# fc hypermesh Octave package, User's Guide [*]

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**Abstract**

This object-oriented Octave package allows to generate conforming meshes of hypercubes, hyperrectangles or of any $d$-orthotopes by simplices or orthotopes with their $m$-faces. It was created to show the implementation of the algorithms of [1]. The fc hypermesh package uses Octave objects and is provided with meshes visualisation tools for dimension less than or equal to 3.

## 0 Contents

[*] Compiled with Octave 4.2.1, packages `fc_hypermesh`-'0.0.7' and `fc_tools`-'0.0.21' under cosmos computer

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# 1 Introduction

The fc hypermesh package contains a simple class object OrthMesh which permits, in any dimension $d \geqslant 1$, to obtain a simplicial mesh or orthotope mesh with all their $m$-faces, $0 \leqslant m < d$. It is also possible with the method function `plotmesh` of the class object OrthMesh to represent a mesh or its $m$-faces for $d \leqslant 3$.

This package was tested under

**Windows 10.0.16299:** with Octave 4.2.0, 4.2.1 and 4.2.2

**macOS High Sierra 10.13.4:** with Octave 4.2.1 (installed with homebrew)

**Ubuntu 16.04.3 LTS:** with Octave 4.2.0, 4.2.1 and 4.2.2 (all compiled from source)

**Ubuntu 17.10:** with Octave 4.2.0 and 4.2.1 and 4.2.2 (all compiled from source)

**centOS 7.4:** with Octave 4.2.0, 4.2.1 and 4.2.2 (all compiled from source)

**Fedora 27:** with Octave 4.2.0, 4.2.1 and 4.2.2 (all compiled from source)

**OpenSUSE Leap 42.3:** with Octave 4.2.0, 4.2.1 and 4.2.2 (all compiled from source)

It is not compatible with Octave 4.0.x and previous.

In the following section, the class object OrthMesh is presented. Thereafter some warning statements on the memory used by these objects in high dimension are given. Finally computation times for orthotope meshes and simplicial meshes are given in dimension $d \in [\![1, 5]\!]$.

# 2 Installation

Here are two methods of installations. The first uses the Octave `pkg` command and the second a provided Octave script.

# 3 Installation via pkg command

- Download the packages. For example, in a terminal:

```
# wget http://www.math.univ-paris13.fr/~cuvelier/software/codes/
    Octave/fc-tools/0.0.21/fc-tools-0.0.21.tar.gz
# wget http://www.math.univ-paris13.fr/~cuvelier/software/codes/
    Octave/fc-hypermesh/0.0.7/fc-hypermesh-0.0.7.tar.gz
```

- Under Octave :

```
>> pkg install fc-tools-0.0.21.tar.gz
>> pkg install fc-hypermesh-0.0.7.tar.gz
```

- Now to use `fc-hypermesh` in any Octave session, it is necessary to load the package:

```
>> pkg load fc-hypermesh
```

- To try the package, one can launch a demo:

```
>> fc_hypermesh.demo01
```

For uninstalling the package, just do in an Octave session:

```
>> pkg uninstall fc-hypermesh
>> pkg uninstall fc-tools
```

## 4 All-in-one installation

For this method, one just has to get/download the install file

ofc_hypermesh_install.m

or get it on the dedicated web page. Thereafter, it should be run under Octave. This command downloads, extracts and configures the *fc-hypermesh* and the required *fc-tools* packages in the current directory.

For example, to install this package in directory `~/Octave/packages`, in a terminal one can do:

```
# mkdir -p ~/Octave/packages
# cd ~/Octave/packages
# wget http://www.math.univ-paris13.fr/~cuvelier/software/codes/Octave/fc
    -hypermesh/0.0.7/ofc_hypermesh_install.m
```

Then in a Octave terminal run the following commands

```
>> cd ~/Octave/packages
>> ofc_hypermesh_install
```

This is the output of the `ofc_hypermesh_install` command:

```
Parts of the GNU Octave <fc-hypermesh> package.
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1- Downloading and extracting the packages
2- Setting the <fc-hypermesh> package
Write in ~/Octave/packages/fc-hypermesh-full/fc_hypermesh-0.0.7/
     configure_loc.m ...
3- Using toolboxes :
   ->            fc-tools : 0.0.21
   ->         fc-hypermesh : 0.0.7
*** Using instructions
   To use the <fc-hypermesh> toolbox:
   addpath('~/Octave/packages/fc-hypermesh-full/fc_hypermesh-0.0.7')
   fc_hypermesh.init()

   See ~/Octave/packages/ofc_hypermesh_set.m
```

The complete package (i.e. with all the other needed packages) is stored in the directory `~/Octave/packages/fc-hypermesh-full` and, for each Octave session, one has to set the package by:

```
>> addpath('~/Octave/packages/fc-hypermesh-full/ofc-hypermesh-0.0.7')
>> fc_hypermesh.init()
```

To **uninstall**, one just has to delete directory `~/Octave/packages/fc-hypermesh-full`

# 5 Using the fc hypermesh package

First of all, the main class object `OrthMesh` is presented. Thereafter some usage samples are given.

## 5.1 Class object `OrthMesh`

The aim of the class object `OrthMesh` is to efficiently create an object which contains a mesh of a `d`-orthotope and all its `m`-face meshes. An elementary mesh class object `EltMesh` is used to store only one mesh, the main mesh as well as any of the `m`-face meshes. This class `EltMesh` also simplifies (for me) the codes writing and its fields are the following:

- `d`, space dimension
- `m`, kind of mesh ($\mathtt{m} = \mathtt{d}$ for the main mesh)
- `type`, 0 for simplicial mesh or 1 for orthotope mesh
- `nq`, number of vertices
- `q`, vertices array of dimension `d`-by-`nq`
- `nme`, number of mesh elements
- `me`, connectivity array of dimension $(\mathtt{d}+1)$-by-`nme` for simplices elements or $2^{\mathtt{d}}$-by-`nme` for orthotopes elements
- `toGlobal`, index array linking local array `q` to the one of the main mesh

- `label`, name/number of this elementary mesh
- `color`, color of this elementary mesh (for plotting purpose)

Let the `d`-orthotope defined by $[a_1, b_1] \times \cdots \times [a_{\mathtt{d}}, b_{\mathtt{d}}]$. The class object `OrthMesh` corresponding to this `d`-orthotope contains the main mesh and all its `m`-face meshes, $0 \leqslant \mathtt{m} < \mathtt{d}$. Its Fields are the following

- `d`: space dimension
- `type`: string `'simplicial'` or `'orthotope'` mesh
- `Mesh`: main mesh as an `EltMesh` object
- `Faces`: list of arrays of `EltMesh` objects such that `Faces(1)` is an array of all the $(\mathtt{d}-1)$-face meshes, `Faces(2)` is an array of all the $(\mathtt{d}-2)$-face meshes, and so on
- `box`: a `d`-by-2 array such that `box(i,1)` $= a_i$ and `box(i,2)` $= b_i$.

### 5.1.1 Constructor

```
Oh = OrthMesh(d,N)
Oh = OrthMesh(d,N, key,value, ...)
```

**Description**

```
Oh = OrthMesh(d,N)
```

Genrerates the `OrthMesh` object `Oh` which contains which contains a simplicial mesh of the unit `d`-orthotope and all its `m`-face meshes.

```
Oh = OrthMesh(d,N, key,value, ...)
```

Some optional `key`/`value` pairs arguments are available with `key`:

- `'type'` : used to select the kind of elements used for meshing. The default `value` is `'simplicial'` and otherwise `'orthotope'` can be used.

  ```
  Oh = OrthMesh(3,10, 'type','orthotope')
  ```

- `'box'` : used to specify the `d`-orthotope $[a_1, b_1] \times \ldots \times [a_{\mathtt{d}}, b_{\mathtt{d}}]$ by setting `value` as an `d`-by-2 array such that $a_i =$ `value(i,1)` and $b_i =$ `value(i,2)`.

  ```
  Oh = OrthMesh(3,10, 'box',[-1 1;-2 2;0 3])
  ```

- `'m_min'` : used to only mesh the `m`-Faces for `m` in $[\![\mathtt{m}, \mathtt{d}]\!]$. Default `value` is 0.

  ```
  Oh = OrthMesh(3,10, 'm_min',2)
  ```

- `'mapping'` : used to apply on the mesh a mapping function given by a function handle.

  ```
  Oh = OrthMesh(3,10, 'mapping',@(q) [q(1,:)+sin(q(2,:));q(2,:);q(3,:)])
  ```

### 5.1.2 `plotmesh` method

The `plotmesh()` member function can be used to represent the mesh given by an `OrthMesh` object if the space dimension is less than or equal to 3.

#### Syntaxe

```
obj.plotmesh()
obj.plotmesh(key, value, ...)
```

#### Description

```
obj.plotmesh()
```

```
obj.plotmesh(key, value, ...)
```

Some optional `key`/`value` pairs arguments are available with `key`:

- `'legend'` : if `value` is `True`, a legend is displayed. Default is `False`.
- `'m'` : plots all the `m`-faces of the mesh. Default `m = d` i.e. the main mesh. ($0 \leqslant$ `m` $\leqslant$ `d`)
- `'color'` : use to specify the color of the mesh.
- ...

Other `key`/`value` pairs arguments can be used depending of `obj.d` and `obj.m` values and they are those of the plotting function used:

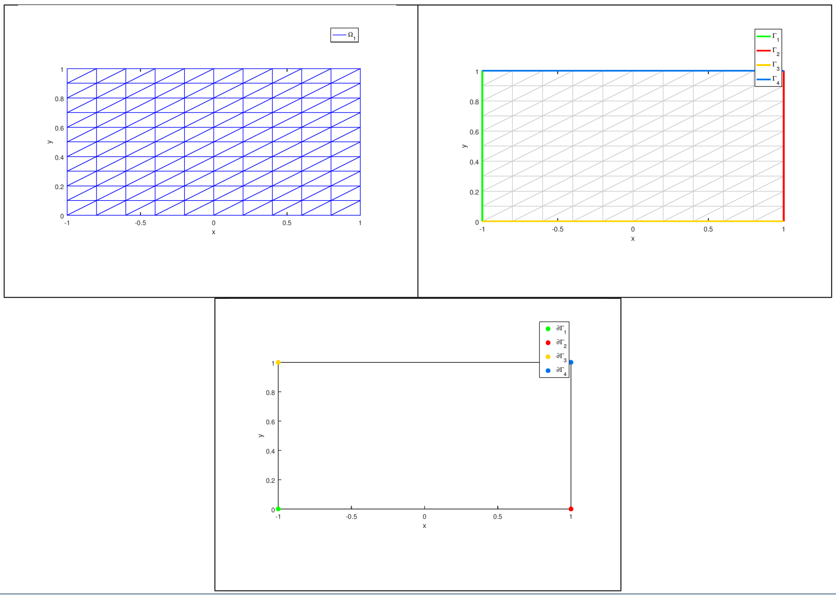
- with `obj.d=3` and `obj.m=3`, `patch` function is used;
- with `obj.d=3` and `obj.m=2`, `trimesh` function is used for simplicial mesh and `patch` function is used for orthotope mesh;
- with `obj.d=3` and `obj.m=1`, `line` function is used;
- with `obj.d=3` and `obj.m=0`, `scatter3` function is used;
- with `obj.d=2` and `obj.m=2`, `triplot` function is used for simplicial mesh and `patch` function is used for orthotope mesh;
- with `obj.d=2` and `obj.m=1`, `line` function is used;
- with `obj.d=2` and `obj.m=0`, `scatter` function is used;
- with `obj.d=1` and `obj.m=1`, `line` function is used;
- with `obj.d=1` and `obj.m=0`, `scatter` function is used;

## 5.2 2d-orthotope meshing by simplices

In Listing 1, an `OrthMesh` object is built under Octave by using command

```
Oh=OrthMesh(2,10,'box',[-1,1;0,1])
```

So the Oh object is the tessellations of the orthotope $[-1,1]\times[0,1]$ with simplicial elements. In each direction $10+1(=11!)$ points are taken. So we have $11^2$ vertices in this mesh. The main mesh and all the m-face meshes of the resulting object are plotted by using plotmesh method.

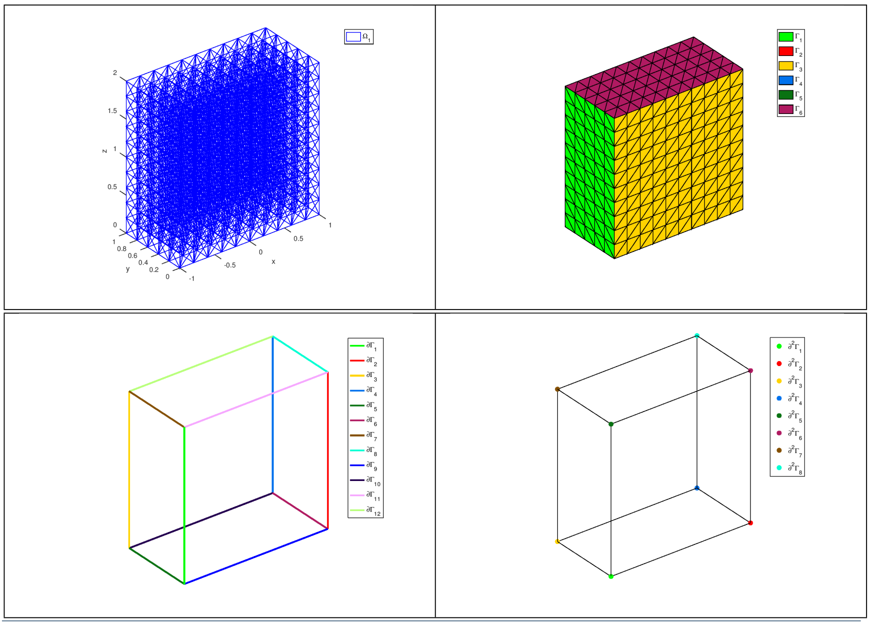


```
Oh=OrthMesh(2,10,'box',[-1,1;0,1])
% plot the main mesh
figure(1)
Oh.plotmesh('legend',true)
% plot the 1-face meshes
figure(2)
Oh.plotmesh('color',[0.8,0.8,0.8])
hold on
Oh.plotmesh('m',1,'Linewidth',2,'legend',true)
% plot the 0-face meshes
figure(3)
Oh.plotmesh('m',1,'color','k')
hold on
Oh.plotmesh('m',0,'legend',true)
```

Listing 1: 2D simplicial OrthMesh object with Octave 4.2.1, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

## 5.3 3d-orthotope meshing by simplices

In Listing 1, an OrthMesh object is built under Octave for the orthotope $[-1,1]\times[0,1]\times[0,2]$ with simplicial elements and N=[10,5,10]. The main mesh and all the m-face meshes of the resulting object are plotted.

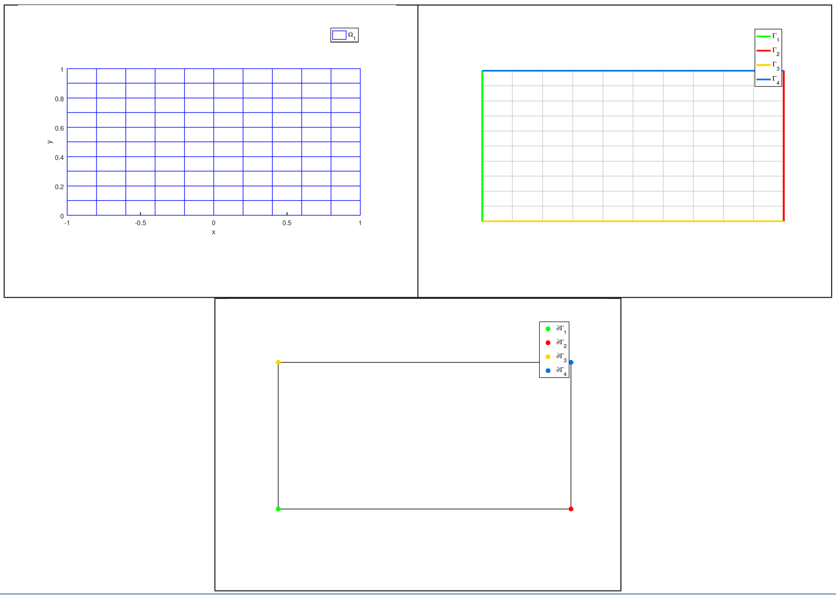


```
Oh=OrthMesh(3,[10,5,10],'box',[-1,1;0,1;0,2])
% plot the main mesh
figure(1)
Oh.plotmesh('legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 2-face meshes
figure(2)
Oh.plotmesh('m',2,'legend',true)
axis equal;axis off
% plot the 1-face meshes
figure(3)
Oh.plotmesh('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plotmesh('m',1,'Linewidth',2,'legend',true)
axis equal;axis off
% plot the 0-face meshes
figure(4)
Oh.plotmesh('m',1,'color','k')
hold on
Oh.plotmesh('m',0,'legend',true)
axis equal;axis off
```

Listing 2: 3D simplicial `OrthMesh` object with Octave 4.2.1, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

## 5.4 2d-orthotope meshing by orthotopes

In Listing 1, an `OrthMesh` object is built under Octavefor the orthotope $[-1,1]\times[0,1]\times[0,2]$ with orthotope elements and `N=[10,5,10]`. The main mesh and all the `m`-face meshes of the resulting object are plotted.

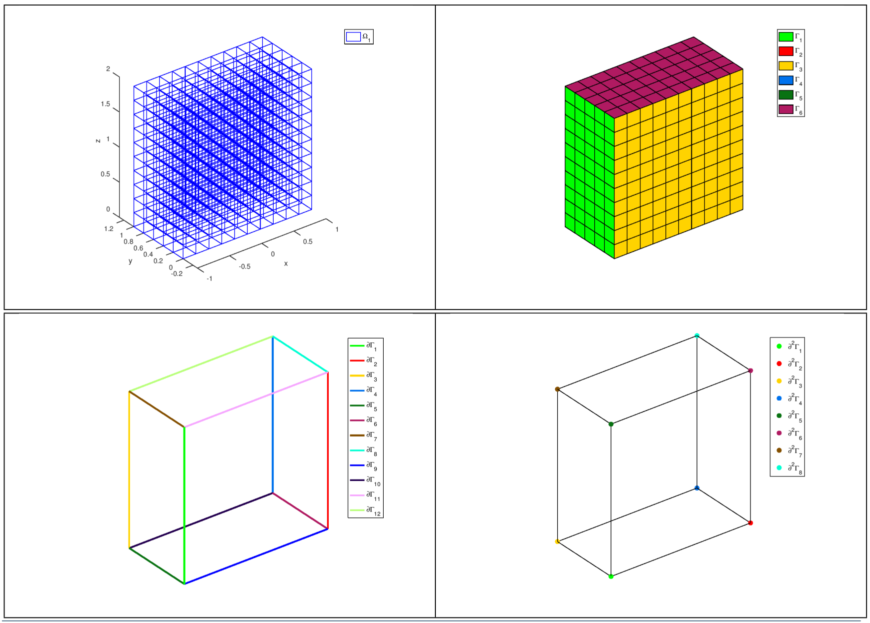


```
Oh=OrthMesh(2,10,’box’,[-1,1;0,1],’type’,’orthotope’)
% plot the main mesh
figure(1)
Oh.plotmesh(’legend’,true)
axis equal;xlabel(’x’);ylabel(’y’)
% plot the 1-face meshes
figure(2)
Oh.plotmesh(’color’,[0.8,0.8,0.8])
hold on
Oh.plotmesh(’m’,1,’Linewidth’,2,’legend’,true)
axis equal;axis off
% plot the 0-face meshes
figure(3)
Oh.plotmesh(’m’,1,’color’,’k’)
hold on
Oh.plotmesh(’m’,0,’legend’,true)
axis equal;axis off
```

Listing 3: 2D orthotope `OrthMesh` object with Octave 4.2.1, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

## 5.5 3d-orthotope meshing by orthotopes

In Listing 1, an `OrthMesh` object is built under Octave for the orthotope $[-1,1]\times[0,1]\times[0,2]$ with orthotope elements and `N=[10,5,10]`. The main mesh and all the `m`-face meshes of the resulting object are plotted.



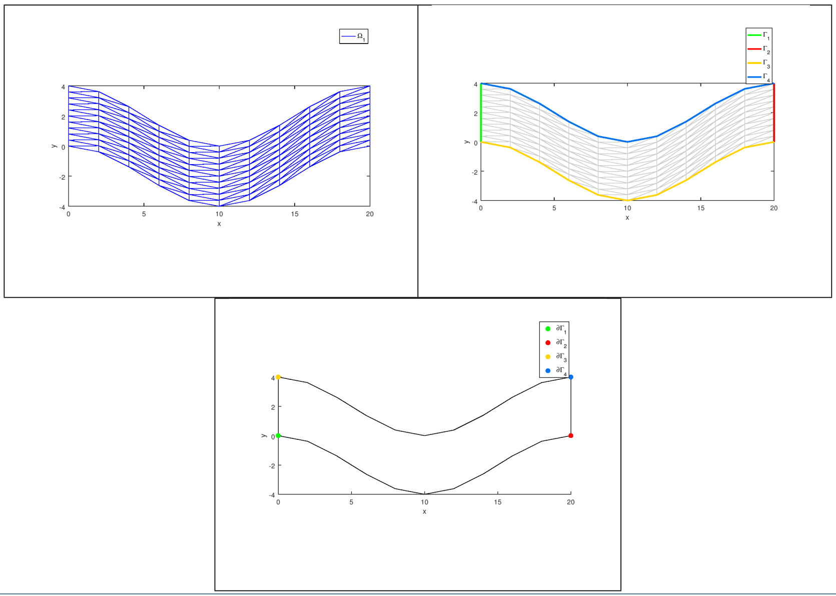
```
Oh=OrthMesh(3,[10,5,10],'box',[-1,1;0,1;0,2], 'type','orthotope')
% plot the main mesh
figure(1)
Oh.plotmesh('legend',true)
axis equal;xlabel('x');ylabel('y');zlabel('z')
% plot the 2-face meshes
figure(2)
Oh.plotmesh('m',2,'legend',true)
axis equal;axis off
% plot the 1-face meshes
figure(3)
Oh.plotmesh('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plotmesh('m',1,'Linewidth',2,'legend',true)
axis equal;axis off
% plot the 0-face meshes
figure(4)
Oh.plotmesh('m',1,'color','k')
hold on
Oh.plotmesh('m',0,'legend',true)
axis equal;axis off
```

Listing 4: 3D orthotope `OrthMesh` object with Octave 4.2.1, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

## 5.6 Mapping of a 2d-orthotope meshing by simplices

For example, the following 2D geometrical transformation allows to deform the reference unit hypercube.

$$\begin{array}{ccc}
[0,1]\times[0,1] & \longrightarrow & \mathbb{R}^2 \\
\begin{pmatrix} x \\ y \end{pmatrix} & \longrightarrow & F(x,y) = \begin{pmatrix} 20\,x \\ 2\,(2\,y-1+\cos(2\pi x)) \end{pmatrix}
\end{array}$$



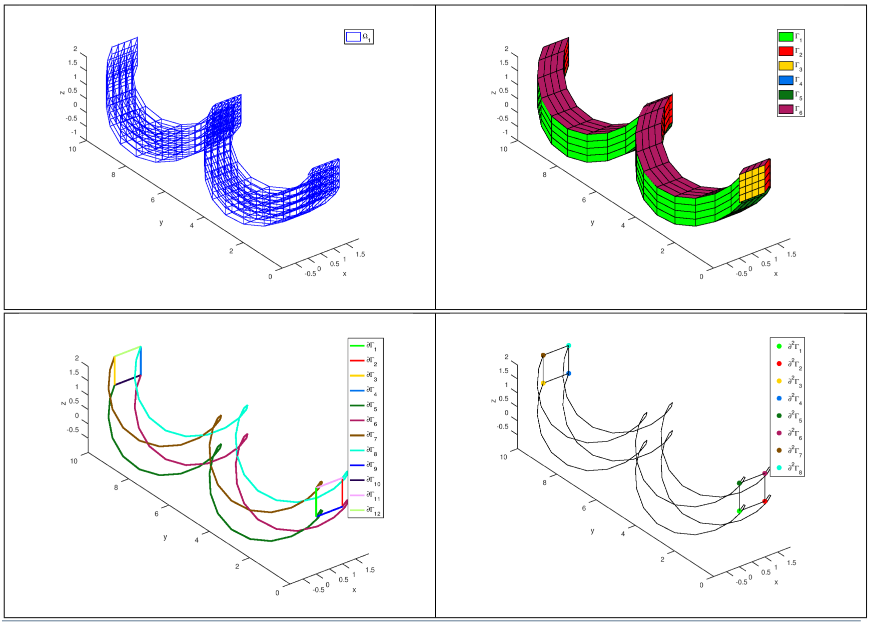
```
Oh=OrthMesh(2,10,’mapping’,@(q) [20*q(1,:); ...
    2*(2*q(2,:)-1+cos(2*pi*q(1,:)))]);
% plot the main mesh
figure(1)
Oh.plotmesh(’legend’,true)
% plot the 1-face meshes
figure(2)
Oh.plotmesh(’color’,[0.8,0.8,0.8])
hold on
Oh.plotmesh(’m’,1,’Linewidth’,2,’legend’,true)
% plot the 0-face meshes
figure(3)
Oh.plotmesh(’m’,1,’color’,’k’)
hold on
Oh.plotmesh(’m’,0,’legend’,true)
```

Listing 5: Mapping of a 2D simplicial `OrthMesh` object with Octave 4.2.1, main mesh (upper left), 1-face meshes (upper right), and 0-face meshes (bottom)

## 5.7 3d-orthotope meshing by orthotopes

For example, the following 3D geometrical transformation allows to deform the reference unit hypercube.

$$\begin{array}{ccc}[0,1]\times[0,1]\times[0,1] & \longrightarrow & \mathbb{R}^2\\ \begin{pmatrix}x\\y\\z\end{pmatrix} & \longrightarrow & F(x,y,y)=\begin{pmatrix}x+\sin(4\pi y)\\10y\\z+\cos(4\pi y)\end{pmatrix}\end{array}$$



```
Map=@(q) [q(1,:)+ sin(4*pi*q(2,:)); 10*q(2,:); q(3,:)+cos(4*pi*q(2,:))];
Oh=OrthMesh(3,[4,25,4],'mapping',Map,'type','orthotope');
% plot the main mesh
figure(1)
Oh.plotmesh()
legend('show')
% plot the 2-face meshes
figure(2)
Oh.plotmesh('m',2)
legend('show')
% plot the 1-face meshes
figure(3)
Oh.plotmesh('m',2,'color',[0.8,0.8,0.8],'EdgeAlpha',0.2, ...
    'FaceColor','none')
hold on
Oh.plotmesh('m',1,'Linewidth',2,'legend',true)
% plot the 0-face meshes
figure(4)
Oh.plotmesh('m',1,'color','k')
hold on
Oh.plotmesh('m',0,'legend',true)
```

Listing 6: Mapping of a 3D orthotope `OrthMesh` object with Octave 4.2.1, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

# 6 Benchmarking

## 6.1 `fc_bench.bench01` function

The `fc_bench.bench01` function can be used to obtain computationnal times of the `OrthMesh` constructor.

### Syntaxe

```
fc_bench.bench01(d,ctype,Box,LN)
```

### Description

```
fc_bench.bench01(d,ctype,Box,LN)
```

displays computationnal times of the `OrthMesh` constructor as follows

`ts=tic();Oh=OrthMesh(d,N,'box',Box,'type',ctype);tcpu=toc(ts);`

for each `N` in `LN`.

## 6.2 Examples

Listing 7: : Computationnal times of `OrthMesh` constructor in dimension `d=3` (simplicial mesh)

```
fc_hypermesh.bench01(3,'simplicial',[-1 1;-1 1;-1 1],25:25:175)
```

Output

```
# BENCH in dimension 3 with simplicial mesh
#d: 3
#type: simplicial
#box: [ -1 1; -1 1; -1 1]
#desc: N        nq       nme    time(s)
       25    17576     93750    0.258
       50   132651    750000    0.317
       75   438976   2531250    0.513
      100  1030301   6000000    0.839
      125  2000376  11718750    1.424
      150  3442951  20250000    2.285
      175  5451776  32156250    3.808
```

Listing 8: : Computationnal times of `OrthMesh` constructor in dimension `d=5` (orthotope mesh)

```
fc_hypermesh.bench01(5,'orthotope',[-1 1;-1 1;-1 1;-1 1;-1 ...
    1],[5:5:25,27])
```

Output

```
# BENCH in dimension 5 with orthotope mesh
#d: 5
#type: orthotope
#box: [ -1 1; -1 1; -1 1; -1 1; -1 1]
#desc: N        nq       nme    time(s)
        5     7776      3125    1.446
       10   161051    100000    1.511
       15  1048576    759375    2.017
       20  4084101   3200000    3.575
       25 11881376   9765625    9.561
       27 17210368  14348907    11.171
```

# 7 References

[1] François Cuvelier and Gilles Scarella. Vectorized algorithms for regular tessellations of d-orthotopes and their faces. *HAL archives ouvertes*, November 2017. preprint.