FC-HYPERMESH Octave package, User's Guide [*]

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Abstract

This object-oriented Octave package allows to mesh any d-orthotopes (hyperrectangle in dimension d) and their m-faces by simplices or orthotopes. It was created to show the implementation of the algorithms of the report Vectorized algorithms for regular tessellations of d-orthotopes and their faces. The FC-HYPERMESH package uses Octave objects and is provided with meshes visualisation tools for dimension leather or equal to 3.

Contents

[*] Compiled with Octave 4.2.0

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1 Introduction

The FC-HYPERMESH package contains a simple class object `OrthMesh` which permits, in any dimension $\mathrm{d} \geqslant 1$, to obtain a simplicial mesh or orthotope mesh with all their m-faces, $0 \leqslant m < \mathrm{d}$. It is also possible with the method function `plot` of the class object `OrthMesh` to represent a mesh or its m-faces for $\mathrm{d} \leqslant 3$.

In the folowing section, the class object `OrthMesh` is presented. Thereafter some warning statements on the memory used by these objects in high dimension are given. Finally computation times for orthotope meshes and simplicial meshes are given in dimension $\mathrm{d} \in [\![1, 5]\!]$.

2 Installation of the fc-hypermesh package

For a better presentation, the following notation is used in various linux command

`<HREF>` equal to
`http://www.math.univ-paris13.fr/~cuvelier/software/codes/`

2.1 Installation from packages (recommanded)

- Download the packages. For example, in a terminal:

Terminal
```
# wget <HREF>/fc-tools/0.0.14/fc-tools-0.0.14.tar.gz
# wget <HREF>/fc-hypermesh/0.0.2/fc-hypermesh-0.0.2.tar.gz
```

- Under Octave :

GNU Octave
```
▷ pkg install fc-tools-0.0.14.tar.gz
▷ pkg install fc-hypermesh-0.0.2.tar.gz
```

- Now to use `fc-hypermseh` in any Octave session, it is necessary to load the package:

GNU Octave
```
▷ pkg load fc-hypermesh
```

In an Octave session, the **unistalling** of the `fc-hypermseh` package is done by:

GNU Octave

```
▷ pkg uninstall fc-hypermseh
```

2.2 Automatic installation

to do.

2.3 Manual installation

to do.

3 Using the fc-hypermesh package

First of all, the main class object `OrthMesh` is presented. A complete report was written to described the algorithms used []. Thereafter some usage samples are given.

3.1 Class object `OrthMesh`

The aim of the class object `OrthMesh` is to efficiently create an object which contains a mesh of a d-orthotope and all its m-face meshes. An elementary mesh class object `EltMesh` is used to store only one mesh, the main mesh as well as any of the m-face meshes. This class `EltMesh` also simplify (for me) the codes writing. Its fields are the following:

- d, space dimension
- m, kind of mesh ($m = \mathrm{d}$ for the main mesh)
- type, 0 for simplicial mesh or 1 for orthotope mesh
- $\mathrm{n_q}$, number of vertices
- q, vertices array of dimension d-by-$\mathrm{n_q}$
- $\mathrm{n_{me}}$, number of mesh elements
- me, connectivity array of dimension $(\mathrm{d}+1)$-by-$\mathrm{n_{me}}$ for simplices elements or 2^{d}-by-$\mathrm{n_{me}}$ for orthotopes elements
- `toGlobal`, index array linking local array q to the one of the main mesh
- label, name/number of this elementary mesh
- color, color of this elementary mesh (for plotting purpose)

Let the d-orthotope defined by $[a_1, b_1] \times \cdots \times [a_{\mathrm{d}}, b_{\mathrm{d}}]$. The class object `OrthMesh` corresponding to this d-orthotope contains the main mesh and all its m-face meshes, $0 \leqslant m < \mathrm{d}$. Its Fields are the following

- d: space dimension
- type: string 'simplicial' or 'orthotope' mesh
- Mesh: main mesh as an `EltMesh` object
- Faces: list of arrays of `EltMesh` objects such that Faces(1) is an array of all the (d − 1)-face meshes, Faces(2) is an array of all the (d − 2)-face meshes, and so on
- box: a d-by-2 array such that $\mathrm{box}(i, 1) = a_i$ and $\mathrm{box}(i, 2) = b_i$.

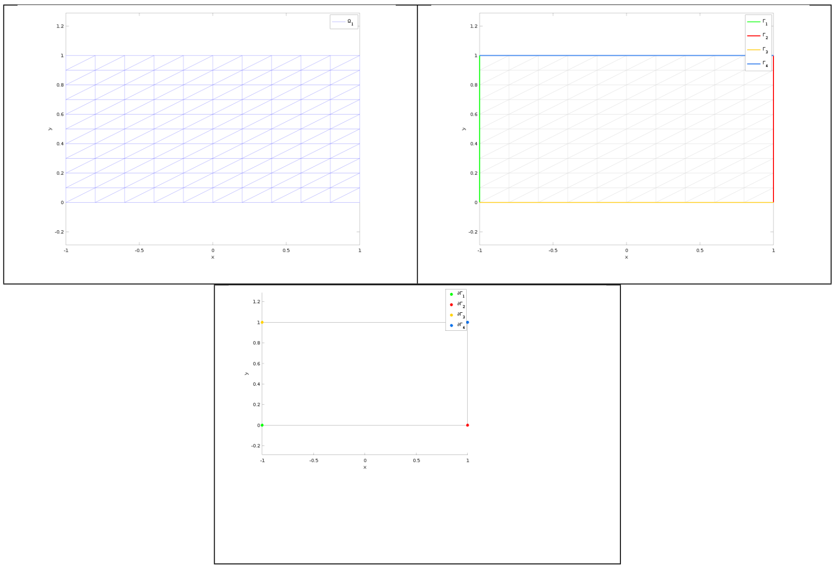
The `OrthMesh` constructor is

$$Oh \leftarrow \textsc{OrthMesh}(\mathrm{d}, \mathbf{N}, < \mathrm{box} >, < \mathrm{type} >)$$

where $\mathbf{N}$ is either a 1-by-d array such that $\mathbf{N}(i)$ is the number of discretization for $[a_i, b_i]$ or either an integer if the the number of discretization is the same in all space directions. The optional parameter box previously described as for default value $a_i = 0$ and $b_i = 1$. The default value for optional parameter type is 'simplicial' and ortherwise 'orthotope' can be used.

3.2 2d-orthotope meshing by simplices

In Listing 22, an `OrthMesh` object is built under Octavefor the orthotope $[-1, 1] \times [0, 1] \times [0, 2]$ with simplicial elements and $\mathbf{N} = (10, 5, 10)$. The main mesh and all the m-face meshes of the resulting object are plotted.

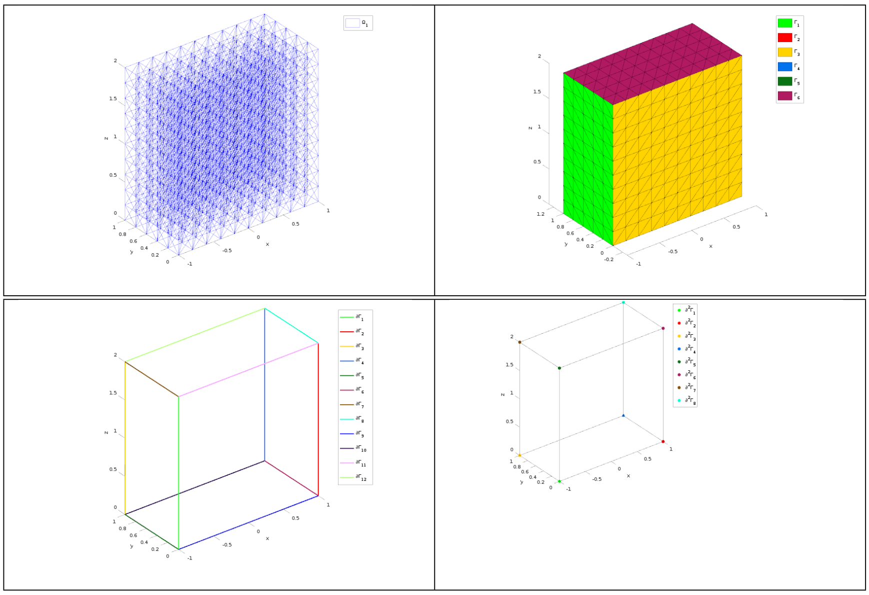


```
Oh=OrthMesh(2,10,'box',[-1,1;0,1])
% plot the main mesh
figure(1)
Oh.plot('legend',true)
axis equal;xlabel('x');ylabel('y')
% plot the 1-face meshes
figure(2)
Oh.plot('color',[0.8,0.8,0.8])
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
axis equal;xlabel('x');ylabel('y');
% plot the 0-face meshes
figure(3)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
axis equal;xlabel('x');ylabel('y');
```

Listing 1: 2D simplicial `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

3.3 3d-orthotope meshing by simplices

In Listing 22, an `OrthMesh` object is built under Octavefor the orthotope $[-1,1]\times[0,1]\times[0,2]$ with simplicial elements and $\mathbf{N}=(10,5,10)$. The main mesh and all the m-face meshes of the resulting object are plotted.

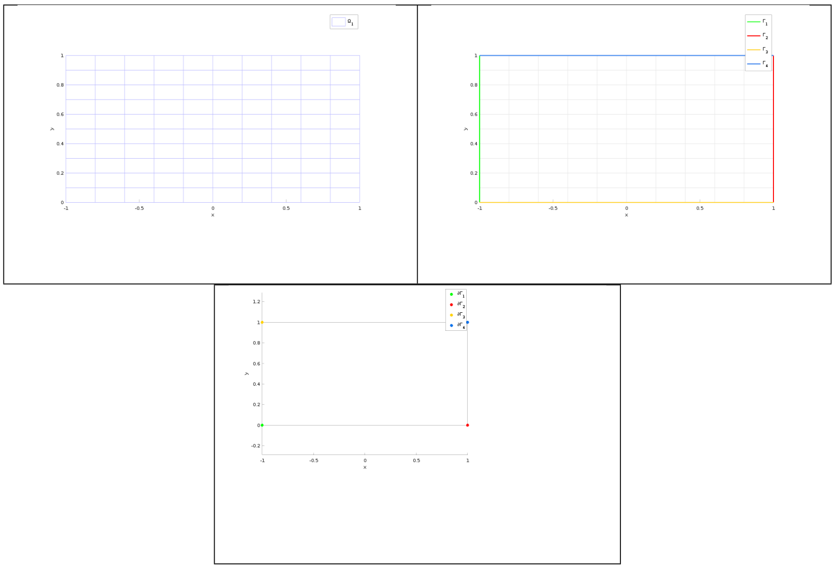


```
Oh=OrthMesh(3,[10,5,10],'box',[-1,1;0,1;0,2])
% plot the main mesh
figure(1)
Oh.plot('legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 2-face meshes
figure(2)
Oh.plot('m',2,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 1-face meshes
figure(3)
Oh.plot('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 0-face meshes
figure(4)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
```

Listing 2: 3D simplicial `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

3.4 2d-orthotope meshing by orthotopes

In Listing 22, an `OrthMesh` object is built under Octavefor the orthotope $[-1,1]\times[0,1]\times[0,2]$ with orthotope elements and $\mathbf{N}=(10,5,10)$. The main mesh and all the m-face meshes of the resulting object are plotted.

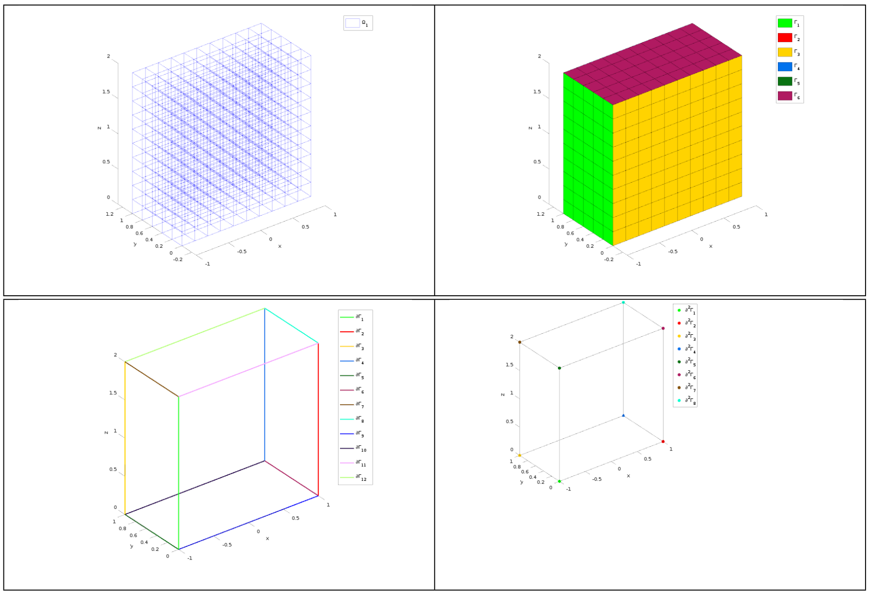


```
Oh=OrthMesh(2,10,'box',[-1,1;0,1],'type','orthotope')
% plot the main mesh
figure(1)
Oh.plot('legend',true)
axis equal;xlabel('x');ylabel('y')
% plot the 1-face meshes
figure(2)
Oh.plot('color',[0.8,0.8,0.8])
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
axis equal;xlabel('x');ylabel('y');
% plot the 0-face meshes
figure(3)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
axis equal;xlabel('x');ylabel('y');
```

Listing 3: 2D orthotope `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

3.5 3d-orthotope meshing by orthotopes

In Listing 22, an `OrthMesh` object is built under Octavefor the orthotope $[-1,1]\times[0,1]\times[0,2]$ with orthotope elements and $\mathbf{N} = (10,5,10)$. The main mesh and all the m-face meshes of the resulting object are plotted.



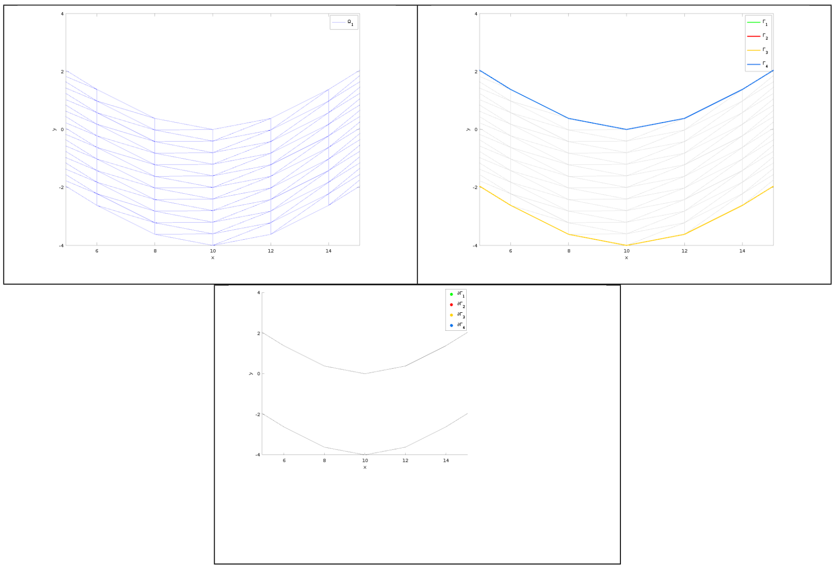
```
Oh=OrthMesh(3,[10,5,10],'box',[-1,1;0,1;0,2], ...
    'type','orthotope')
% plot the main mesh
figure(1)
Oh.plot('legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 2-face meshes
figure(2)
Oh.plot('m',2,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 1-face meshes
figure(3)
Oh.plot('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 0-face meshes
figure(4)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
```

Listing 4: 3D orthotope `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

3.6 Mapping of a 2d-orthotope meshing by simplices

For example, the following 2D geometrical transformation allows to deform the reference unit hypercube.

$$\begin{array}{ccc} [0,1]\times[0,1] & \longrightarrow & \mathbb{R}^2 \\ \begin{pmatrix} x \\ y \end{pmatrix} & \longrightarrow & F(x,y) = \begin{pmatrix} 20\,x \\ 2\,(2\,y-1+\cos(2\pi x)) \end{pmatrix} \end{array}$$



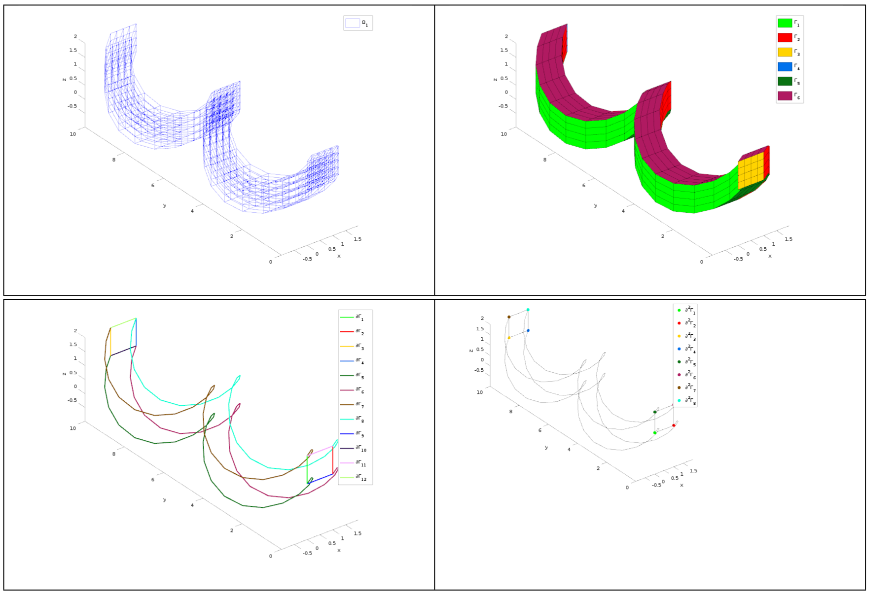
```
Oh=OrthMesh(2,10,'mapping',@(q) [20*q(1,:); ...
    2*(2*q(2,:)-1+cos(2*pi*q(1,:)))]);
% plot the main mesh
figure(1)
Oh.plot('legend',true)
axis equal;xlabel('x');ylabel('y')
% plot the 1-face meshes
figure(2)
Oh.plot('color',[0.8,0.8,0.8])
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
axis equal;xlabel('x');ylabel('y');
% plot the 0-face meshes
figure(3)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
axis equal;xlabel('x');ylabel('y');
```

Listing 5: Mapping of a 2D simplicial `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

3.7 3d-orthotope meshing by orthotopes

For example, the following 3D geometrical transformation allows to deform the reference unit hypercube.

$$\begin{array}{rcl} [0,1]\times[0,1]\times[0,1] & \longrightarrow & \mathbb{R}^2 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \longrightarrow & F(x,y,y) = \begin{pmatrix} x+\sin(4\pi y) \\ 10y \\ z+\cos(4\pi y) \end{pmatrix} \end{array}$$



```
Map=@(q) [q(1,:)+ sin(4*pi*q(2,:)); 10*q(2,:); ...
    q(3,:)+cos(4*pi*q(2,:))];
Oh=OrthMesh(3,[4,25,4],'mapping',Map,'type','orthotope');
% plot the main mesh
figure(1)
Oh.plot('legend',true)
xlabel('x');ylabel('y');zlabel('z');axis image;axis equal;
% plot the 2-face meshes
figure(2)
Oh.plot('m',2,'legend',true)
xlabel('x');ylabel('y');zlabel('z');axis image;axis equal;
% plot the 1-face meshes
figure(3)
Oh.plot('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plot('m',1,'Linewidth',2,'legend',true)
xlabel('x');ylabel('y');zlabel('z');axis image;axis equal;
% plot the 0-face meshes
figure(4)
Oh.plot('m',1,'color','k')
hold on
Oh.plot('m',0,'legend',true)
xlabel('x');ylabel('y');zlabel('z');axis image;axis equal;
```

Listing 6: Mapping of a 3D orthotope `OrthMesh` object with Octave 4.2.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)