



Python package, User's Guide *

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Abstract

The  Python package allows to benchmark functions and much more

*Compiled with Python 3.6.5, packages `fc_bench-0.0.3rc0` and `fc_tools-0.0.16`

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1 Introduction

The  Python package aims to perform simultaneous benchmarks of several functions performing the same tasks but implemented in different ways.

We will illustrate its possibilities on an example. This one will focus on different ways of coding the Lagrange interpolation polynomial. We first recall some generalities about this polynomial.

Let $\mathbf{X}$ and $\mathbf{Y}$ be 1-by- $(n+1)$ arrays where no two $\mathbf{X}(j)$ are the same. The Lagrange interpolating polynomial is the polynomial $P(t)$ of degree $\leq n$ that passes through the $(n+1)$ points $(\mathbf{X}(j), \mathbf{Y}(j))$ and is given by

$$P(t) = \sum_{j=1}^{n+1} Y(j) \prod_{k=1, k \neq j} \frac{t - \mathbf{X}(k)}{\mathbf{X}(j) - \mathbf{X}(k)}.$$

Three different functions have been implemented to compute this polynomial. They all have the same header given by

```
def fun(X,Y,x):
```

where $\mathbf{x}$ is a 1-by- m array and the returned value $\mathbf{y}$ is a 1-by- m so that

$$\mathbf{y}(i) = P(\mathbf{x}(i)).$$

These functions are

- `fc_bench.demos.Lagrange`, a simplistic writing;
- `fc_bench.demos.lagint`, an optimized writing ;

- `fc_bench.demos.scipyLagrange`, using the `barycentric_interpolate` function of the `scipy.interpolate` module.

Their source codes are in file `demos.py` of the package. The last function needed the `scipy` package [3] to be installed.

To run benchmarks, the main tool is `fc_bench.bench` function described in section 3. To use it, you must first write a function to initialize the input datas of the Lagrange function: it is given in Listing 1 by the `setLagrange00` function. This function returns the inputs datas and a list of `bData` objects (one object by column to print). More information is provided in section 3.1. Then the `setLagrange00` function can be used as second argument of the `fc_bench.bench` function while the first one contains the three handle functions to benchmark. A complete script is given in Listing 2 with its displayed output.

```
def setLagrange00(N,verbose,**kwargs):
    from fc_bench.bench import bData
    n=N[0] # degree of the interpolating polynomial
    m=N[1] # number of interpolate values
    a=0;b=2*np.pi
    X=np.linspace(a,b,n+1)
    Y=np.cos(X)
    x=a+(b-a)*np.random.rand(m)
    bDs=[]
    bDs.append(bData('{:>8}'.format('m'),m,'{:5}')) # 1st column
    bDs.append(bData('{:>8}'.format('n'),n,'{:5}')) # 2nd column
    return ((X,Y,x),bDs)
```

Listing 1: `fc_bench.demos.setLagrange00` function

Listing 2: `fc_bench.demos.bench_Lagrange00` function

```
def bench_Lagrange00():
    import fc_bench
    import numpy as np
    Lfun=[Lagrange,lagint,scipyLagrange]
    n=[5,9,15]; m=[100,500,1000];
    N,M=np.meshgrid(n,m)
    LN=np.vstack((N.flatten(),M.flatten())).T
    fc_bench.bench(Lfun,setLagrange00,LN=LN)
```

Output

```
#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#   (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
#date:2018/05/18 13:02:48
#nbruns:5
#numpy:      i4      i4      f4      f4      f4
#format:    {:5}    {:5}    {:13.3f}  {:11.3f}  {:18.3f}
#labels:      m      n  Lagrange(s)  lagint(s)  scipyLagrange(s)
100      5      0.012      0.014      0.000
100      9      0.020      0.024      0.000
100     15      0.035      0.039      0.000
500      5      0.056      0.074      0.000
500      9      0.099      0.121      0.000
500     15      0.183      0.195      0.000
1000     5      0.111      0.148      0.000
1000     9      0.200      0.245      0.001
1000    15      0.363      0.398      0.000
```

We now propose a slightly more elaborate version of the initialization function that allows to display some informations and to choose certain parameters when generating inputs datas. This new version named `fc_bench.demos.setLagrange` is given in Listing 3. A complete script is given in Listing 4 with its displayed output. In this script some options of the `fc_bench.bench` function are used `'error'`, `'info'`, `'labelsinfo'`, jointly with those of the `fc_bench.demos.setLagrange`: `'a'`, `'b'` and `'fun'`. One must be careful not to take as an option name for the initialization function one of those used in `fc_bench.bench` function. More details are given in section 3.

```
def setLagrange(N,verbose,**kwargs):
    import fc_tools
    a=kwargs.pop('a',0)
    b=kwargs.pop('b',2*np.pi)
    sfun=kwargs.pop('fun','lambda x: np.cos(x)')
    fun=eval(sfun)
    Print=kwargs.get('Print',lambda s: print(s))
    from fc_bench.bench import bData
    n=N[0] # degree of the interpolating polynomial
    m=N[1] # number of interpolate values
    X=np.linspace(a,b,n+1)
    Y=fun(X)
    x=a+(b-a)*np.random.rand(m)
    Error=lambda y: np.linalg.norm(y-fun(x),np.inf)
    if verbose:
        Print('# Setting inputs of Lagrange polynomial functions:
              y=LAGRANGE(X,Y,x)')
        Print('# where X is numpy.linspace(a,b,n+1), Y=fun(X) and x is random
              values on [a,b]')
        Print('# n is the order of the Lagrange polynomial')
        Print('# fun function is: %s'%sfun)
        Print('# [a,b]=[%g,%g]'%(a,b))
        Print('# X: (n+1,) numpy array')
        Print('# Y: (n+1,) numpy array')
        Print('# x: (m,) numpy array')
        Print('# Error[i] computed with fun[i] output:')
        Print('# %s'%fc_tools.others.func2str(Error))

    bDs=[]
    bDs.append(bData('{:>5}'.format('m'),m,'{:>5}'))
    bDs.append(bData('{:>5}'.format('n'),n,'{:>5}'))
    return ((X,Y,x,Error),bDs)
```

Listing 3: `fc_bench.demos.setLagrange` function

Listing 4: : `fc_bench.demos.bench_Lagrange` function

```
def bench_Lagrange():
    import fc_bench
    import numpy as np
    Lfun=[Lagrange, lagint]
    names=['Lag', 'lagint']
    error=lambda o1,o2: np.linalg.norm(o1-o2, np.inf)
    n=[5,9,15]; m=[100,500,1000];
    N,M=np.meshgrid(n,m)
    LN=np.vstack((N.flatten(),M.flatten())).T
    fc_bench.bench(Lfun, setLagrange, LN=LN, error=error, names=names,
        info=False, labelsinfo=True, a=-1, b=1, fun='lambda x: np.sin(x)')
```

Output

```
#-----
# Benchmarking functions:
# fun[0],          Lag: fc_bench.demos.Lagrange
# fun[1],          lagint: fc_bench.demos.lagint
# cmpErr[i], error between fun[0] and fun[i] outputs computed with function
# lambda o1,o2: np.linalg.norm(o1-o2, np.inf)
# where
#   - 1st input parameter is the output of fun[0]
#   - 2nd input parameter is the output of fun[i]
#-----
# Setting inputs of Lagrange polynomial functions: y=LAGRANGE(X,Y,x)
# where X is numpy.linspace(a,b,n+1), Y=fun(X) and x is random values on [a,b]
# n is the order of the Lagrange polynomial
# fun function is: lambda x: np.sin(x)
# [a,b]=[-1,1]
# X: (n+1,) numpy array
# Y: (n+1,) numpy array
# x: (m,) numpy array
# Error[i] computed with fun[i] output:
# lambda y: np.linalg.norm(y-fun(x), np.inf)
#-----
#date:2018/05/18 13:03:05
#nbruns:5
#numpy:      i4      i4      f4      f4      f4      f4      f4
#format: {:>5} {:>5} {:8.3f} {:10.3e} {:11.3f} {:10.3e} {:11.3e}
#labels:      m      n      Lag(s)      Error[0]      lagint(s)      Error[1]      cmpErr[1]
100          5      0.012      1.162e-05      0.015      1.162e-05      3.331e-16
100          9      0.021      2.890e-10      0.024      2.890e-10      1.554e-15
100          15     0.037      1.565e-14      0.040      1.565e-14      1.499e-14
500          5      0.058      1.163e-05      0.074      1.163e-05      5.551e-16
500          9      0.103      2.902e-10      0.123      2.902e-10      1.443e-15
500          15     0.185      3.142e-14      0.201      3.142e-14      3.963e-14
1000         5      0.116      1.163e-05      0.149      1.163e-05      5.551e-16
1000         9      0.207      2.901e-10      0.253      2.901e-10      1.776e-15
1000         15     0.367      2.243e-14      0.396      2.243e-14      2.842e-14
```

2 Installation

The  Python package was tested on various OS and Python versions:

- **CentOS 7**^[7]
 - with Python 2.7.14 compiled from source ^[8]
 - with Python 3.5.5 compiled from source ^[8]
 - with Python 3.6.4 compiled from source ^[8]
- **Fedora 27**^[6]
 - with Python 2.7.14 compiled from source ^[8]
 - with Python 3.5.5 compiled from source ^[8]
 - with Python 3.6.4 compiled from source ^[8]

- **MacOS High Sierra**[1]
 - with Miniconda Python 2.7 distribution (release 2.7.14) [9]
 - with Miniconda Python 3.6 distribution (release 3.6.4) [9]
 - with Python 2.7.14 [8]
 - with Python 3.5.4 [8]
 - with Python 3.6.5 [8]
- **openSUSE Leap 42.3** 7[4]
 - with Python 2.7.14 compiled from source [8]
 - with Python 3.5.5 compiled from source [8]
 - with Python 3.6.4 compiled from source [8]
- **Ubuntu 18.04 LTS, 17.10, 16.04 LTS** [10]
 - with Python 2.7.x (x=13,14,15) compiled from source [8]
 - with Python 3.5.5 compiled from source [8]
 - with Python 3.6.x (x=3,4,5) compiled from source [8]
- **Windows 10** [5]
 - with Miniconda Python 2.7 distribution (release 2.7.14) [9]
 - with Miniconda Python 3.6 distribution (release 3.6.4) [9]
 - with Python 2.7.15 [8]
 - with Python 3.5.4 [8]
 - with Python 3.6.5 [8]

Installation : The  Python package can be installed by using **pip** (or **pip3**) command [2].

- For an installation which isolated to the current user, one can do:

```
$ pip install -U --user fc_bench
```

- For an installation for all users, one can do:

```
$ sudo pip install -U fc_bench
```

An other way is to download the required archive and to make the installation from the downloaded file.

- For an installation which isolated to the current user, one can do:

```
$ pip install <PATH_TO_FOLDER>/fc_bench-<VERSION>.tar.gz --user -U
```

where **<PATH_TO_FOLDER>** will be replaced by the path to the saved archive and **<VERSION>** by the version of the archive.

- For an installation for all users, one can do:

```
$ sudo pip install <PATH_TO_FOLDER>/fc_bench-<VERSION>.tar.gz -U
```

Uninstall : To uninstall this package, you only have to execute one of these commands depending on the type of installation performed

```
$ pip uninstall fc_bench
```

or

```
$ sudo pip uninstall fc_bench
```

3 `fc_bench.bench` function

The `fc_bench.bench` function run benchmark

Syntaxe

```
fc_bench.bench(Lfun, setfun)
fc_bench.bench(Lfun, setfun, key=value, ...)
R=fc_bench.bench(Lfun, setfun)
R=fc_bench.bench(Lfun, setfun, key=value, ...)
```

Description

```
fc_bench.bench(Lfun, setfun)
```

Runs benchmark for each function given in the list `Lfun`. The function `setfun` is used to set input datas to these functions. There is the imposed syntax:

```
def setfun(N, verbose, **kwargs):
    ...
    return ((In1, In2, ...), bDs)
```

By default, for all `N` in `[5,10,15]`, computational time in second of each function in `Lfun` is evaluated by `time.time()` function:

```
t=time.time(); out=Lfun[i]( *Inputs ); tcpu=time.time()-t
```

where `Inputs` is given by

```
[Inputs, Bdatas]=setfun(N, verbose, **kwargs)
```

```
fc_bench.bench(Lfun, setfun, key=value, ...)
```

Some optional key are available:

- `LN`, to set values of the first input of the `setfun` function of the `n` benchmark to be run. For `i`-th benchmark, the `setfun` function is used with the `i`-th value given by `value[i]`. `value` can be a list of length `n` or a numpy array with shape `(n,)` or `(n,m)`. By default, `value` is `[5,10,15]`.

- `names` , set the names that will be displayed during the benchmarks to name each of the functions of `Lfun` . By default `value` is `None` and all the names are guessed from the functions of `Lfun` . Otherwise, `value` is a list with same length as `Lfun` such that `value[i]` is the string name associated with `Lfun[i]` function. If `value[i]` is the empty string, then the name is guessed from the function `Lfun[i]` .
- `nbruns` , to set number of benchmark runs for each case and the mean of computational times is taken. Default `value` is `5`. In fact, `value+2` benchmarks are executed and the two worst are forgotten (see `fc_bench.mean_run` function)
- `comment` , string or list of strings displayed before running the benchmarks. If `value` is a list of strings, after printing the `value[i]`, a line break is performed.
- `info` , if `value` is `True`(default), some informations on the computer and the system are displayed.
- `labelsinfo` , if `value` is `True`, some informations on the labels of the columns are automatically displayed. Default is `False`.
- `savefile` , if `value` is a not empty string, then displayed results are saved in directory `benchs` with `value` as filename. One can used `savedir` option to change the directory.
- `savedir` , if `value` is a not empty string, then when using `savefile` , the file is saved in directory `value`.
- `error` , to use when compative errors between the various functions are desired when displaying. In this case a function must be given which returns error (as scalar) between the output of the first function `Lfun[0]` and one of the others.

3.1 bData object

The `bData` object is used to described and print the columns of the `bench` output. Its contructor is:

Syntaxe

```
bData(name,value,sformat)
```

where `name` is the label of the column which appears in line `#labels:` ... of the `bench` output, `value` is the corresponding value to print and `sformat` is the format used for formatting the column by using the `str.format()` method. The string print is `sformat.format(value)`. In the `setfun` function one only has to choose datas to be print as first columns: columns associated with each of the functions to be tested are automatically added next.

3.2 Matricial product examples

Let X be a (m,n) numpy array and Y be a (n,p) numpy array. We want to measure efficiency of the matricial product by using functions that we have

written or `numpy.matmul(X,Y)` (function version) or `X@Y` (operator version for Python ≥ 3.5) with various values of `m`, `n` and `p`. When using the  `cibench` package with Python version < 3.5 , efficiency is measured only for the function `numpy.matmul` and the ones we wrote.

3.2.1 Square matrices: `fc_bench.demos.bench_MatProd00` function

Let $m = n = p$. As a first step, we want to measure the performance of the function `numpy.matmul(X,Y)`.

```
def setMatProd00(m, verbose, **kwargs):
    from fc_bench.bench import bData
    X=np.random.randn(m,m)
    Y=np.random.randn(m,m)
    bDs=[bData('f:>8').format('m'),m,'f:>8')]
    return ((X,Y),bDs)
```

Listing 5: `fc_bench.demos.setMatProd00` function

The `fc_bench.demos.setMatProd00` function given in Listing 5 is used in `fc_bench.demos.bench_MatProd00` function (file `demos.py` of the package directory)

Listing 6: : `fc_bench.demos.bench_MatProd00` function

```
def bench_MatProd00():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul]
    LN=np.arange(500,4001,500)
    fc_bench.bench(Lfun,setMatProd00,LN=LN)
```

Output

```
#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#             (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
#
#date:2018/05/18 13:03:23
#nbruns:5
#numpy:           i4           f4
#format:   {:>8}   {:11.3f}
#labels:      m      matmul(s)
#           500      0.002
#           1000     0.005
#           1500     0.016
#           2000     0.036
#           2500     0.073
#           3000     0.122
#           3500     0.186
#           4000     0.268
```

3.2.2 Square matrices: `fc bench.demos.bench MatProd01` function

Let $m = n = p$.

Some improvements are made to the `fc_bench.demos.setMatProd00` function (Listing 5) by using `verbose` flag to display some information: this gives the `fc_bench.demos.setMatProd01` function presented in Listing 7. From the previous example, we also add the use of the options `comment` and `savefile` to the

function `fc_bench.bench`: this gives the function `fc_bench.demos.bench_MatProd01` presented in Listing 8 that uses the `fc_bench.demos.setMatProd01` function.

```
def setMatProd01(m, verbose, **kwargs):
    from fc_bench.bench import bData
    X=np.random.randn(m,m)
    Y=np.random.randn(m,m)
    if verbose:
        print('# 1st input parameter: (m,m) Numpy array')
        print('# 2nd input parameter: (m,m) Numpy array')
    bDs=[bData('{:>8}'.format('m'),m,'{:>8}')]
    return ((X,Y),bDs)
```

Listing 7: `fc_bench.demos.setMatProd01` function

Listing 8: `fc_bench.demos.bench_MatProd01` function

```
def bench_MatProd01():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul]
    comment=['# Benchmarking function numpy.matmul',
            '# where X and Y are (m,m) Numpy arrays']
    LN=np.arange(500,4001,500)
    fc_bench.bench(Lfun,setMatProd01,LN=LN, comment=comment,
        savefile='MadProd01.out')
```

Output

```
#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#             (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are (m,m) Numpy arrays
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#benchfile: benches/MadProd01.out
#date:2018/05/18 13:03:33
#nbruns:5
#numpy:      i4      f4
#format:    {:>8}    {:11.3f}
#labels:    m      matmul(s)
#           500     0.002
#           1000    0.005
#           1500    0.016
#           2000    0.036
#           2500    0.074
#           3000    0.123
#           3500    0.188
#           4000    0.268
```

```

#-----
# computer: cosmos
# system: Ubuntu 17.10 (x86_64)
# processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
# (1 procs/14 cores by proc/2 threads by core)
# RAM: 62.6 Go
# software: Python
# release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are (m,m) Numpy arrays
#-----
#benchfile: benches/MatProd01.out
#date:2018/05/18 13:03:33
#nbruns:5
#numpy:      i4      f4
#format:  {:>8}  {:11.3f}
#labels:      m      matmul(s)
      500      0.002
      1000     0.005
      1500     0.016
      2000     0.036
      2500     0.074
      3000     0.123
      3500     0.188
      4000     0.268

```

Listing 9: Output file benches/MatProd01.out

As we can see information print from the `fc_bench.demos.setMatProd01` function are missing in output file `benches/MatProd01.out`. We will see how to print them also in the output file.

3.2.3 Square matrices: `fc_bench.demos.bench_MatProd02` function

Let $m = n = p$.

To have information print from the `fc_bench.demos.setMatProd01` also saved in the output file we just add the following line:

```
Print=kwargs.get('Print',lambda s: print(s))
```

This gives the `fc_bench.demos.setMatProd02` function presented in Listing 10. This function is then used in `fc_bench.demos.bench_MatProd02` given in Listing 11.

```

def setMatProd02(m,verbose,**kwargs):
    Print=kwargs.get('Print',lambda s: print(s))
    from fc_bench.bench import bData
    X=np.random.randn(m,m)
    Y=np.random.randn(m,m)
    if verbose:
        Print('# 1st input parameter: (m,m) Numpy array')
        Print('# 2nd input parameter: (m,m) Numpy array')
    bDs=[bData('{:>8}'.format('m'), m, '{:>8}')]
    return ((X,Y),bDs)

```

Listing 10: `fc_bench.demos.setMatProd02` function

Listing 11: : `fc_bench.demos.bench_MatProd02` function

```
def bench_MatProd02():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul]
    comment=['# Benchmarking function numpy.matmul',
            '# where X and Y are (m,m) Numpy arrays']
    LN=np.arange(500,4001,500)
    fc_bench.bench(Lfun,setMatProd02,LN=LN, comment=comment,
        savefile='MadProd02')
```

Output

```
#-----
# computer: cosmos
# system: Ubuntu 17.10 (x86_64)
# processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
# (1 procs/14 cores by proc/2 threads by core)
# RAM: 62.6 Go
# software: Python
# release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are (m,m) Numpy arrays
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#benchfile: benches/MadProd02.out
#date:2018/05/18 13:03:42
#nbruns:5
#numpy:      i4      f4
#format:    {:>8}    {:<11.3f}
#labels:      m      matmul(s)
#
#      500      0.001
#     1000      0.005
#     1500      0.016
#     2000      0.036
#     2500      0.074
#     3000      0.124
#     3500      0.187
#     4000      0.268
```

```
#-----
# computer: cosmos
# system: Ubuntu 17.10 (x86_64)
# processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
# (1 procs/14 cores by proc/2 threads by core)
# RAM: 62.6 Go
# software: Python
# release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are (m,m) Numpy arrays
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#benchfile: benches/MadProd02.out
#date:2018/05/18 13:03:42
#nbruns:5
#numpy:      i4      f4
#format:    {:>8}    {:<11.3f}
#labels:      m      matmul(s)
#
#      500      0.001
#     1000      0.005
#     1500      0.016
#     2000      0.036
#     2500      0.074
#     3000      0.124
#     3500      0.187
#     4000      0.268
```

Listing 12: Output file `benches/MadProd02.out`

3.2.4 Square matrices: `fc_bench.demos.bench_MatProd03` and `04` functions

Let $m = n = p$.

We want to compare computationnal times between the `numpy.matmul(X,Y)` function, the `X@Y` command (`@` operator is defined since version 3.5 of Python) and the `fc_bench.demos.matprod01` function given in Listing 13.

```
def matprod01(A,B):
    import numpy as np
    (n,m)=A.shape
    (p,q)=B.shape
    assert m==p, 'shapes %s and %s not aligned: %d (dim 1) != %d (dim
        0)'%(str(A.shape),str(B.shape),A.shape[1],B.shape[0])
    C=np.zeros((n,q))
    for i in np.arange(n):
        for j in np.arange(q):
            for k in np.arange(m):
                C[i,j]+=A[i,k]*B[k,j]
    return C
```

Listing 13: `fc_bench.demos.matprod01` function

The `fc_bench.demos.setMatProd02` function given in Listing 10 is used in `fc_bench.demos.bench_MatProd03` function (file `demos_op.py`).

Listing 14: : `fc_bench.demos.bench_MatProd03` function

```
def bench_MatProd03():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul,lambda X,Y: X@Y,fc_bench.demos.matprod01]
    comment=['# Benchmarking function numpy.matmul',
        '# where X and Y are m-by-m Numpy arrays']
    LN=np.arange(50,201,50)
    fc_bench.bench(Lfun,fc_bench.demos.setMatProd02,LN=LN,
        comment=comment)
```

Output

```
#-----
# computer: cosmos
# system: Ubuntu 17.10 (x86_64)
# processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
# (1 procs/14 cores by proc/2 threads by core)
# RAM: 62.6 Go
# software: Python
# release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are m-by-m Numpy arrays
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#date:2018/05/18 13:03:52
#nbruns:5
#numpy:      i4      f4      f4      f4
#format:  {:>8}  {:11.3f}  {:13.3f}  {:14.3f}
#labels:      m  matmul(s)  <lambda>(s)  matprod01(s)
#-----
# 50      0.000      0.000      0.061
# 100     0.000      0.000      0.479
# 150     0.000      0.000      1.601
# 200     0.000      0.000      3.764
```

As the second function in `Lfun` has no name, the guess name is `<lambda>`. One can set a more convenient name by using the `names` option: this is the object of Listing 15. When empty value is set in `names` list then a guessed name is used.

Listing 15: : `fc_bench.demos.bench_MatProd04` function

```
def bench_MatProd04():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul, lambda X,Y: X@Y, fc_bench.demos.matprod01]
    names=['matmul(X,Y)', 'X@Y', '']
    comment=['# Benchmarking function numpy.matmul',
            '# where X and Y are m-by-m Numpy arrays']
    LN=np.arange(50,201,50)
    fc_bench.bench(Lfun, fc_bench.demos.setMatProd02, LN=LN,
                  comment=comment, names=names)
```

Output

```
#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#             (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
# Benchmarking function numpy.matmul
# where X and Y are m-by-m Numpy arrays
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#date:2018/05/18 13:04:34
#nbruns:5
#numpy:      i4          f4      f4          f4
#format:  {:>8}      {:16.3f}  {:8.3f}    {:14.3f}
#labels:    m    matmul(X,Y) (s)  X@Y (s)    matprod01 (s)
#           50      0.000      0.000      0.059
#           100     0.000      0.000      0.469
#           150     0.000      0.000      1.583
#           200     0.000      0.000      3.751
```

With Python version < 3.5, the `lambda X,Y: X@Y` function must be omitted in `Lfun`.

3.2.5 Square matrices: `fc_bench.demos.bench_MatProd05` function

As previous section, we want to compare computationnal times between the `numpy.matmul(X,Y)` function, the `X@Y` command and the `fc_bench.demos.matprod01` function given in Listing 13. In addition, we also want to display errors between the outputs of the functions. The first function given in `Lfun` is the reference one and errors are always computed by using output of this reference function and output of the other functions.

Two examples are proposed that use the `fc_bench.bench` function with `error` option to display comparative errors. They both use the `fc_bench.demos.setMatProd02` function given in Listing 10. The first one given in Listing 16 uses the `comment` option and a manual writing to print some informations about labels columns. The second one given in Listing 17 uses the `labelsinfo` option to automatically print some informations about labels columns.

Listing 16: : fc_bench.demos.bench_MatProd05 function

```

def bench_MatProd05():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul,lambda X,Y: X@Y ,fc_bench.demos.matprod02]
    error=lambda o1,o2: np.linalg.norm(o1-o2,np.inf)
    names=['matmul(X,Y)', 'X@Y', '']
    comment=['# Benchmarking functions:',
            '#      A1=numpy.matmul(X,Y) (reference)',
            '#      A2= X@Y',
            '#      A3= fc_bench.demos.matprod02(X,Y)',
            '# where X and Y are m-by-m Numpy arrays',
            '# cmpErr[1] is the norm(A1-A2,Inf)',
            '# cmpErr[2] is the norm(A1-A3,Inf)']
    LN=np.arange(100,401,100)
    fc_bench.bench(Lfun,fc_bench.demos.setMatProd02,LN=LN,
                  comment=comment, names=names, info=False, error=error)

```

Output

```

#-----
# Benchmarking functions:
#   A1=numpy.matmul(X,Y) (reference)
#   A2= X@Y
#   A3= fc_bench.demos.matprod02(X,Y)
# where X and Y are m-by-m Numpy arrays
# cmpErr[1] is the norm(A1-A2,Inf)
# cmpErr[2] is the norm(A1-A3,Inf)
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#date:2018/05/18 13:05:15
#nbruns:5
#numpy:      i4          f4          f4          f4          f4          f4
#format:  {:>8}      {:16.3f}  {:8.3f}    {:11.3e}    {:14.3f}    {:11.3e}
#labels:      m    matmul(X,Y)(s)  X@Y(s)  cmpErr[1]  matprod02(s)  cmpErr[2]
#-----
100          0.000    0.000    0.000e+00    0.007    3.692e-13
200          0.000    0.000    0.000e+00    0.033    1.337e-12
300          0.001    0.001    0.000e+00    0.078    2.310e-12
400          0.002    0.001    0.000e+00    0.159    4.023e-12

```

Listing 17: : `fc_bench.demos.bench_MatProd05bis` function

```
def bench_MatProd05bis():
    import fc_bench
    import numpy as np
    # with labelsinfo, <lambda> function must be alone on a code line
    # (otherwise not well guessed)
    f=lambda X,Y: X@Y
    Lfun=[np.matmul,f,fc_bench.demos.matprod02]
    error=lambda o1,o2: np.linalg.norm(o1-o2,np.inf)
    names=['matmul(X,Y)', 'X@Y', '']
    LN=np.arange(100,401,100)
    fc_bench.bench(Lfun,fc_bench.demos.setMatProd02,LN=LN, names=names,
        info=False,labelsinfo=True, error=error)
```

Output

```
#-----
# Benchmarking functions:
# fun[0], matmul(X,Y): numpy.core.multiarray.matmul
# fun[1], X@Y: lambda X,Y: X@Y
# fun[2], matprod02: fc_bench.demos.matprod02
# cmpErr[i], error between fun[0] and fun[i] outputs computed with function
# lambda o1,o2: np.linalg.norm(o1-o2,np.inf)
# where
# - 1st input parameter is the output of fun[0]
# - 2nd input parameter is the output of fun[i]
#-----
# 1st input parameter: (m,m) Numpy array
# 2nd input parameter: (m,m) Numpy array
#-----
#date:2018/05/18 13:05:18
#nbruns:5
#numpy: i4 f4 f4 f4 f4 f4
#format: {:>8} {:>16.3f} {:>8.3f} {:>11.3e} {:>14.3f} {:>11.3e}
#labels: m matmul(X,Y)(s) X@Y(s) cmpErr[1] matprod02(s) cmpErr[2]
```

100	0.000	0.000	0.000e+00	0.008	3.573e-13
200	0.000	0.000	0.000e+00	0.033	1.458e-12
300	0.001	0.001	0.000e+00	0.078	2.355e-12
400	0.002	0.001	0.000e+00	0.158	4.086e-12

3.2.6 Non-square matrices: `fc_bench.demos.bench_MatProd06` function

As previous section, we want to compare computationnal times between the `numpy.matmul(X,Y)` function, the `X@Y` command and the `fc_bench.demos.matprod01` function given in Listing 13 but this time with non-square matrices. In addition, we also want to display errors between the outputs of the functions. The first function is the reference one and errors are always computed by using output of this reference function and output of the functions.

```

def setMatProd03(N, verbose, **kwargs):
    assert len(N)==3 or len(N)==1
    Print=kwargs.get('Print', lambda s: print(s))
    ldtype=kwargs.get('ldtype', )
    rdtype=kwargs.get('rdtype', )
    lcomplex=kwargs.get('lcomplex', False)
    rcomplex=kwargs.get('rcomplex', False)

    from fc_bench.bench import bData
    if len(N)==1:
        m=n=p=N
    else:
        [m,n,p]=N

    X=genMat(m,n,ldtype,lcomplex)
    Y=genMat(n,p,rdtype,rcomplex)

    if verbose:
        Print('# 1st input parameter: (m,n) Numpy array [%s]'%str(X.dtype))
        Print('# 2nd input parameter: (n,p) Numpy array [%s]'%str(Y.dtype))
    bDs=[bData('{:>7}'.format('m'), m, '{:>7}')]
    bDs.append(bData('{:>7}'.format('n'), n, '{:>7}'))
    bDs.append(bData('{:>7}'.format('p'), p, '{:>7}'))
    return ((X,Y),bDs)

```

Listing 18: fc_bench.demos.setMatProd03 function

```

def genMat(m,n,dtype,isComplex):
    M=0;
    if isComplex:
        M=1j*np.random.randn(m,n)
    M+=np.random.randn(m,n)
    return M

```

Listing 19: fc_bench.demos.genMat function

The `fc_bench.demos.setMatProd03` function given in Listing 18 is used in `fc_bench.demos.bench_MatProd06` function.

Listing 20: : `fc_bench.demos.bench_MatProd06` function

```

def bench_MatProd06():
    import fc_bench
    import numpy as np
    Lfun=[np.matmul,lambda X,Y: X@Y,fc_bench.demos.matprod02]
    error=lambda o1,o2: np.linalg.norm(o1-o2,np.inf)
    names=['matmul(X,Y)', 'X@Y', '']
    comment=['# Benchmarking functions:',
            '#      A1=numpy.matmul(X,Y) (reference)',
            '#      A2= X@Y',
            '#      A3= fc_bench.demos.matprod02(X,Y)',
            '# where X and Y are respectively (m,n) and (n,p) Numpy',
            '# arrays',
            '# cmpErr[1] is the norm(A1-A2,Inf)',
            '# cmpErr[2] is the norm(A1-A3,Inf)']
    LN=[ [100,50,100], [150,50,100], [200,50,100], [150,100,300]]
    fc_bench.bench(Lfun,fc_bench.demos.setMatProd03,LN=LN, lcomplex=True,
                  rtype=np.dtype('f4'),
                  comment=comment, names=names, info=False, error=error)

```

Output

```

#-----
# Benchmarking functions:
#      A1=numpy.matmul(X,Y) (reference)
#      A2= X@Y
#      A3= fc_bench.demos.matprod02(X,Y)
# where X and Y are respectively (m,n) and (n,p) Numpy arrays
# cmpErr[1] is the norm(A1-A2,Inf)
# cmpErr[2] is the norm(A1-A3,Inf)
#-----
# 1st input parameter: (m,n) Numpy array [complex128]
# 2nd input parameter: (n,p) Numpy array [float64]
#-----
#date:2018/05/18 13:05:21
#nbruns:5
#numpy:      i4      i4      i4      f4      f4      f4      f4      f4
#format:  {:>7}  {:>7}  {:>7}  {:16.3f}  {:8.3f}  {:11.3e}  {:14.3f}  {:11.3e}
#labels:      m      n      p  matmul(X,Y)(s)  X@Y(s)  cmpErr[1]  matprod02(s)  cmpErr[2]
      100      50      100      0.000      0.000      0.000e+00      0.012      2.594e-13
      150      50      100      0.000      0.000      0.000e+00      0.017      2.835e-13
      200      50      100      0.000      0.000      0.000e+00      0.023      2.597e-13
      150      100     300      0.001      0.001      0.000e+00      0.056      1.586e-12

```

3.3 LU factorization examples

Let A be (m,m) Numpy array. The function `fc_bench.demos.permLU` computes the permuted LU factorization of A and returns the three (m,m) Numpy array L , U and P which are respectively a lower triangular matrix with unit diagonal, an upper triangular matrix and a permutation matrix so that

$$P@A == L@U$$

Its header is given in Listing 21.

```

def permLU(A):
    (m,n)=A.shape
    assert m==n
    U=A.copy()
    p=np.arange(n)
    L=np.eye(m)
    for k in np.arange(m-1):
        mu=np.argmax(abs(U[k:,k]))+k
        if abs(U[mu,k])> 1e-15:
            if mu!=k:
                U[[k,mu],k:]=U[[mu,k],k:]
                L[[k,mu],:k]=L[[mu,k],:k]
                p[[k,mu]]=p[[mu,k]]
            for j in np.arange(k+1,m):
                L[j,k]=U[j,k]/U[k,k]
                U[j,k:]+= -L[j,k]*U[k,k:]
    return L,U,permInd2Mat(p)

```

Listing 21: `fc_bench.demos.permLU` function

3.3.1 `fc_bench.demos.bench_LU00`

We present a very simple benchmark of the `fc_bench.demos.permLU` function. The `fc_bench.demos.setLU00` function given in Listing 22 is used in the `fc_bench.demos.bench_LU00` function (file `demos.py`). The source code and the printed output are given in Listing 23.

```

def setLU00(m,verbose,**kwargs):
    Print=kwargs.get('Print',lambda s: print(s))
    from fc_bench.bench import bData
    A=np.random.randn(m,m)
    if verbose:
        Print('# input parameter: (m,m) Numpy array')

    bDs=[bData('{:>8}'.format('m'),m,'{:>8}')]
    return ((A),bDs)

```

Listing 22: `fc_bench.demos.setLU00` function

Listing 23 : `fc_bench.demos.bench_LU00` function

```
def bench_LU00():
    import fc_bench
    import numpy as np
    Lfun=[permLU]
    comment=['# Benchmarking function fc_bench.demos.permLU function (LU
            factorization)']
    LN=np.arange(100,401,100)
    fc_bench.bench(Lfun,setLU00,LN=LN, comment=comment)
```

Output

```
#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#   (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
# Benchmarking function fc_bench.demos.permLU function (LU factorization)
#-----
# input parameter: (m,m) Numpy array
#-----
#date:2018/05/18 13:05:22
#nbruns:5
#numpy:      i4      f4
#format:  {:>8}    {:11.3f}
#labels:    m    permLU(s)
#           100    0.014
#           200    0.056
#           300    0.129
#           400    0.233
```

3.3.2 `fc_bench.demos.bench_LU01`

We return to the previous benchmark example to which we want to add for each `m` value the error committed:

```
np.linalg.norm(np.matmul(L,U)-np.matmul(P,A),np.inf).
```

The syntax of the `fc_bench.demos.permLU` function is

```
L,U,P=fc_bench.demos.permLU(A).
```

So we can defined, for each input matrix `A`, an `Error` function which only depends on the outputs (with same order)

```
Error=lambda L,U,P: np.linalg.norm(np.matmul(L,U)-np.matmul(P,A),np.inf)
```

This command is written in the initialization function (after initialization of returned input datas) and the `Error` function is appended at the end of the `Inputs` tuple. The initialization function named `fc_bench.demos.setLU01` is provided in Listing 24.

```

def setLU01(m,verbose,**kwargs):
    import fc_tools
    Print=kwargs.get('Print',lambda s: print(s))
    from fc_bench.bench import bData
    A=np.random.randn(m,m)
    Error=lambda L,U,P: np.linalg.norm(np.matmul(L,U)-np.matmul(P,A),np.inf);
    if verbose:
        Print('# input parameter: (m,m) Numpy array')
        Print('# Outputs are [L,U,P] such that P*A=L*U')
        Print('# Error[i] computed with fun[i] outputs :\n#
              %s'%fc_tools.others.func2str(Error,source=False))
    bDs=[bData('{:>8}'.format('m'),m,'{:>8}')]
    return ((A,Error),bDs)

```

Listing 24: `fc_bench.demos.setLU01` function

Listing 25: `fc_bench.demos.bench_LU01` function

```

def bench_LU01():
    import fc_bench
    import numpy as np
    Lfun=[permLU]
    comment=['# Benchmarking function fc_bench.demos.permLU function (LU
            factorization)']
    LN=np.arange(100,401,100)
    fc_bench.bench(Lfun,setLU01,LN=LN, comment=comment)

```

Output

```

#-----
#   computer: cosmos
#   system: Ubuntu 17.10 (x86_64)
#   processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
#             (1 procs/14 cores by proc/2 threads by core)
#   RAM: 62.6 Go
#   software: Python
#   release: 3.6.5
#-----
# Benchmarking function fc_bench.demos.permLU function (LU factorization)
#-----
# input parameter: (m,m) Numpy array
# Outputs are [L,U,P] such that P*A=L*U
# Error[i] computed with fun[i] outputs :
#   np.linalg.norm(np.matmul(L,U)-np.matmul(P,A),np.inf);
#-----
#date:2018/05/18 13:05:26
#nbruns:5
#numpy:      i4          f4          f4
#format:  {:>8}    {:11.3f}  {:10.3e}
#labels:    m   permLU(s)  Error[0]
#-----
#   100      0.014  9.031e-14
#   200      0.057  3.645e-13
#   300      0.131  7.629e-13
#   400      0.234  1.482e-12

```

3.3.3 `fc_bench.demos.bench_LU02`

We now want to add to previous example the computationnal times of the `lu` Scipy function provides by `scipy.linalg` module. This function accepts various number of inputs and outputs but the command

$$P,L,U=lu(A)$$

returns the three (m,m) Numpy arrays where

$$A == P @ L @ U$$

where P is the transpose of that obtained by the function `fc_bench.demos.permLU`. So we must write a wrapper function to fit with the `Error` function and order of

parameters: this is done in `fc_bench.demos.scipyLU` function given in Listing 26. Thereafter these two functions are used in `fc_bench.demos.bench_LU02` function given in Listing 27 with its output.

```
def scipyLU(A):
    from scipy.linalg import lu
    P,L,U=lu(A)
    return L,U,P.T
```

Listing 26: `fc_bench.demos.scipyLU` function

Listing 27: `fc_bench.demos.bench_LU02` function

```
def bench_LU02():
    import fc_bench
    import numpy as np
    Lfun=[scipyLU,permLU]
    comment=['# Benchmarking functions (LU factorization):',
            '# fun[1]: fc_bench.demos.scipyLU',
            '# fun[1]: fc_bench.demos.permLU']
    cmpErr=lambda o1,o2: np.linalg.norm(o1[0]-o2[0],np.inf)+\
                        np.linalg.norm(o1[1]-o2[1],np.inf)+\
                        np.linalg.norm(o1[2]-o2[2],np.inf)
    LN=np.arange(100,401,100)
    fc_bench.bench(Lfun,setLU01,LN=LN, comment=comment,error=cmpErr)
```

Output

```
#-----
# computer: cosmos
# system: Ubuntu 17.10 (x86_64)
# processor: Intel(R) Core(TM) i9-7940X CPU @ 3.10GHz
# (1 procs/14 cores by proc/2 threads by core)
# RAM: 62.6 Go
# software: Python
# release: 3.6.5
#-----
# Benchmarking functions (LU factorization):
# fun[1]: fc_bench.demos.scipyLU
# fun[1]: fc_bench.demos.permLU
#-----
# input parameter: (m,m) Numpy array
# Outputs are [L,U,P] such that P*A=L*U
# Error[i] computed with fun[i] outputs :
# np.linalg.norm(np.matmul(L,U)-np.matmul(P,A),np.inf);
#-----
#date:2018/05/18 13:05:30
#nbruns:5
#numpy:      i4      f4      f4      f4      f4      f4
#format:  {:>8}  {:12.3f}  {:10.3e}  {:11.3f}  {:10.3e}  {:11.3e}
#labels:    m  scipyLU(s)  Error[0]  permLU(s)  Error[1]  cmpErr[1]
#         100      0.000  5.889e-14  0.021  5.889e-14  6.372e-13
#         200      0.001  2.838e-13  0.060  2.838e-13  6.590e-12
#         300      0.002  7.127e-13  0.138  7.127e-13  1.693e-11
#         400      0.004  1.388e-12  0.250  1.388e-12  4.196e-11
```

3 References

- [1] Apple. **macOS** high sierra. <https://www.apple.com/macOS/high-sierra/>.
- [2] Python Software Foundation. Pypi, the python package index. <https://pypi.python.org/>, 2003–.
- [3] Eric Jones, Travis Oliphant, Pearu Peterson, et al. SciPy: Open source scientific tools for Python. <http://www.scipy.org/>, 2001–.

- [4] SUSE LLC. **openSUSE** linux distribution. <https://www.opensuse.org/>.
- [5] Microsoft. **Windows 10**. <https://www.microsoft.com/windows>.
- [6] Fedora Project. **Fedora** linux distribution. <https://getfedora.org/>.
- [7] The CentOS Project. **CentOS** linux distribution. <https://www.centos.org/>.
- [8] Python.org. Python. <http://www.python.org/>, 2013.
- [9] Conda team. Miniconda, minimal anaconda distribution. <https://conda.io/miniconda.html>, 2017.
- [10] Canonical Ltd Ubuntu Foundation. **Ubuntu** linux distribution. <https://www.ubuntu.com/>.

Informations for developers/maintainers of the Python package

git informations on the packages used to build this manual

```
name: fc-bench
tag: 0.0.3rc0
commit: 6fabfb9ab5d08281670bb13131980bdec58012
date: 2018-05-18
time: 12-55-12
status: 0
```

```
name: fc-tools
tag: 0.0.16
commit: fbe68454862f64c1b1822eb6f5f82c7e19317ad2
date: 2018-05-06
time: 04-50-39
status: 0
```

git informations on the L^AT_EX package used to build this manual

```
name: fctools
tag:
commit: 72693985daa7d84c61906a71c61d15f33893c3f6
date: 2018-05-09
time: 13-36-42
status: True
```

git informations on the fc_config package used to build this distribution

```
name: fc-config
tag:
commit: 4b173524b68fe2d93a619fc3f019057e8b44ec2f
date: 2018-05-15
time: 18-39-03
status: False
```