000 4      4]
#-------------------------------------------------------------------------------
#date:2020/01/01 15:22:24
#nbruns:5
#numpy:       i4         f4
#format:      %d       %.3f
#labels:       N   mtimes(s)
          200000      0.152
          400000      0.290
          600000      0.444
          800000      0.578
         1000000      0.929
```

8.3 LU Factorization

Let `A` be a N-by-m-by-m `amat` object. The `[L,U,P]=lu(A)` command returns three N-by-m-by-m `amat` objects where `L` , `U` and `P` are respectively a unit lower triangular `amat` , an upper triangular `amat` and a permutation `amat` such that

$$\texttt{P*A=L*U} \quad \text{or} \quad \texttt{A=P'*L*U}\,. \tag{7}$$

Here, operator `*` is the `amat` matricial product, i.e.

$$\forall \texttt{k} \in \texttt{1:N}, \quad P(k) * A(k) = L(k) * U(k).$$

Explanations on programming techniques can be found in [1].

Syntaxe Let `A` be a `N`-by-`m`-by-`m` `amat` object.

```
[L,U,P]=lu(A)
[L,U,P]=lu(A,type)
```

Description

`[L,U,P]=lu(A)`

returns three `N`-by-`m`-by-`m` `amat` objects where `L`, `U` and `P` are respectively a unit lower triangular `amat`, an upper triangular `amat` and a permutation `amat` such that

$$\texttt{P*A=L*U} \text{ or } \texttt{A=P'*L*U}. \tag{8}$$

Here operator `*` is the `amat` matricial product, i.e.

$$\forall \texttt{k} \in \texttt{1:N}, \quad \texttt{P(k)*A(k)=L(k)*U(k)}.$$

`[L,U,P]=lu(A,type)`

- If `type` is `'amat'`, then the command is equivalent to `[L,U,P]=lu(A)`.
- If `type` is `'vector'` or `'matrix'` then, returns the permutation information `P` as a `N`-by-`m` matrix instead of an `amat`. If so, the permutation `amat` object can be build with the `fc_amat.permind2amat(P)` command.

In Listing 51, some examples are provided.

Listing 51: : examples of `lu` method usage

```
A=complex(fc_amat.random.randn(100,3,3),fc_amat.random.randn(100,3,3));
info(A)
[L,U,P]=lu(A);
info(L);info(U);info(P);
E=P*A-L*U;
disp(E);
```

Output

```
A is a 100x3x3 amat[complex double] object
L is a 100x3x3 amat[complex double] object
U is a 100x3x3 amat[complex double] object
P is a 100x3x3 amat[double] object
E is a 100x3x3 amat[complex double] object
  E(1)=
   1.0e-15 *

   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.1110 - 0.0069i 0.0000 + 0.0000i 0.0000 + 0.0000i

  E(2)=
   1.0e-15 *

   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.1110 + 0.0000i -0.2220 + 0.0000i
   0.0000 + 0.0000i 0.0555 + 0.0000i 0.0278 + 0.0278i

  ...

  E(99)=
   1.0e-15 *

   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i -0.1110 + 0.0278i 0.0000 + 0.0000i
   0.0000 + 0.0000i -0.0278 + 0.0000i 0.0000 + 0.0000i

  E(100)=
   1.0e-15 *

   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 - 0.2220i 0.0000 - 0.0833i 0.0000 + 0.0000i
```

8.3.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.lu` can be used and is described in Section 8.3.2. This function uses the FC-BENCH Matlab toolbox described in [2] and performs all computational times of this section.

Let `A` be a `N`-by-`d`-by-`d amat` object, in Table 3 computational times in seconds of `[L,U,P]=lu(A)` are given.In Figure 2, computational times in seconds for a given `N` are represented in function of very small values of `d`.

N	d=2	d=4	d=6	d=8	d=10
200 000	0.069(s)	0.244(s)	0.638(s)	1.200(s)	2.007(s)
400 000	0.108(s)	0.543(s)	1.286(s)	2.478(s)	4.168(s)
600 000	0.169(s)	0.809(s)	2.017(s)	3.824(s)	6.476(s)
800 000	0.225(s)	1.104(s)	2.795(s)	5.355(s)	9.295(s)
1 000 000	0.289(s)	1.437(s)	3.736(s)	7.260(s)	11.042(s)

Table 3: Computational times in seconds of `[L,U,P]=lu(A)` where `A` is a `N`-by-`d`-by-`d amat` object.

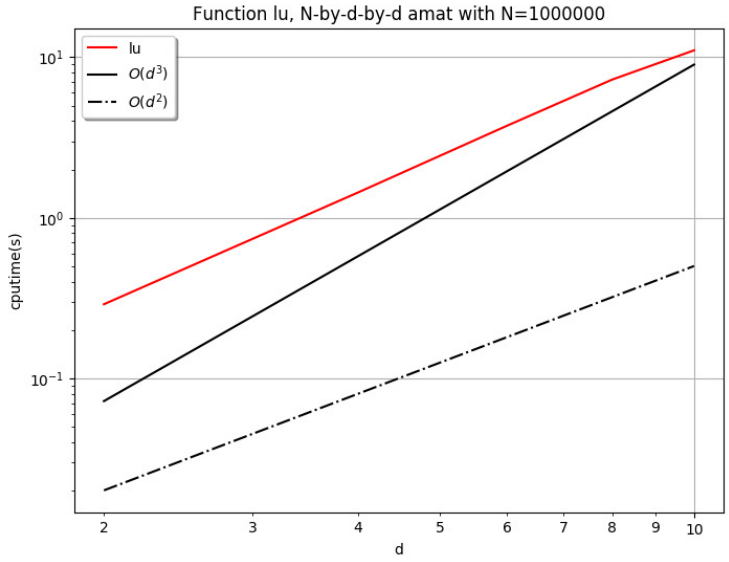

Figure 2: Computational times in seconds of of `[L,U,P]=lu(A)` where `A` is a `N`-by-`d`-by-`d` `amat` object.

8.3.2 Benchmark function

The function `fc_amat.benchs.lu` measures performance of LU factorization `[L,U,P]=lu(A)` where the input `A` is a `N`-by-`d`-by-`d` `amat` object. When running this function the `d` value is fixed, the number `N` varies and it is given by a list of values `LN`.

Syntaxe

```
fc_amat.benchs.lu(LN)
fc_amat.benchs.lu(LN,key,value,...)
```

Description

`fc_amat.benchs.lu(LN)`

runs a benchmark of the `lu` method on a `N`-by-`2`-by-`2` `amat` object for all `N` in `LN`.

`fc_amat.benchs.lu(LN,key,value,...)`

Optional key/value pairs arguments are available and can modify the input `N`-by-`d`-by-`d` `amat` object of the `lu` function. `key` can be one of the following strings

- `'d'`, to set `d` (default `value` is `2`)

- 'class' , to set classname of the input amat object. Value can be 'double' (default) or 'single' .
- 'complex' , if true the input amat object is complex (default value is false).

In Listings 52 and 53 two examples with outputs are provided.

Listing 52: : Benchmarking [L,U,P]=lu(A) with A a N-by-4-by-4 matrix amat object

```
LN=10^5*[2:2:10];
fc_amat.benchs.lu(LN,'d',4);
```

Output

```
#------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#------------------------------------------------------------------------------
# input parameter is :
# -> amat[double] with (N,nr,nc)=(200000,4,4), size=[200000 4      4]
#------------------------------------------------------------------------------
#date:2020/01/01 15:45:48
#nbruns:5
#numpy:       i4     f4          f4
#format:      %d    %.3f        %.3e
#labels:       N   lu(s)    Error[0]
          200000  0.276   1.388e-15
          400000  0.548   2.304e-15
          600000  0.823   1.985e-15
          800000  1.223   2.415e-15
         1000000  1.495   2.776e-15
```

Listing 53: : Benchmarking [L,U,P]=lu(A) where A is N-by-3-by-3 amat object with complex single values.

```
LN=10^5*[2:2:10];
fc_amat.benchs.lu(LN,'d',3,'complex',true,'class','single', ...
    'info',false);
```

Output

```
#------------------------------------------------------------------------------
# input parameter is :
# -> amat[complex single] with (N,nr,nc)=(200000,3,3), size=[200000 3      3]
#------------------------------------------------------------------------------
#date:2020/01/01 15:46:34
#nbruns:5
#numpy:       i4     f4          f4
#format:      %d    %.3f        %.3e
#labels:       N   lu(s)    Error[0]
          200000  0.179   8.525e-07
          400000  0.323   1.035e-06
          600000  0.524   9.562e-07
          800000  0.802   9.058e-07
         1000000  0.913   8.937e-07
```

8.4 Cholesky Factorization

The chol(A) command returns the positive Cholesky factorization of symmetric (or hermitian) positive definite amat object A as a upper triangular amat object with strictly positive diagonal entries. Explanations on programming techniques can be found in [1].

Syntaxe Let `A` be a `N`-by-`d`-by-`d` symmetric (or hermitian) positive definite `amat` object.

```
B=chol(A)
B=chol(A,type)
```

Description

`B=chol(A)`

returns the positive Cholesky factorization of `A` as a `N`-by-`d`-by-`d` upper triangular `amat` object `B` with strictly positive diagonal entries such that

$$\texttt{A=B'*B} \tag{9}$$

Here, operator `*` is the `amat` matricial product, i.e.

$$\forall \texttt{k} \in \texttt{1:N}, \quad \texttt{A(k)=B(k)'*B(k)} \ .$$

`B=chol(A,type)`

- If `type` is `'upper'`, then the command is equivalent to `B=chol(A)`.
- If `type` is `'lower'`, then `B` is a `N`-by-`d`-by-`d` lower triangular `amat` object with strictly positive diagonal entries such that

$$\texttt{A=B*B'} \tag{10}$$

Here, operator `*` is the `amat` matricial product, i.e.

$$\forall \texttt{k} \in \texttt{1:N}, \quad \texttt{A(k)=B(k)*B(k)'} \ .$$

In Listing 54, some examples are provided.

Listing 54: : examples of `chol` method usage

```
A=fc_amat.random.randnherpd(100,3);
info(A)
B=chol(A);
info(B);
E=A-B'*B;
disp(E);
```

Output

```
A is a 100x3x3 amat[complex double] object
B is a 100x3x3 amat[complex double] object
E is a 100x3x3 amat[complex double] object
  E(1)=
   1.0e-14 *

   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.3553 + 0.0000i -0.1776 + 0.0000i
   0.0000 + 0.0000i -0.1776 + 0.0000i 0.3553 + 0.0000i

  E(2)=
   1.0e-14 *

  -0.3553 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i

  ...

  E(99)=
   1.0e-14 *

  -0.3553 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
   0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.0444i
   0.0000 + 0.0000i 0.0000 + 0.0444i -0.3553 + 0.0000i

  E(100)=
   1.0e-14 *

   0.0000 + 0.0000i 0.0000 - 0.1776i 0.0000 - 0.0028i
   0.0000 + 0.1776i 0.0000 + 0.0000i 0.0000 + 0.1776i
   0.0000 + 0.0028i 0.0000 - 0.1776i 0.0000 + 0.0000i
```

8.4.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.chol` can be used and is described in Section 8.4.2. This function uses the FC-BENCH Matlab toolbox described in [2] and performs all computational times of this section.

Let `A` be a `N`-by-`d`-by-`d` symmetric (or hermitian) positive definite `amat` object, in Table 4 computational times in seconds of `B=chol(A)` are given.In Figure 3, computational times in seconds for a given `N` are represented in fonction of very small values of `d` .

N	d=2	d=4	d=6	d=8	d=10
200 000	0.007(s)	0.021(s)	0.062(s)	0.124(s)	0.238(s)
400 000	0.009(s)	0.047(s)	0.123(s)	0.248(s)	0.428(s)
600 000	0.013(s)	0.074(s)	0.195(s)	0.396(s)	0.708(s)
800 000	0.018(s)	0.101(s)	0.274(s)	0.562(s)	0.987(s)
1 000 000	0.024(s)	0.130(s)	0.380(s)	0.719(s)	1.316(s)

Table 4: Computational times in seconds of `B=chol(A)` where `A` is a `N`-by-`d`-by-`d` symmetric positive definite `amat` object.

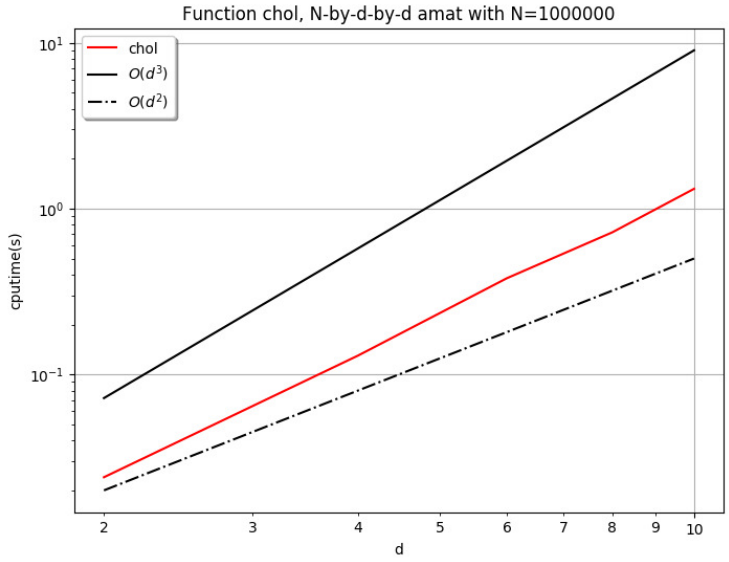

Figure 3: Computational times in seconds of of `B=chol(A)` where `A` is a `N`-by-`d`-by-`d` symmetric positive definite `amat` object.

8.4.2 Benchmark function

The function `fc_amat.benchs.chol` measures performance of Cholesky factorization `B=chol(A)` where the input `A` is a `N`-by-`d`-by-`d` symmetric (or hermitian) positive definite `amat` object. When running this function the `d` value is fixed, the number `N` varies and it is given by a list of values `LN`.

Syntaxe

```
fc_amat.benchs.chol(LN)
fc_amat.benchs.chol(LN,key,value,...)
```

Description

`fc_amat.benchs.chol(LN)`

runs a benchmark of the `chol` method on a `N`-by-`2`-by-`2` symmetric positive definite `amat` object for all `N` in `LN`.

`fc_amat.benchs.chol(LN,key,value,...)`

Optional key/value pairs arguments are available and can modify the input `N`-by-`d`-by-`d` `amat` object of the `chol` function. `key` can be one of the following strings

- `'d'`, to set `d` (default `value` is `2`)

- 'kind' , to set the kind of the square output amat object. If value is 'lower' , then the output is a lower triangular amat object with strictly positive diagonal entries. Default value is 'upper' . d (default value is 2)
- 'class' , to set classname of the input amat object. Value can be 'double' (default) or 'single' .
- 'complex' , if true the input amat object is Hermitian positive definite (default value is false).

In Listings 55 and 56 two examples with outputs are provided.

Listing 55: : Benchmarking B=chol(A) with A a N-by-4-by-4 matrix amat object

```
LN=10^5*[2:2:10];
fc_amat.benchs.chol(LN,'d',4,'kind','lower');
```

Output

```
#----------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#----------------------------------------------------------------------------
# Symmetric Positive Definite matrices
# -> amat[double] with (N,m,n)=(N,4,4)
# Error function: @(X)max(norm(X*X'-A))+all(~istril(X),0)
#----------------------------------------------------------------------------
# Benchmarking command: @(A) chol(A,'lower');
#----------------------------------------------------------------------------
#date:2020/01/01 15:54:24
#nbruns:5
#numpy:      i4       f4         f4
#format:     %d     %.3f       %.3e
#labels:      N   chol(s)    Error[0]
          200000    0.024   2.132e-14
          400000    0.047   2.842e-14
          600000    0.074   2.842e-14
          800000    0.102   3.197e-14
         1000000    0.121   3.197e-14
```

Listing 56: : Benchmarking B=chol(A) where A is N-by-3-by-3 amat object with complex single vacholes.

```
LN=10^5*[2:2:10];
fc_amat.benchs.chol(LN,'d',3,'complex',true,'class','single', ...
    'info',false);
```

Output

```
#----------------------------------------------------------------------------
# Hermitian Positive Definite matrices
# -> amat[complex single] with (N,m,n)=(N,3,3)
# Error function: @(X)max(norm(X'*X-A))+all(~istriu(X),0)
#----------------------------------------------------------------------------
# Benchmarking command: @(A) chol(A,'upper');
#----------------------------------------------------------------------------
#date:2020/01/01 15:54:42
#nbruns:5
#numpy:      i4       f4         f4
#format:     %d     %.3f       %.3e
#labels:      N   chol(s)    Error[0]
          200000    0.049   1.024e-05
          400000    0.085   1.157e-05
          600000    0.151   1.141e-05
          800000    0.204   1.257e-05
         1000000    0.262   1.188e-05
```

8.5 Determinants

The `det(A)` command returns determinants of the matrices of the square `amat` object. Explanations on programming techniques can be found in [1].

Syntaxe Let `A` be a `N`-by-`d`-by-`d` `amat` object.

```
D=det(A)
D=det(A,'select',value)
```

Description

`D=det(A)`

returns determinants of the matrices of `A` as a `N`-by-`1`-by-`1` `amat` object `D` such that

$$\forall k \in 1{:}N, \quad D(k)=\det(A(k)) \ .$$

`D=det(A,'select',value)`

when `value` is

- `'lu'` , uses LU factorizations.
- `'laplace'` , uses vectorized Laplace expansion.
- `'auto'` (default), uses vectorized Laplace expansion for `d<=5` and LU factorization otherwise.

In Listing 57, some examples are provided.

Listing 57: : examples of `det` method usage

```
A=complex(fc_amat.random.randn(100,3),fc_amat.random.randn(100,3));
info(A)
D1=det(A);
info(D1);
D2=det(A,'select','lu');
info(D2);
D3=det(A,'select','laplace');
info(D3);
E=abs(D1-D2)+abs(D1-D3);
disp(E)
```

Output

```
A is a 100x3x3 amat[complex double] object
D1 is a 100x1x1 amat[complex double] object
D2 is a 100x1x1 amat[complex double] object
D3 is a 100x1x1 amat[complex double] object
E is a 100x1x1 amat[double] object
  E(1)=
   4.0030e-16

  E(2)=
   7.8505e-16

  ...

  E(99)=
   6.2804e-16

  E(100)=
   2.5121e-15
```

8.5.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.det` can be used and is described in Section 8.5.2. This function uses the FC-BENCH Matlab toolbox described in [2] and performs all computational times of this section.

Let `A` be a `N`-by-`d`-by-`d` `amat` object, in Table 5 computational times in seconds of `B=det(A)` are given.In Figure 4, computational times in seconds for a given `N` are represented in function of very small values of `d` .

N	d=2	d=4	d=6	d=8	d=10
200 000	0.076(s)	0.228(s)	0.542(s)	1.129(s)	1.967(s)
400 000	0.108(s)	0.480(s)	1.233(s)	2.362(s)	4.028(s)
600 000	0.171(s)	0.750(s)	1.879(s)	3.633(s)	6.911(s)
800 000	0.245(s)	1.023(s)	2.658(s)	5.351(s)	9.107(s)
1 000 000	0.288(s)	1.414(s)	3.345(s)	6.655(s)	12.060(s)

Table 5: Computational times in seconds of `B=det(A)` where `A` is a `N`-by-`d`-by-`d` `amat` object.

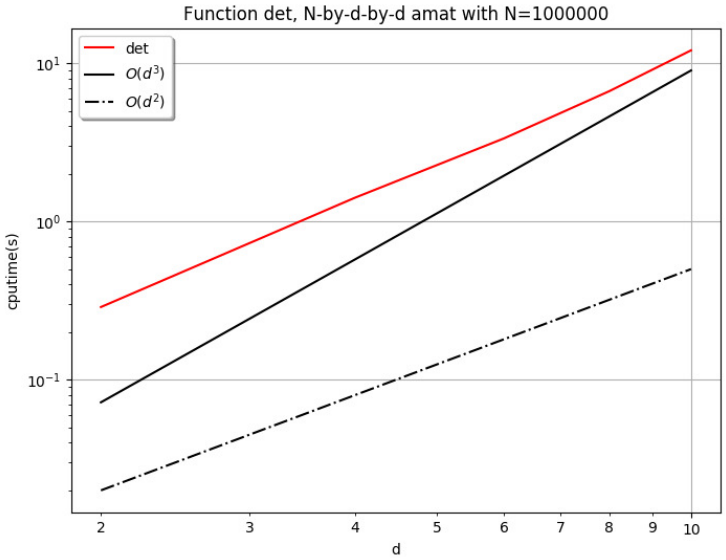

Figure 4: Computational times in seconds of of `B=det(A)` where `A` is a `N`-by-`d`-by-`d` `amat` object.

8.5.2 Benchmark function

The function `fc_amat.benchs.det` measures performance of `B=det(A)` where the input `A` is a `N`-by-`d`-by-`d` `amat` object. When running this function the `d` value is fixed, the number `N` varies and it is given by a list of values `LN` .

Syntaxe

```
fc_amat.benchs.det(LN)
fc_amat.benchs.det(LN,key,value,...)
```

Description

`fc_amat.benchs.det(LN)`

runs a benchmark of the `det` method on a `N`-by-2-by-2 `amat` object for all `N` in `LN`.

`fc_amat.benchs.det(LN,key,value,...)`

Optional key/value pairs arguments are available and can modify the input `N`-by-`d`-by-`d` `amat` object of the `det` function. `key` can be one of the following strings

- `'d'`, to set `d` (default `value` is `2`)
- `'select'`, to set the `'select'` option of the `'det'` function: `value` can be `'lu'` (default), `'laplace'` or `'auto'`.
- `'class'`, to set classname of the input `amat` object. Value can be `'double'` (default) or `'single'`.
- `'complex'`, if `true` the input `amat` object is complex (default `value` is `false`).

In Listings 58 and 59 two examples with outputs are provided.

Listing 58: : Benchmarking `D=det(A)` with `A` a `N`-by-4-by-4 matrix `amat` object

```
LN=10^5*[2:2:10];
fc_amat.benchs.det(LN,'d',4,'select','lu');
```

Output

```
#------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#             (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#------------------------------------------------------------------------------
# input parameter for N=200000 is :
# -> amat[double] with (N,nr,nc)=(200000,4,4), size=[200000 4      4]
#------------------------------------------------------------------------------
# Benchmarking command: @(A) det(A,'select','lu');
#------------------------------------------------------------------------------
#date:2020/01/01 16:13:53
#nbruns:5
#numpy:       i4      f4
#format:      %d    %.3f
#labels:       N   det(s)
          200000   0.285
          400000   0.521
          600000   0.819
          800000   1.178
         1000000   1.472
```

Listing 59: : Benchmarking B=det(A) where A is N-by-3-by-3 amat object with complex single vadetes.

```
LN=10^5*[2:2:10];
fc_amat.benchs.det(LN,'d',3,'complex',true,'class','single', ...
    'info',false);
```

Output

```
#-------------------------------------------------------------------------------
# input parameter for N=200000 is :
# -> amat[complex single] with (N,nr,nc)=(200000,3,3), size=[200000 3     3]
#-------------------------------------------------------------------------------
# Benchmarking command: @(A) det(A,'select','lu');
#-------------------------------------------------------------------------------
#date:2020/01/01 16:14:33
#nbruns:5
#numpy:       i4      f4
#format:      %d    %.3f
#labels:       N   det(s)
          200000   0.165
          400000   0.315
          600000   0.474
          800000   0.673
         1000000   0.884
```

8.6 Solving particular linear systems

There are three functions to solve linear systems A*X=B where A is a particular (regular) amat object.

- X=solvetriu(A,B) , if A is an upper triangular amat object.
- X=solvetril(A,B) , if A is a lower triangular amat object.
- X=solvediag(A,B) , if A is a diagonal amat object.

Explanations on programming techniques can be found in [1]. We only describe the solvetriu function because the two others are used similarly.

The X=solvetriu(A,B) command returns solutions of the linear systems A*X=B where A is a regular upper triangular amat object. If A is not upper triangular, then X is solution of triu(A)*X=B .

Description

X=solvetriu(A,B)

The input A supposes to be a N-by-d-by-d regular upper triangular amat object and B is either a N-by-d-by-n amat object or a d-by-n matrix. Then, the ouput X is the N-by-d-by-n amat object such that

$$\forall k \in 1{:}N, \quad A(k)*X(k) = \begin{cases} B(k) & \text{if } B \text{ is an amat object} \\ B & \text{if } B \text{ is a matrix} \end{cases}.$$

In Listing 60, some examples are provided.

Listing 60: : examples of `solvetriu` method usage

```
N=100; d=3;
A=fc_amat.random.randtriu(N,d);
info(A)
B1=fc_amat.random.randn(N,d,4);
info(B1)
X1=solvetriu(A,B1);
info(X1)
fprintf('Max. of errors in Inf. norm: %.4e\n',max(norm(A*X1-B1)))
B2=randn(d,1);
X2=solvetriu(A,B2);
info(X2)
E2=A*X2-B2;
disp(E2)
```

Output

```
A is a 100x3x3 amat[double] object
B1 is a 100x3x4 amat[double] object
X1 is a 100x3x4 amat[double] object
Max. of errors in Inf. norm: 1.3822e-14
X2 is a 100x3x1 amat[double] object
E2 is a 100x3x1 amat[double] object
  E2(1)=
   1.0e-15 *

   -0.1110
         0
         0

  E2(2)=
   1.0e-15 *

    0.1665
    0.2220
         0

  ...

  E2(99)=
   1.0e-15 *

    0.4996
         0
         0

  E2(100)=
   1.0e-15 *

   -0.6106
         0
         0
```

8.6.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.solvetriu` can be used and is described in Section 8.6.2. This function uses the FC-BENCH Matlab toolbox described in [2] and performs all computational times of this section.

Let `A` be a `N`-by-`d`-by-`d` regular triangular upper `amat` object and `B` be a `N`-by-`d`-by-`1` `amat` object. In Table 6 computational times in seconds of `X=solvetriu(A,B)` are given. In Figure 5, computational times in seconds for a given `N` are represented in function of very small values of `d`.

8.6.2 Benchmark function

The function `fc_amat.benchs.solvetriu` measures performance of `X=solvetriu(A,B)` where the input `A` is a `N`-by-`d`-by-`d` regular triangular upper `amat` object and

N	solvetriu	Error
200 000	0.020(s)	$6.2170e-15$
400 000	0.024(s)	$5.9950e-15$
600 000	0.038(s)	$7.6610e-15$
800 000	0.056(s)	$6.8830e-15$
1 000 000	0.081(s)	$1.1770e-14$
5 000 000	0.673(s)	$1.2550e-14$
10 000 000	1.334(s)	$1.1550e-14$

Table 6: Computational times in seconds of `X=solvetriu(A,B)` where `A` is a `N`-by-`d`-by-`d` `amat` object and `B` is a `N`-by-`d`-by-`1` `amat` object with `d=4`.

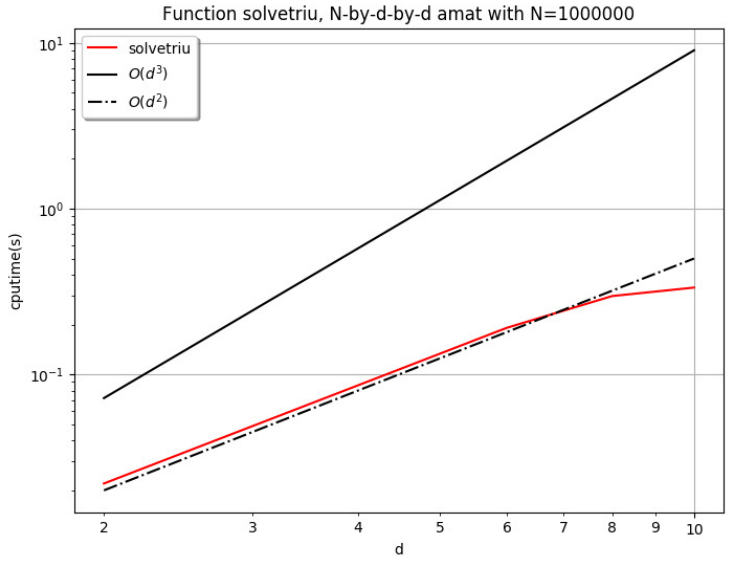

Figure 5: Computational times in seconds of of `X=solvetriu(A,B)` where `A` is a `N`-by-`d`-by-`d` `amat` object and `B` is a `N`-by-`d`-by-`1` `amat` object.

`B` is either a `N`-by-`d`-by-`n` `amat` object or a `d`-by-`n` matrix. When running this function the `d` and `n` value are fixed, the number `N` varies and it is given by a list of values `LN` .

Syntaxe

```
fc_amat.benchs.solvetriu(LN)
fc_amat.benchs.solvetriu(LN,key,value,...)
```

Description

`fc_amat.benchs.solvetriu(LN)`

runs a benchmark of the `X=solvetriu(A,B)` command where `A` is a `N`-by-`2`-by-`2` regular triangular upper `amat` object and `B` is a `N`-by-`2`-by-`1` `amat` object for all `N` in `LN` . So, by default `d=2` and `n=1` .

`fc_amat.benchs.solvetriu(LN,key,value,...)`

Optional key/value pairs arguments are available and can modify inputs of the `solvetriu` function. `key` can be one of the following strings

- `'d'` , to set `d` (default `value` is `2`)
- `'n'` , to set `n` (default `value` is `1`)
- `'rhstype'` , to set the kind of `B` : `'amat'` (default) for `amat` object and `'mat'` for matrix
- `'class'` , to set classname of the two inputs. Value can be `'double'` (default) or `'single'` .
- `'complex'` , if `true` the inputs are complex (default `value` is `false`).
- `'a'` , to set the lower bound of the interval of the uniform distribution used to generate input data (default `value` is `0.5`).
- `'b'` , to set `b` the upper bound of the interval of the uniform distribution used to generate input data (default `value` is `2`).

In Listings 61 and 62 two examples with outputs are provided.

Listing 61: : Benchmarking `X=solvetriu(A,B))` with `A` a `N`-by-`4`-by-`4` matrix `amat` object and `B` a `N`-by-`4`-by-`5` matrix `amat` object.

```
LN=10^5*[2:2:10];
fc_amat.benchs.solvetriu(LN,'d',4,'n',5, 'rhstype','mat');
```

Output

```
#------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[double] with (N,m,n)=(N,4,4)
#    containing upper triangular matrices
# 2nd parameter is :
# -> matrix[double] with (m,n)=(4,5), size=[4 5]
# Error function: @(X)max(norm(A*X-B))
#------------------------------------------------------------------------------
# Benchmarking command: @(A,B) solvetriu(A,B);
#------------------------------------------------------------------------------
#date:2020/01/01 16:17:12
#nbruns:5
#numpy:       i4           f4          f4
#format:      %d         %.3f        %.3e
#labels:       N   solvetriu(s)   Error[0]
          200000        0.061   7.161e-15
          400000        0.112   2.526e-14
          600000        0.178   6.505e-15
          800000        0.241   2.337e-14
         1000000        0.311   1.554e-14
```

Listing 62: : Benchmarking `X=solvetriu(A,B)` where `A` is `N`-by-`3`-by-`3` `amat` object and `B` is `N`-by-`3`-by-`1` `amat` object with both `complex single` values.

```
LN=10^5*[2:2:10];
fc_amat.benchs.solvetriu(LN,'d',3,'complex',true,'class','single', ...
    'info',false);
```

Output

```
#------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[complex single] with (N,m,n)=(N,3,3)
#    containing upper triangular matrices
# 2nd parameter is :
# -> amat[complex single] with (N,nr,nc)=(200000,3,1), size=[200000 3      1]
# Error function: @(X)max(norm(A*X-B))
#------------------------------------------------------------------------------
# Benchmarking command: @(A,B) solvetriu(A,B);
#------------------------------------------------------------------------------
#date:2020/01/01 16:17:32
#nbruns:5
#numpy:       i4           f4          f4
#format:      %d         %.3f        %.3e
#labels:       N   solvetriu(s)   Error[0]
          200000        0.026   3.198e-06
          400000        0.039   1.862e-06
          600000        0.056   2.282e-06
          800000        0.093   2.102e-06
         1000000        0.118   2.188e-06
```

8.7 Solving linear systems

The `X=mldivide(A,B)` or `X=A\B` commands return solutions of the linear systems `A*X=B` where `A` is a regular `amat` object. Explanations on programming techniques can be found in [1].

Description

`X=mldivide(A,B)` or `X=A\B`

The input `A` supposes to be a `N`-by-`d`-by-`d` regular `amat` object and `B` is either a `N`-by-`d`-by-`n` `amat` object or a `d`-by-`n` matrix. Then, the ouput `X` is the `N`-by-`d`-by-`n` `amat` object such that

$$\forall k \in 1{:}N, \quad A(k)*X(k) = \begin{cases} B(k) & \text{if } B \text{ is an } amat \text{ object} \\ B & \text{if } B \text{ is a matrix} \end{cases}.$$

In Listing 63, some examples are provided.

Listing 63: : examples of `mldivide` method operator usage

```
N=100; d=3;
A=fc_amat.random.randnsdd(N,d);
info(A)
B1=fc_amat.random.randn(N,d,4);
info(B1)
X1=mldivide(A,B1); % using function
info(X1)
fprintf('Max. of errors in Inf. norm: %.4e\n',max(norm(A*X1-B1)))
B2=randn(d,1);
X2=A\B2; % using operator
info(X2)
E2=A*X2-B2;
disp(E2)
```

Output

```
A is a 100x3x3 amat[double] object
B1 is a 100x3x4 amat[double] object
X1 is a 100x3x4 amat[double] object
Max. of errors in Inf. norm: 3.4417e-15
X2 is a 100x3x1 amat[double] object
E2 is a 100x3x1 amat[double] object
  E2(1)=
   1.0e-15 *

    0.0139
   -0.1110
         0

  E2(2)=
   1.0e-16 *

    0.4163
         0
   -0.2082

  ...

  E2(99)=
   1.0e-15 *

   -0.0139
   -0.1110
   -0.0486

  E2(100)=
   1.0e-16 *

   -0.1388
         0
   -0.2082
```

8.7.1 Efficiency

For benchmarking purpose the function `fc_amat.benchs.mldivide` can be used and is described in Section 8.7.2. This function uses the FC-BENCH Matlab

toolbox described in [2] and performs all computational times of this section.

Let `A` be a `N`-by-`d`-by-`d` regular triangular upper `amat` object and `B` be a `N`-by-`d`-by-`1` `amat` object. In Table 7 computational times in seconds of `X=mldivide(A,B)` are given.In Figure 6, computational times in seconds for a given `N` are represented in function of very small values of `d`.

N	d=2	d=4	d=6	d=8	d=10
200 000	0.098(s)	0.292(s)	0.710(s)	1.308(s)	2.109(s)
400 000	0.155(s)	0.658(s)	1.478(s)	2.691(s)	4.747(s)
600 000	0.254(s)	1.041(s)	2.546(s)	4.624(s)	7.913(s)
800 000	0.334(s)	1.494(s)	3.493(s)	7.100(s)	10.643(s)
1 000 000	0.418(s)	1.821(s)	4.587(s)	8.335(s)	12.801(s)

Table 7: Computational times in seconds of `X=mldivide(A,B)` where `A` is a `N`-by-`d`-by-`d` `amat` object and `B` is a `N`-by-`d`-by-`1` `amat` object.

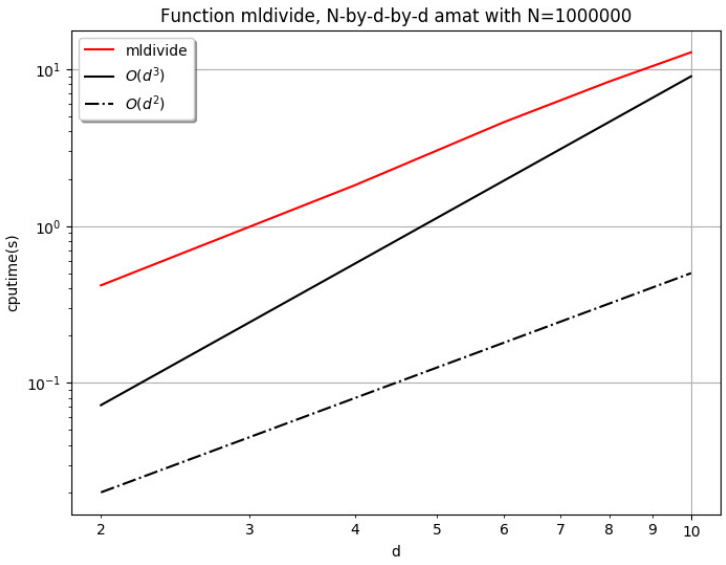

Figure 6: Computational times in seconds of of `X=mldivide(A,B)` where `A` is a `N`-by-`d`-by-`d` `amat` object and `B` is a `N`-by-`d`-by-`1` `amat` object.

8.7.2 Benchmark function

The function `fc_amat.benchs.mldivide` measures performance of `X=mldivide(A,B)` where the input `A` is a `N`-by-`d`-by-`d` regular triangular upper `amat` object and `B` is either a `N`-by-`d`-by-`n` `amat` object or a `d`-by-`n` matrix. When running this function the `d` and `n` value are fixed, the number `N` varies and it is given by a list of values `LN`.

Syntaxe

```
fc_amat.benchs.mldivide(LN)
fc_amat.benchs.mldivide(LN,key,value,...)
```

Description

`fc_amat.benchs.mldivide(LN)`

runs a benchmark of the `X=mldivide(A,B)` command where `A` is a `N`-by-`2`-by-`2` regular triangular upper `amat` object and `B` is a `N`-by-`2`-by-`1` `amat` object for all `N` in `LN`. So, by default `d=2` and `n=1`.

`fc_amat.benchs.mldivide(LN,key,value,...)`

Optional key/value pairs arguments are available and can modify inputs of the `mldivide` function. `key` can be one of the following strings

- `'d'`, to set `d` (default `value` is `2`)
- `'n'`, to set `n` (default `value` is `1`)
- `'rhstype'`, to set the kind of `B` : `'amat'` (default) for `amat` object and `'mat'` for matrix
- `'class'`, to set classname of the two inputs. Value can be `'double'` (default) or `'single'`.
- `'complex'`, if `true` the inputs are complex (default `value` is `false`).
- `'a'`, to set the lower bound of the interval of the uniform distribution used to generate input datas (default `value` is `0.5`).
- `'b'`, to set `b` the upper bound of the interval of the uniform distribution used to generate input datas (default `value` is `2`).

In Listings 64 and 65 two examples with outputs are provided.

Listing 64: : Benchmarking `X=mldivide(A,B)` with `A` a `N`-by-`4`-by-`4` matrix `amat` object and `B` a `N`-by-`4`-by-`5` matrix `amat` object.

```
LN=10^5*[2:2:10];
fc_amat.benchs.mldivide(LN,'d',4,'n',5, 'rhstype','mat');
```

Output

```
#-------------------------------------------------------------------------------
#    computer: rhum-ubuntu18-04
#      system: Ubuntu 18.04.3 LTS (x86_64)
#   processor: Intel(R) Xeon(R) CPU E5-2667 0 @ 2.90GHz
#              (2 procs/6 cores by proc/2 threads by core)
#         RAM: 62.9 Go
#    software: Matlab
#     release: 2019a
#  MaxThreads: 12
#-------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[double] with (N,m,n)=(N,4,4)
#    containing strictly diagonally dominant matrices
# 2nd parameter is :
# -> matrix[double] with (m,n)=(4,5), size=[4 5]
# Error function: @(X)max(norm(A*X-B))
#-------------------------------------------------------------------------------
#date:2020/01/01 16:41:25
#nbruns:5
#numpy:       i4           f4          f4
#format:      %d         %.3f        %.3e
#labels:       N   mldivide(s)    Error[0]
          200000        0.506    1.099e-14
          400000        1.121    1.510e-14
          600000        1.721    1.928e-14
          800000        2.348    2.792e-14
         1000000        2.966    2.011e-14
```

Listing 65: : Benchmarking `X=mldivide(A,B)` where `A` is `N`-by-`3`-by-`3` `amat` object and `B` is `N`-by-`3`-by-`1` `amat` object with both `complex single` values.

```
LN=10^5*[2:2:10];
fc_amat.benchs.mldivide(LN,'d',3,'complex',true,'class','single', ...
    'info',false);
```

Output

```
#-------------------------------------------------------------------------------
# 1st parameter is :
# -> amat[complex single] with (N,m,n)=(N,3,3)
#    containing strictly diagonally dominant matrices
# 2nd parameter is :
# -> amat[complex single] with (N,nr,nc)=(200000,3,1), size=[200000 3      1]
# Error function: @(X)max(norm(A*X-B))
#-------------------------------------------------------------------------------
#date:2020/01/01 16:42:40
#nbruns:5
#numpy:       i4           f4          f4
#format:      %d         %.3f        %.3e
#labels:       N   mldivide(s)    Error[0]
          200000        0.249    2.458e-06
          400000        0.469    2.387e-06
          600000        0.796    1.966e-06
          800000        1.062    2.588e-06
         1000000        1.452    2.110e-06
```

8 References

[1] François Cuvelier. Efficient algorithms to perform linear algebra operations on 3d arrays in vector languages. 2018.

[2] Francois Cuvelier. fc-bench: Matlab toolbox for benckmarking. http://www.math.univ-paris13.fr/~cuvelier/software/Matlab/fc-bench.html, 2018.