



FC_SIMESH package, User's Guide *

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Abstract

This object-oriented Python package allows to use simplices meshes generated from `gmsh` (in dimension 2 or 3) or an hypercube triangulation (in any dimension).

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1 Introduction

The **experimental** `fc_simesh` package does not contain graphical tools natively. However two add-ons were written to integrate graphical tools : the `fc_simesh_mayavi` package and the `fc_simesh_matplotlib` package which use respectively Mayavi ($\geq 4.5.0$) and matplotlib ($\geq 2.0.0$) packages. Installation of Mayavi package for Python 3.6.0 under the linux distributions CentOS 7 and Ubuntu 14.04 LTS is presented on my web page <http://www.math.univ-paris13.fr/~cuvelier> in the section *Informatique/Python*.

1.1 Prerequisite

This package needs the `fc_tools`, `fc_oogmsh` and `fc_hypermesh` Python packages.

1.2 Installation

See <http://www.math.univ-paris13.fr/~cuvelier/software/fc-simesh-Python.html>

1.3 Remarks

In all this report, for graphical representation, we use the `fc_simesh_matplotlib` package with `siplt` namespace for graphical functions using matplotlib and the `fc_simesh_mayavi` package with `simlab` namespace for graphical functions using matplotlib. There is the python code previously used before some of the listings given thereafter:

```
from fc_oogmsh import gmsh
from fc_simesh.siMesh import siMesh, HyperCube
# Graphical packages :
import matplotlib.pyplot as plt
import fc_simesh_matplotlib.siMesh as siplt
from mayavi import mlab
import fc_simesh_mayavi.siMesh as simlab
```

2 Mesh Objects

In geometry, a simplex is a generalization of the notion of a triangle or tetrahedron to arbitrary dimensions. Specifically, a k -simplex in \mathbb{R}^{\dim} , $k \leq \dim$, is a polytope which is the convex hull of its $k + 1$ vertices of \mathbb{R}^{\dim} . More formally, suppose the $k + 1$ vertices $q^0, \dots, q^k \in \mathbb{R}^{\dim}$ such that $q^1 - q^0, \dots, q^k - q^0$ are linearly independent. Then, the k -simplex K determined by them is the set of points

$$K = \left\{ \sum_{i=0}^k \lambda_i q^i \mid \lambda_i \geq 0, i \in \llbracket 0, k \rrbracket, \text{ with } \sum_{i=0}^k \lambda_i = 1 \right\}.$$

We denote by k -**simplicial elementary mesh** in \mathbb{R}^{\dim} , $k \leq \dim$, a mesh with **unique label** only composed with k -simplices.

A d -**simplicial mesh** in \mathbb{R}^{\dim} , $d \leq \dim$, is an union of k -simplicial elementary meshes with $k \in \llbracket 0, d \rrbracket$.

2.1 `siMESH` object

An elementary d -simplicial mesh in dimension \dim is represented by the class `siMESH`. We give properties of this class :

Properties of `siMESH` object for d -simplicial elementary meshes in \mathbb{R}^{\dim}

<code>dim</code>	: integer space dimension
<code>d</code>	: integer ($0 \leq d \leq \dim$)
<code>n_q</code>	: integer number of vertices
<code>n_{me}</code>	: integer number of elements (d -simplices)
<code>q</code>	: <code>n_q</code> -by- <code>dim</code> numpy array of reals array of vertex coordinates
<code>me</code>	: $(d + 1)$ -by- <code>n_{me}</code> numpy array of integers connectivity array for mesh elements
<code>vols</code>	: <code>n_{me}</code> numpy array of reals array of mesh element volumes
<code>h</code>	: double mesh step size (=maximum edge length in the mesh)
<code>toGlobal</code>	: <code>n_q</code> numpy array of integers convert from local to global mesh vertices numbering
<code>toParent</code>	: <code>n_q</code> array of integers convert from local to parent mesh vertices numbering (same as global if not part of a partitioned mesh)

More precisely

- $q[j, \nu]$ is the ν -th coordinate of the $(j + 1)$ -th vertex, $\nu \in \{0, \dots, \dim - 1\}$, $j \in \{0, \dots, n_q - 1\}$. The $(j + 1)$ -th vertex will be also denoted by $q^{j+1} = q[j]$.
- $me[\beta, k]$ is the storage index of the $(\beta + 1)$ -th vertex of the $k + 1$ -th element (d -simplex), in the array q , for $\beta \in \{0, \dots, d\}$ and $k \in \{0, \dots, n_{me} - 1\}$. So $q[me[\beta, k]]$ represents the coordinates of the $(\beta + 1)$ -th vertex of the $(k + 1)$ -th mesh element.
- $vols[k]$ is the volume of the $(k + 1)$ -th d -simplex .

2.2 `siMESH` object

A d -simplicial mesh in dimension \dim , represented as an `siMESH` object, is an union of `siMESH` objects which are elementary l -simplicial meshes ($l \leq d$) in space dimension \dim .



siMESH object properties

dim	: integer space dimension
d	: integer d-dimensional simplicial mesh
sTh	: list of siMESH _{ELT} objects
nsTh	: number of siMESH _{ELT} objects
sThsimp	: numpy array of nsTh integers <i>i</i> -th siMESH _{ELT} object in sTh is a sThsimp[<i>i</i> - 1]- simplicial elementary mesh
sThlab	: numpy array of nsTh integers in sTh label of <i>i</i> -th siMESH _{ELT} object in sTh is number sThlab[<i>i</i> - 1]
n _q	: integer number of vertices in the mesh
toGlobal	: numpy array of n _q integers convert from local to global mesh vertices numbering
toParent	: numpy array of n _q integers convert from local to parent mesh vertices numbering (same as global if not part of a partitioned mesh)

Let \mathcal{T}_h be a siMESH object. The global dim-by- $\mathcal{T}_h.n_q$ numpy array q of mesh vertices is not explicitly stored in \mathcal{T}_h , however one can easily build it if necessary:

$$q[\mathbf{p}^i] = \mathcal{T}_h.sTh[i].q, \quad \forall i \in \{0, \dots, \mathcal{T}_h.nsTh - 1\} \quad (1)$$

where $\mathbf{p}^i = \mathcal{T}_h.sTh[i].toParent$.

2.3 Mesh samples

2.3.1 2-simplicial mesh in \mathbb{R}^2

Listing 1: : 2D siMESH object from sample20.geo

```
# generate .msh file from sample20.geo by using gmsh
meshfile=gmsh.buildmesh2d('sample20',20,verbose=0)
# generate a siMesh object from .msh file
Th=siMesh(meshfile)
print(Th)
```

Output

```
siMesh object
  d : 2
  dim : 2
  nq : 2558
  nme : 4954
  sTh : list of 11 siMeshElt
  nsTh : 11
  sThsimp : (11,) ndarray
  [1 1 1 1 1 1 1 2 2 2 2]
  sThlab : (11,) ndarray
  [ 1  2 20 101 102 103 104 1  2 10 20]
```

For example, from output of Listing 1 or Figure 1 the complete domain is

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_{10} \cup \Omega_{20}$$

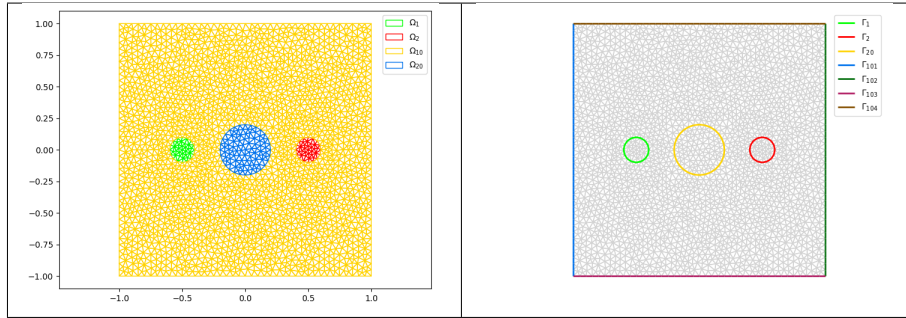


Figure 1: Mesh from `sample20.geo`, label of the domains (left) and label of the boundaries (right)

and we note

$$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_{20} \cup \Gamma_{101} \cup \Gamma_{102} \cup \Gamma_{103} \cup \Gamma_{104}.$$

So this mesh is 2-simplicial mesh in \mathbb{R}^2 and is composed of :

- four 2-simplicial elementary meshes : $\Omega_i, \forall i \in \{1, 2, 10, 20\}$
- seven 1-simplicial elementary meshes : $\Gamma_i \forall i \in \{1, 2, 20, 101, 102, 104\}$

2.3.2 Sample of a 3-simplicial mesh in \mathbb{R}^3

```

Listing 2 : 3D siMESH object from quart_sphere2.geo
meshfile=gmsh.buildmesh3d('quart_sphere2',5, verbose=0)
Th=siMesh(meshfile)
print(Th)

Output

Mesh /fcopt/python3.6.0/lib/python3.6/site-packages/meshes/quart_sphere2-5.msh is a
3-dimensional mesh
Force dimension to 3

siMesh object
  d : 3
  dim : 3
  nq : 1123
  nme : 4552
  sTh : list of 18 siMeshElt
  nsTh : 18
  sThsimp : (18,) ndarray
  [1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 3 3]
  sThlab : (18,) ndarray
  [1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 1 2]

```

The mesh obtained from Listing 2 or 2 is a 3-simplicial mesh in \mathbb{R}^3 and is composed of :

- two 3-simplicial elementary meshes : $\Omega_i, \forall i \in \{1, 2\}$
- seven 2-simplicial elementary meshes : $\Gamma_i \forall i \in \llbracket 1, 7 \rrbracket$
- nine 1-simplicial elementary meshes : $\partial\Gamma_i \forall i \in \llbracket 1, 9 \rrbracket$

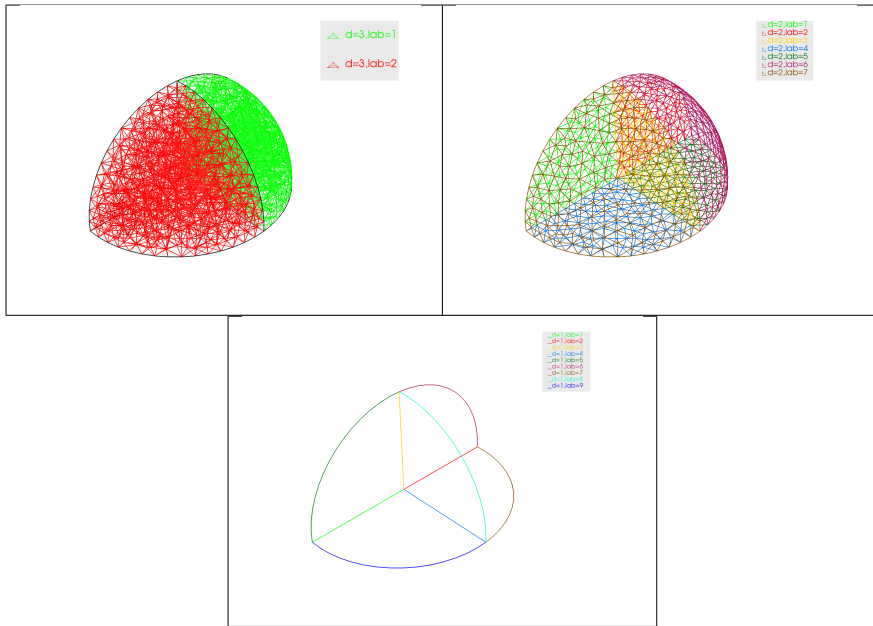


Figure 2: Mesh from `quart_sphere2.geo`, label of the domains (top left) , label of the boundaries (top right) and label of the 1-simplicial elementary meshes (bottom)

2.3.3 Sample of a 2-simplicial mesh in \mathbb{R}^3

```

Listing 3: : 3D surface siMESH object from demisphere4surf.geo
meshfile=gmsht.buildmesh3ds('demisphere4surf',5,verbose=0)
Th=siMesh(meshfile)
print(Th)

Output

Mesh /fcopt/python3.6.0/lib/python3.6/site-packages/meshes/demisphere4surf-5.msh is a
3-dimensional mesh
Force dimension to 3

siMesh object
d : 2
dim : 3
nq : 209
nme : 384
sTh : list of 12 siMeshElt
nsTh : 12
sThsimp : (12,) ndarray
[1 1 1 1 1 1 1 1 2 2 2 2]
sThlab : (12,) ndarray
[1 2 3 4 5 6 7 8 1 2 3 4]

```

The mesh obtained from Listing 3 or 3 is a 2-simplicial mesh in \mathbb{R}^3 and is composed of :

- four 2-simplicial elementary meshes : $\Omega_i, \forall i \in \llbracket 1, 4 \rrbracket$
- eight 1-simplicial elementary meshes : $\Gamma_i \forall i \in \llbracket 1, 8 \rrbracket$

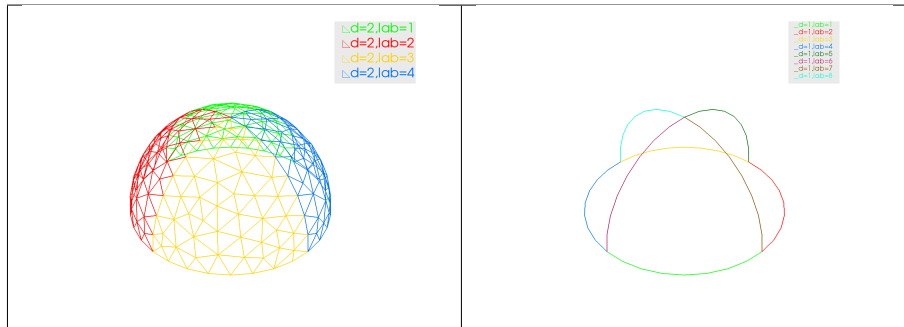


Figure 3: Mesh from `sample20.geo`, label of the domains (left) and label of the boundaries (right)

2.4 siMESH object methods

2.4.1 siMESH constructor

The constructor of the `siMESH` class can initialize the object from a `gmsh` mesh file saved as a `.msh` file

Syntaxe

```
Th=siMesh(meshfile)
Th=siMesh(meshfile,Name=Value)
```

Description

`Th=siMesh(meshfile)` create the `siMESH` object `Th` from the mesh file `meshfile` (`gmsh` format by default).

`Th=siMesh(meshfile,Name=Value, ...)` specifies function options using one or more `Name,Value` pair arguments. The `Name` options can be

- `dim` : to specify the space dimension (default 2),
- `d` : to specify the dimensions of the simplices to read, (default `[dim,dim-1]`)

Examples The following example use the function `gmsh.buildmesh2d` of the `fc_oogmsh` package to build the mesh from the `.geo` file `condenser11.geo`. This `.geo` file is located in the directory `geodir` of the `fc_oogmsh` package.

Listing 4: `siMESH` constructor sample

```
meshfile=gmshtool.buildmesh2d('condenser11',400,force=True,verbose=0)
Th=siMesh(meshfile)
print(Th)
```

Output

```
siMesh object
  d : 2
  dim : 2
  nq : 823086
  nme : 1641970
  sTh : list of 19 siMeshElt
  nsTh : 19
  sThsimp : (19,) ndarray
  [1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1]
  sThlab : (19,) ndarray
  [ 1  2  3  4  5  6  7  8 20 101 102 103 104 2  4  6  8 10
    20]
```

2.4.2 find method

We denote by `Th` a `siMESH` object.

- `Th.find(d)` : returns the indices numpy array of the d -simplicial elementary meshes in the array `Th.sTh`.
- `Th.find(d, labels=value)` : returns the indices of the d -simplicial elementary meshes with label in `value`. `value` could be an integer, a list of integers or a numpy array of integers. If nothing is found then return `None` otherwise returns the returned indices are of the same type as the input optional value.

Several examples are given in functions `siMesh.demos.find2D01()`, `siMesh.demos.find3D01()`, ... We present now some very basic samples.

2.4.3 feval method

Evaluates a function at vertices of the mesh. We denote by `Th` a `siMESH` object.

- `res=Th.feval(fun)` : the input parameter `fun` is either a function or a list of functions for vector-valued functions. If `fun` is a function then the output is a numpy array with shape `(Th.nq)`. If `fun` is a list of functions then the output is a numpy array with shape `(len(fun),Th.nq)`.
- `res=Th.feval(fun,key=value,...)` specifies function options using one or more `key,value` pair arguments. Key options could be
 - `d` : to specify the d -simplicial elementary meshes on which to evaluate the function (default `Th.d`). A zero value is set on all vertices not in these meshes.
 - `labels` : to specify the labels of the elementary meshes on which to evaluate the function (default is all). A zero value is set on all vertices not in these meshes.

Several examples are given in functions `siMesh.demos.feval2D01()`, `siMesh.demos.feval3D01()`, ... We present now some very basic samples.

Sample 1 Let $g : \mathbb{R}^2 \mapsto \mathbb{R}$ defined by $g(x, y) = \cos(x)\sin(y)$. We propose in Listing 5 four approaches to defined this function for using with feval method.

Listing 5: : feval method, four ways to defined a function

```
meshfile=gmesh.buildmesh2d('condenser11',50,verbose=0)
Th=siMesh(meshfile)

import numpy as np
g1=lambda x,y: np.cos(x)*np.sin(y)
g2=lambda X: np.cos(X[0])*np.sin(X[1])

def g3(x,y):
    return np.cos(x)*np.sin(y)

def g4(X):
    return np.cos(X[0])*np.sin(X[1])

z1=Th.feval(g1)
z2=Th.feval(g2)
z3=Th.feval(g3)
z4=Th.feval(g4)

print('max(abs(z2-z1))=%e'%max(abs(z2-z1)))
print('max(abs(z3-z1))=%e'%max(abs(z3-z1)))
print('max(abs(z4-z1))=%e'%max(abs(z4-z1)))
```

Output

```
max(abs(z2-z1))=0.000000e+00
max(abs(z3-z1))=0.000000e+00
max(abs(z4-z1))=0.000000e+00
```

Sample 2

Listing 6: : feval method, functions with parameters

```
meshfile=gmesh.buildmesh2d('condenser11',50,verbose=0)
Th=siMesh(meshfile)

import numpy as np
f=lambda x,y: np.cos(2*x)*np.sin(3*y)

def g(x,y,cx,cy):
    return np.cos(cx*x)*np.sin(cy*y)

g23=lambda x,y: g(x,y,2,3)

z=Th.feval(f)
z23=Th.feval(g23)
print('max(abs(z23-z))=%e'%max(abs(z23-z)))
```

Output

```
max(abs(z23-z))=0.000000e+00
```

Sample 3

Listing 7: : feval method with a vector-valued function

```
meshfile=gmesh.buildmesh2d('condenser11',50,verbose=0)
Th=siMesh(meshfile)

# f : R^2 -> R^3
import numpy as np
f=[lambda x,y: np.cos(2*x)*np.sin(3*y),
   lambda x,y: np.cos(3*x)*np.sin(4*y),
   lambda x,y: np.cos(4*x)*np.sin(5*y)]
z=Th.feval(f)
print('nq='+str(Th.nq)+'_z.shape='+str(z.shape))
```

Output

```
nq=13255 z.shape=(3, 13255)
```

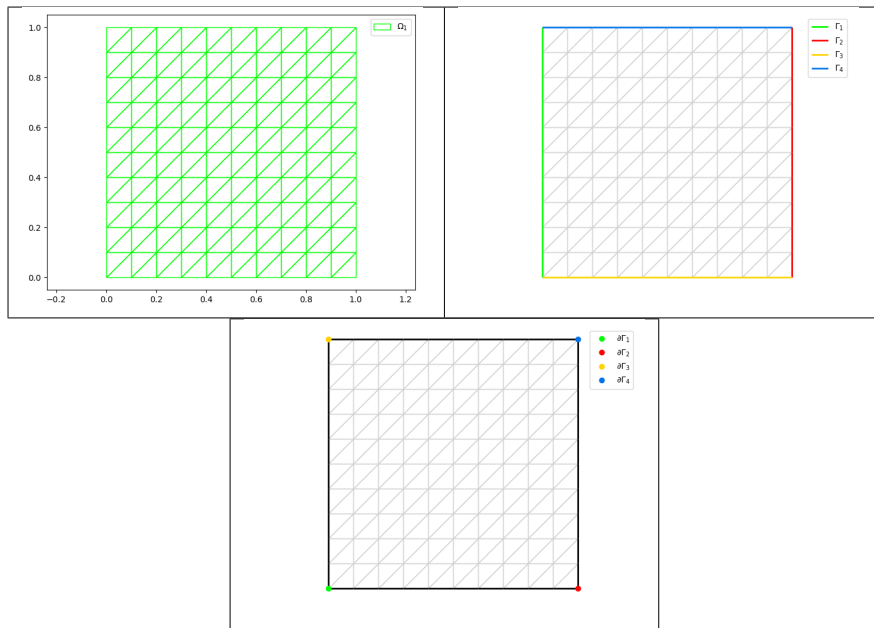


Figure 4: 2D Hypercube `siMESH` object generated with the function `siMesh.HyperCube`, representation of the elementary meshes with 2-simplices (top left), 1-simplices (top right) and 0-simplices (bottom)

2.5 Hypercube as a `siMESH` object

The function `siMesh.HyperCube` allows to create a `siMESH` object representing an hypercube in any dimension. It uses the Python package `fc_hypermesh`.

```

Listing 8: : 2D Hypercube siMESH object generated with the function siMesh.HyperCube
Th=HyperCube(2,10)
print(Th)

Output

siMesh object
  d : 2
  dim : 2
  nq : 121
  nme : 200
  sTh : list of 9 siMeshElt
  nsTh : 9
  sThsimp : (9,) ndarray
           [2 1 1 1 1 0 0 0 0]
  sThlab : (9,) ndarray
           [1 1 2 3 4 1 2 3 4]

```

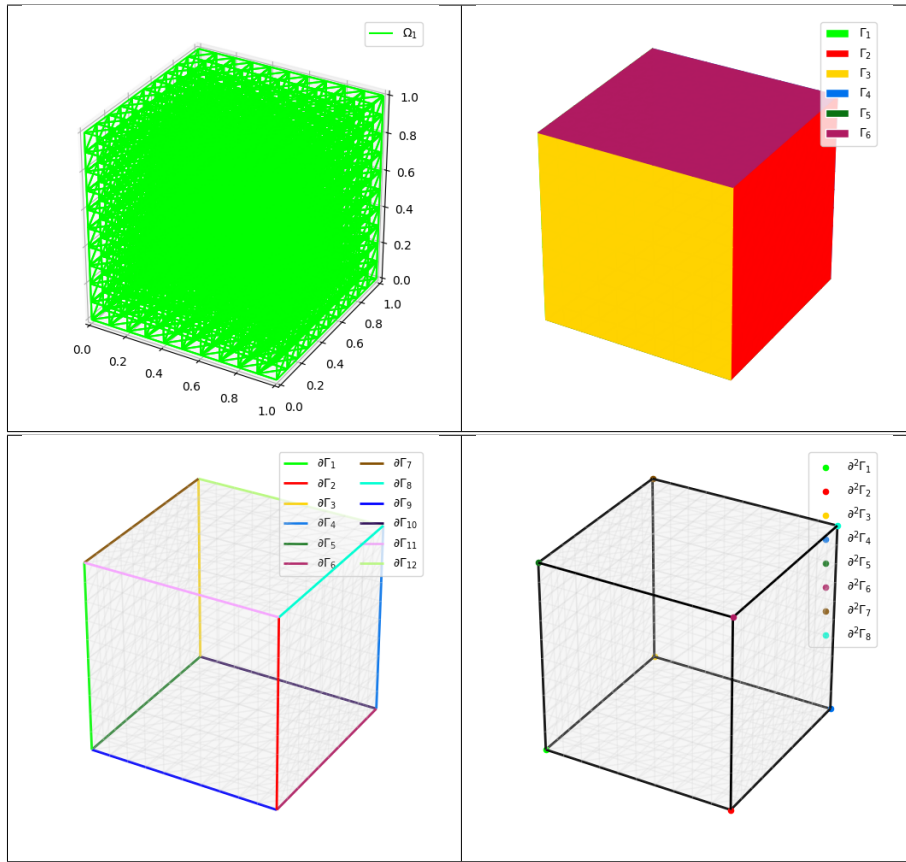


Figure 5: 3D Hypercube `siMESH` object generated with the function `siMesh.HyperCube`, representation of the elementary meshes with 3-simplices (top left), 2-simplices (top right), 1-simplices (bottom left) and 0-simplices (bottom right)

```

Listing 9: : 3D Hypercube siMESH object generated with the function siMesh.HyperCube
Th=HyperCube(3,10)
print(Th)

Output

siMesh object
  d : 3
  dim : 3
  nq : 1331
  nme : 6000
  sTh : list of 27 siMeshElt
  nsTh : 27
sThsimp : (27,) ndarray
[3 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0]
sThlab : (27,) ndarray
[ 1 1 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6
 7 8]

```

Listing 10: : function siMesh.HyperCube

```
Th=HyperCube(4,10)
print(Th)
```

Output

```
siMesh object
  d : 4
  dim : 4
  nq : 14641
  nme : 240000
  sTh : list of 81 siMeshElt
  nsTh : 81
sThsimp : (81,) ndarray
 [4 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1
  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0]
sThlab : (81,) ndarray
 [ 1 1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
  17 18 19 20 21 22 23 24 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
  18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 1 2 3 4 5 6 7 8 9 10
  11 12 13 14 15 16]
```

Listing 11: : function siMesh.HyperCube

```
Th=HyperCube(5,6)
print(Th)
```

Output

```
siMesh object
  d : 5
  dim : 5
  nq : 16807
  nme : 933120
  sTh : list of 243 siMeshElt
  nsTh : 243
sThsimp : (243,) ndarray
 [5 4 4 4 4 4 4 4 4 4 4 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
  3 3 3 3 3 3 3 3 3 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
  2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
  2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
sThlab : (243,) ndarray
 [ 1 1 2 3 4 5 6 7 8 9 10 1 2 3 4 5 6 7 8 9 10 11 12 13 14
  15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39
  40 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
  25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49
  50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74
  75 76 77 78 79 80 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
  20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44
  45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69
  70 71 72 73 74 75 76 77 78 79 80 1 2 3 4 5 6 7 8 9 10 11 12 13 14
  15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32]
```