



FC_HYPERMESH package, User's Guide *

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Abstract

This object-oriented Python package allows to mesh any d-orthotopes (hyperrectangle in dimension d) and their m-faces by simplices or orthotopes. It was created to show the implementation of the algorithms of [1]. The FC_HYPERMESH package uses Python objects and is provided with meshes visualisation tools for dimension leather or equal to 3.

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1 Introduction

The `FC_HYPERMESH` package contains a simple class object `OrthMesh` which permits, in any dimension $d \geq 1$, to obtain a simplicial mesh or orthotope mesh with all their m -faces, $0 \leq m < d$. It is also possible with the method function `plotmesh` of the class object `OrthMesh` to represent a mesh or its m -faces for $d \leq 3$.

In the following section, the class object `OrthMesh` is presented. Thereafter some warning statements on the memory used by these objects in high dimension are given. Finally computation times for orthotope meshes and simplicial meshes are given in dimension $d \in \llbracket 1, 5 \rrbracket$.

2 Contents of the `fc_hypermesh` package

2.1 Class object `EltMesh`

An elementary mesh class object `EltMesh` is used to store only one mesh, the main mesh as well as any of the meshes of the m -faces. This class `EltMesh` also simplify (for me) the codes writing. Its attributes are the following:

- `d` : space dimension
- `m` : kind of mesh corresponding to a m -face, $0 \leq m \leq d$, $m == d$ for the main mesh.
- `type` : 0 for simplicial mesh or 1 for orthotope mesh.
- `nq` : number of vertices.
- `q` : vertices numpy array of dimension d -by-`nq`
- `nme` : number of mesh elements
- `me` : connectivity numpy array of dimension $(d+1)$ -by-`nme` for simplices elements or 2^d -by-`nme` for orthotopes elements
- `toGlobal` : index array linking local array `q` to the one of the main mesh.
- `label` : name/number of this elementary mesh
- `color` : color of this elementary mesh (for plotting purpose)

2.2 Class object `OrthMesh`

The aim of the class object `OrthMesh` is to efficiently create an object which contains a mesh of a d-orthotope and all its m -face meshes.

Let the d-orthotope defined by $[a_1, b_1] \times \dots \times [a_d, b_d]$. The class object `OrthMesh` corresponding to this d-orthotope contains the main mesh and all the meshes of its m -faces, $0 \leq m < d$. Its attributes are the following

- `d`: space dimension
- `type`: string 'simplicial' or 'orthotope' mesh
- `Mesh`: main mesh as an `EltMesh` object
- `Faces`: 2d-list of `EltMesh` objects such that `Faces[0]` is a list of all the meshes of the $(d - 1)$ -faces, `Faces[1]` is a list of all the meshes of the $(d - 2)$ -faces, and so on
- `box`: a d-by-2 numpy array such that `box[i-1][0]` is a_i value and `box[i-1][1]` is b_i value.

2.2.1 Constructor

The `OrthMesh` constructor is :

`Oh = OrthMesh(d,N)`

where `N` is either a 1-by-d array/list such that `N[i-1]` is the number of discretization for $[a_i, b_i]$ or either an integer if the the number of discretization is the same in all space directions.

Some options are proposed with the constructor:

`Oh = OrthMesh(d,N,key=value)`

- `box = value` : where `value` is a d-by-2 list or array such that `value[i-1][0]` is a_i value and `value[i-1][1]` is b_i value. Default is $[0, 1]^d$.
- `type = value` : The default value for optional key parameter `type` is 'simplicial' and otherwise 'orthotope' can be used.

2.2.2 plotmesh property

`plotmesh()`

Used matplotlib to represents the mesh represented by an `OrthMesh` object. Some options are proposed with this property:

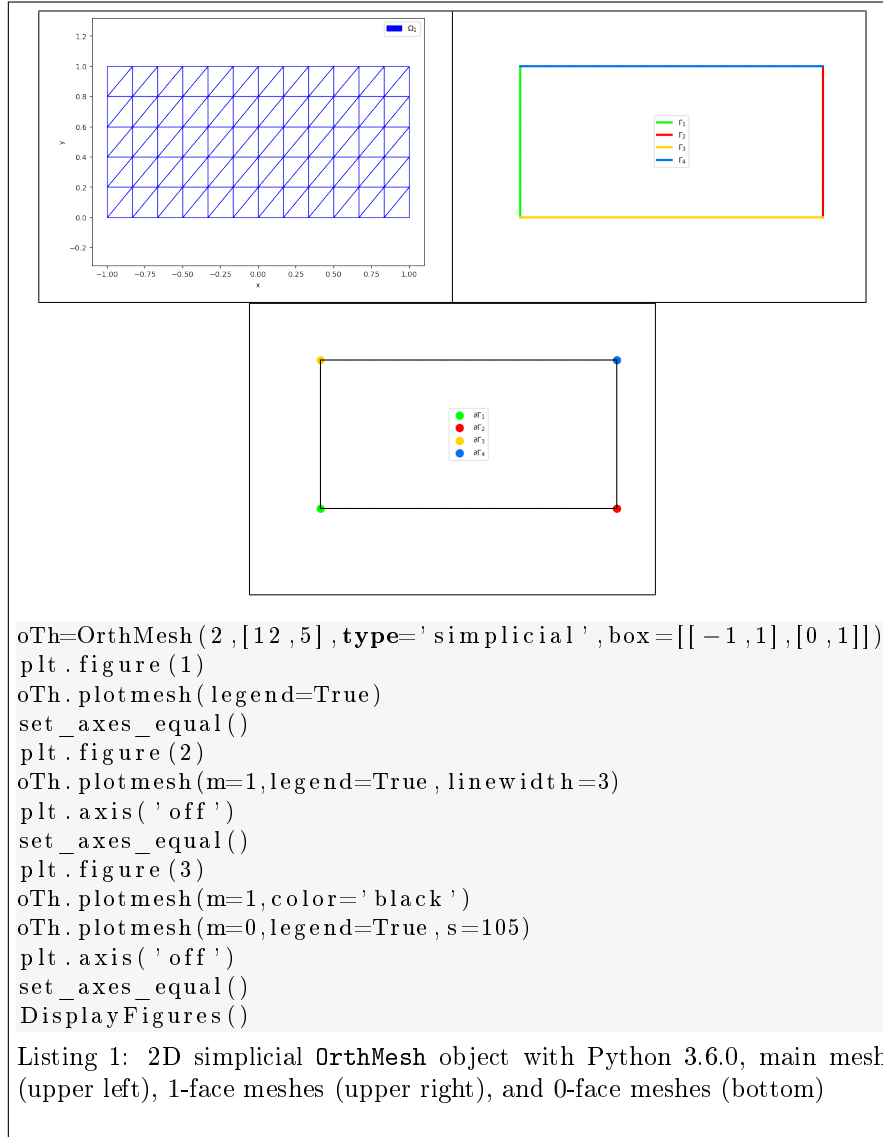
`plotmesh(key=value)`

- `legend = value` : if `value` is True, a legend is displayed. Default is False.
- `m = value` : plots all the m -faces of the mesh. Default `m = d` i.e. the main mesh. ($0 \leq m \leq d$)
- ...

3 Using the fc_hypermesh package

3.1 2d-orthotope meshing by simplices

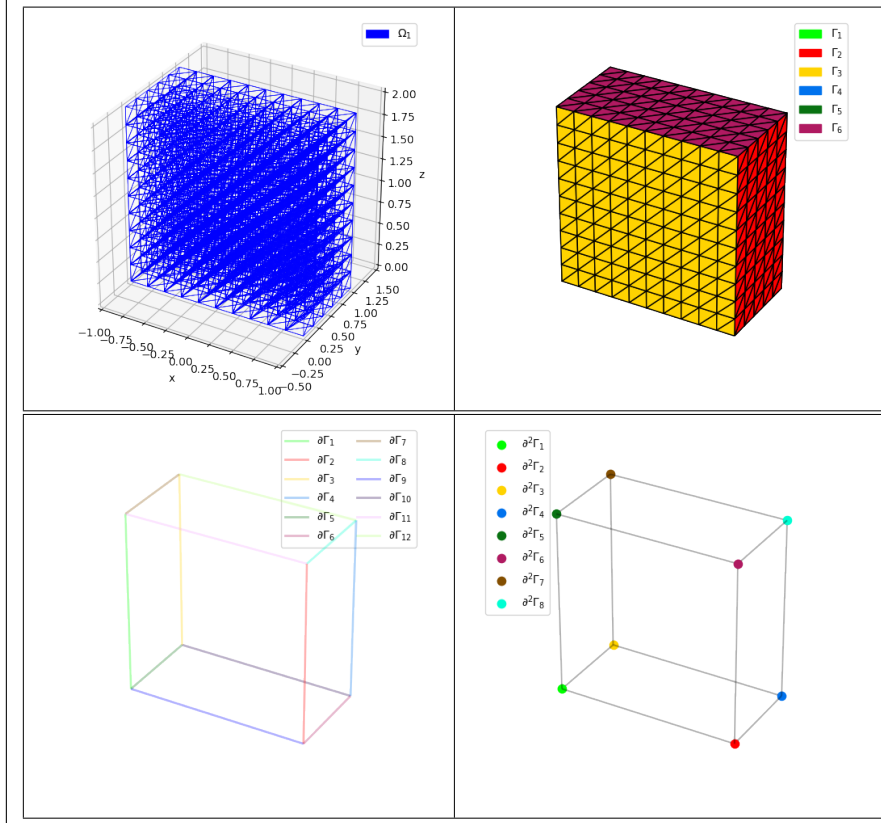
In Listing 1, an `OrthMesh` object is built under Python for the orthotope $[-1, 1] \times [0, 1]$ with simplicial elements and $\mathbf{N} = (12, 5)$. The main mesh and all the m -face meshes of the resulting object are plotted.



3.2 3d-orthotope meshing by simplices

In Listing 2, an `OrthMesh` object is built under Python for the orthotope $[-1, 1] \times [0, 1] \times [0, 2]$ with simplicial elements and $\mathbf{N} = (10, 5, 10)$. The main

mesh and all the m -face meshes of the resulting object are plotted.

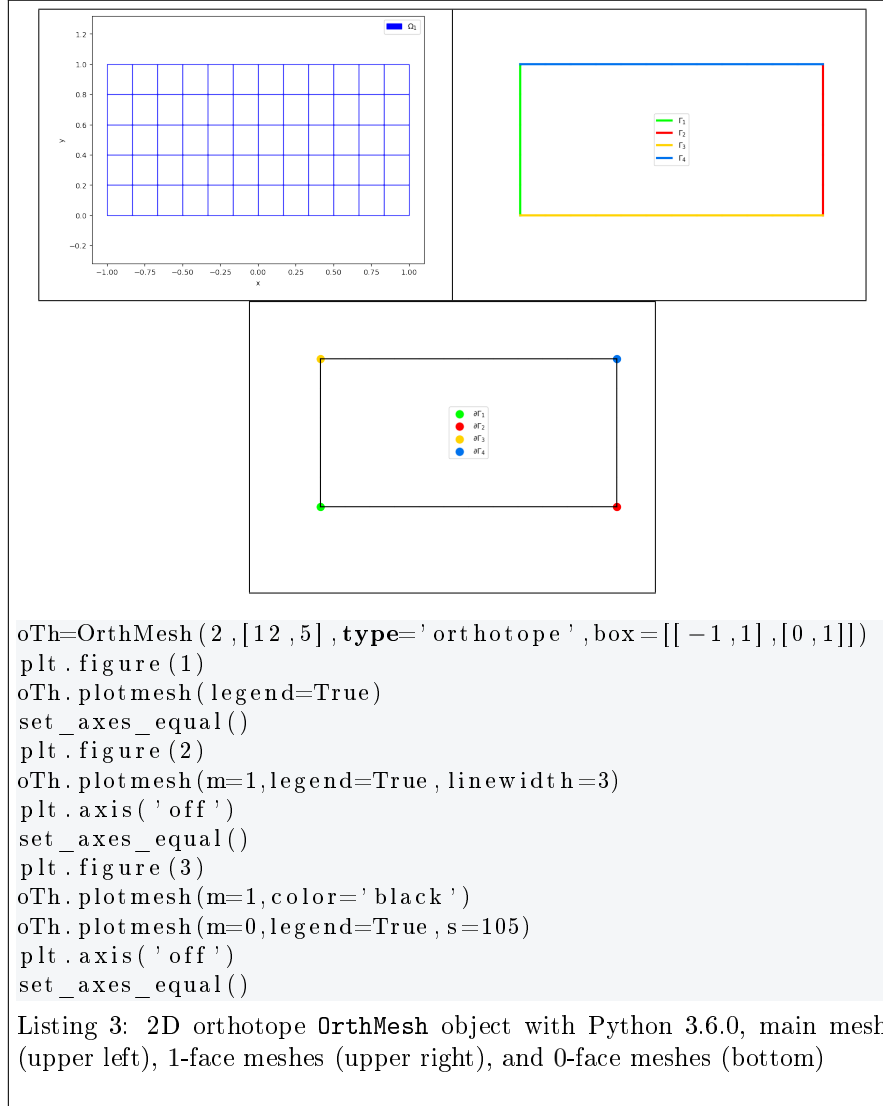


```
oTh=OrthMesh(3,[10,5,10],box=[[-1,1],[0,1],[0,2]])
plt.figure(1)
oTh.plotmesh(legend=True,linewidth=0.5)
set_axes_equal()
plt.figure(2)
oTh.plotmesh(m=2,legend=True,edgecolor=[0,0,0])
plt.axis('off')
set_axes_equal()
plt.figure(3)
oTh.plotmesh(m=2,edgecolor=[0,0,0],color='none')
oTh.plotmesh(m=1,legend=True,linewidth=2,alpha=0.3)
plt.axis('off')
set_axes_equal()
plt.figure(4)
oTh.plotmesh(m=1,color='black',alpha=0.3)
oTh.plotmesh(m=0,legend=True,s=55)
set_axes_equal()
plt.axis('off')
```

Listing 2: 3D simplicial `OrthMesh` object with Python 3.6.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

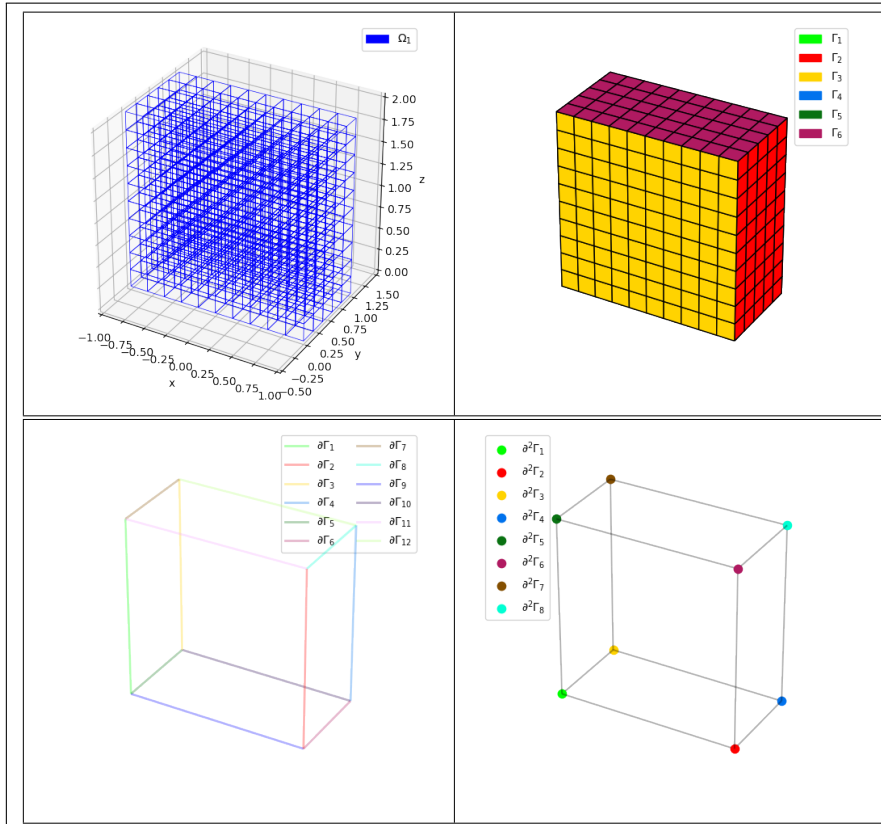
3.3 2d-orthotope meshing by orthotopes

In Listing 3, an `OrthMesh` object is built under Python for the orthotope $[-1, 1] \times [0, 1]$ with orthotope elements and $\mathbf{N} = (10, 5, 10)$. The main mesh and all the m -face meshes of the resulting object are plotted.



3.4 3d-orthotope meshing by orthotopes

In Listing 4, an `OrthMesh` object is built under Python for the orthotope $[-1, 1] \times [0, 1] \times [0, 2]$ with orthotope elements and $\mathbf{N} = (10, 5, 10)$. The main mesh and all the m -face meshes of the resulting object are plotted.



```

oTh=OrthMesh(3,[10,5,10],type='orthotope',
    box=[[-1,1],[0,1],[0,2]])
plt.figure(1)
oTh.plotmesh(legend=True,linewidth=0.5)
set_axes_equal()
plt.figure(2)
oTh.plotmesh(m=2,legend=True,edgecolor=[0,0,0])
plt.axis('off')
set_axes_equal()
plt.figure(3)
oTh.plotmesh(m=2,edgecolor=[0,0,0],color='none')
oTh.plotmesh(m=1,legend=True,linewidth=2,alpha=0.3)
plt.axis('off')
set_axes_equal()
plt.figure(4)
oTh.plotmesh(m=1,color='black',alpha=0.3)
oTh.plotmesh(m=0,legend=True,s=55)
set_axes_equal()
plt.axis('off')

```

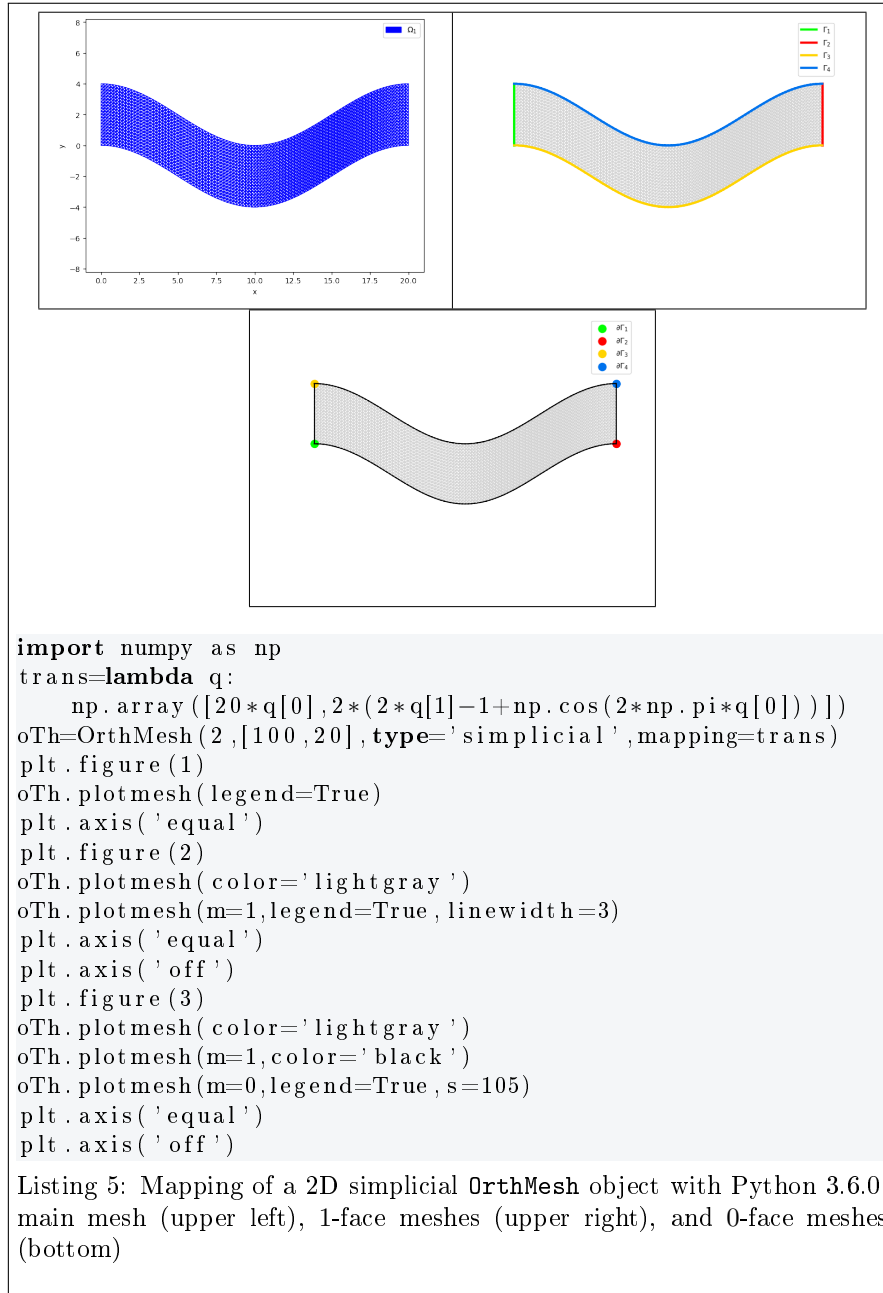
Listing 4: 3D orthotope `OrthMesh` object with Python 3.6.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

3.5 Mapping of a 2d-orthotope meshing by simplices

For example, the following 2D geometrical transformation allows to deform the reference unit hypercube.

$$[0, 1] \times [0, 1] \longrightarrow \mathbb{R}^2$$

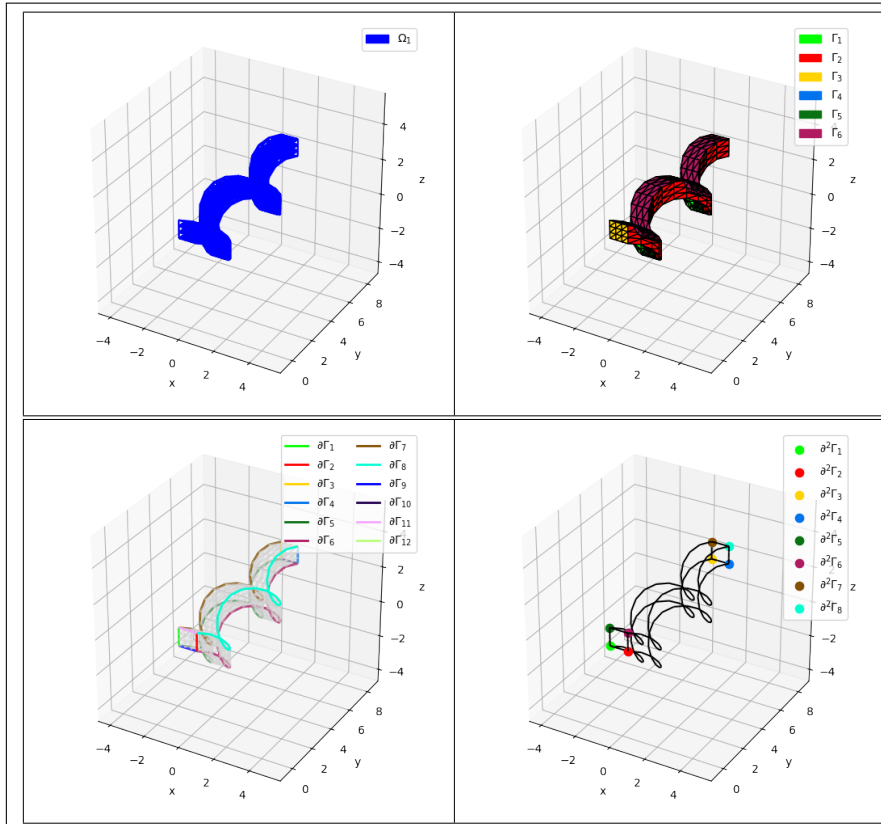
$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow F(x, y) = \begin{pmatrix} 20x \\ 2(2y - 1 + \cos(2\pi x)) \end{pmatrix}$$



3.6 3d-orthotope meshing by orthotopes

For example, the following 3D geometrical transformation allows to deform the reference unit hypercube.

$$\begin{aligned} [0, 1] \times [0, 1] \times [0, 1] &\longrightarrow \mathbb{R}^3 \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} &\longrightarrow F(x, y, z) = \begin{pmatrix} x + \sin(4\pi y) \\ 10y \\ z + \cos(4\pi y) \end{pmatrix} \end{aligned}$$



```

import numpy as np
trans=lambda q: np.array([q[0]+np.sin(4*np.pi*q[1]),
    10*q[1]-1, q[2]+np.cos(4*np.pi*q[1])])
oTh=OrthMesh(3,[3,25,3],type='simplicial',mapping=trans)
plt.figure(1)
oTh.plotmesh(legend=True)
set_axes_equal()
plt.figure(2)
oTh.plotmesh(m=2,legend=True,edgecolor=[0,0,0])
set_axes_equal()
plt.figure(3)
oTh.plotmesh(m=2,edgecolor='lightgray',facecolor=None,alpha=0.3)
oTh.plotmesh(m=1,legend=True,linewidth=2)
set_axes_equal()
plt.figure(4)
oTh.plotmesh(m=1,color='black')
oTh.plotmesh(m=0,legend=True,s=55)
set_axes_equal()

```

Listing 6: Mapping of a 3D orthotope `OrthMesh` object with Python 3.6.0, main mesh (upper left), 2-face meshes (upper right), 1-face meshes (bottom left) and 0-face meshes (bottom right)

4 Memory consuming

Take care when using theses codes with memory consuming : the number of points n_q and the number of elements increases exponentially according to the space dimension d . If $(N + 1)$ points are taken in each space direction, we have

$$n_q = (N + 1)^d, \text{ for both tessellation and triangulation}$$

and

$$\begin{aligned} n_{me} &= N^d, & \text{for tessellation by orthotopes} \\ n_{me} &= d!N^d, & \text{for tessellation by simplices.} \end{aligned}$$

If the array q is stored as *double* (8 octets) then

$$\text{mem. size of } q = d \times n_q \times 8 \text{ octets}$$

and if the array me as *int* (4 octets) then

$$\text{mem. size of } me = \begin{cases} 2^d \times n_{me} \times 4 \text{ octets} & \text{(tessellation by orthotopes)} \\ (d + 1) \times n_{me} \times 4 \text{ octets} & \text{(tessellation by simplices)} \end{cases}$$

For $N = 10$ and $d \in \llbracket 1, 8 \rrbracket$, the values of n_q and n_{me} are given in Table 1. The memory usage for the corresponding array q and array me is available in Table 2.

d	$n_q = (N + 1)^d$	$n_{me} = N^d$ (orthotopes)	$n_{me} = d!N^d$ (simplices)
1	11	10	10
2	121	100	200
3	1 331	1 000	6 000
4	14 641	10 000	240 000
5	161 051	100 000	12 000 000
6	1 771 561	1 000 000	720 000 000
7	19 487 171	10 000 000	50 400 000 000
8	214 358 881	100 000 000	4 032 000 000 000

Table 1: Number of vertices n_q and number of elements n_{me} for the tessellation of an orthotope by orthotopes and by simplices according to the space dimension d and with $N = 10$.

d	q	me (orthotopes)	me (simplices)
1	88 o	80 o	80 o
2	1 ko	1 ko	2 ko
3	31 ko	32 ko	96 ko
4	468 ko	640 ko	4 Mo
5	6 Mo	12 Mo	288 Mo
6	85 Mo	256 Mo	20 Go
7	1 Go	5 Go	1 612 Go
8	13 Go	102 Go	145 152 Go

Table 2: Memory usage of the array q and the array me for the tessellation of an orthotope by orthotopes and by simplices according to the space dimension d and with $N = 10$.

5 Benchmarks

For all the following tables, the computational costs of the `OrthMesh` constructor are given for the orthotope $[-1, 1]^d$ under Python 3.6.0. The computations were done on a laptop with Core i7-4800MQ processor and 16Go of RAM under Ubuntu 14.04 LTS (64bits).

In the following pages, computational costs of the `OrthMesh` constructor will be done by using `bench_gen` function. As sample, we give an example with output. Thereafter, all the output will be presented in tabular form.

```
from fc_hypermesh.benchs import bench_gen
bench_gen(3, 'simplicial', [[-1, 1], [-1, 1], [-1, 1]],
          range(20, 170, 20))
```

Listing 7: bench sample

Output

```
# BENCH in dimension 3 with simplicial mesh
#d: 3
#type: simplicial
#box: [[-1, 1], [-1, 1], [-1, 1]]
#desc: N      nq      nme      time(s)
      20      9261      48000      0.210
      40     68921     384000      0.219
      60    226981    1296000      0.289
      80    531441    3072000      0.380
     100   1030301    6000000      0.506
     120   1771561   10368000      0.697
     140   2803221   16464000      0.972
     160   4173281   24576000      1.337
```

5.1 Tessellation by orthotopes

```
from fc_hypermesh.benchs import bench_gen
bench_gen(2, 'orthotope', [[-1, 1], [-1, 1]], range(1000, 6000, 1000))
```

Listing 8: Tessellation of $[-1, 1]^2$ by orthotopes

N	n_q	n_{me}	Python
1000	1 002 001	1 000 000	0.192
2000	4 004 001	4 000 000	0.293
3000	9 006 001	9 000 000	0.496
4000	16 008 001	16 000 000	0.791
5000	25 010 001	25 000 000	1.211

Table 3: Tessellation of $[-1, 1]^2$ by orthotopes

```

from fc_hypermesh.benchs import bench_gen
bench_gen(3, 'orthotope', [[-1, 1], [-1, 1], [-1, 1]],
          range(50, 400, 50))

```

Listing 9: Tessellation of $[-1, 1]^3$ by orthotopes

N	n_q			n_{me}	Python
50	132	651		125 000	0.228
100	1 030	301		1 000 000	0.299
150	3 442	951		3 375 000	0.461
200	8 120	601		8 000 000	0.75
250	15 813	251		15 625 000	1.24
300	27 270	901		27 000 000	2.106
350	43 243	551		42 875 000	3.788

Table 4: Tessellation of $[-1, 1]^3$ by orthotopes

```

from fc_hypermesh.benchs import bench_gen
bench_gen(4, 'orthotope', [[-1, 1], [-1, 1], [-1, 1], [-1, 1]],
          [10, 20, 30, 40, 50, 62])

```

Listing 10: Tessellation of $[-1, 1]^4$ by orthotopes

N	n_q			n_{me}	Python
10	14	641		10 000	0.299
20	194	481		160 000	0.319
30	923	521		810 000	0.415
40	2 825	761		2 560 000	0.605
50	6 765	201		6 250 000	1.067
62	15 752	961		14 776 336	1.953

Table 5: Tessellation of $[-1, 1]^4$ by orthotopes

```

from fc_hypermesh.benchs import bench_gen
bench_gen(5, 'orthotope', [[-1, 1], [-1, 1], [-1, 1], [-1, 1], [-1, 1]],
          [5, 10, 15, 20, 25, 27])

```

Listing 11: Tessellation of $[-1, 1]^5$ by orthotopes

N	n_q			n_{me}	Python
5	7	776		3 125	0.542
10	161	051		100 000	0.568
15	1 048	576		759 375	0.813
20	4 084	101		3 200 000	1.315
25	11 881	376		9 765 625	2.644
27	17 210	368		14 348 907	4.731

Table 6: Tessellation of $[-1, 1]^5$ by orthotopes

5.2 Tessellation by simplices

```
from fc_hypermesh.benchs import bench_gen
bench_gen(2, 'simplicial', [[-1,1],[-1,1]], range(1000,6000,1000))
```

Listing 12: Tessellation of $[-1,1]^2$ by simplices

N	n_q	n_{me}	Python
1000	1 002 001	2 000 000	0.211
2000	4 004 001	8 000 000	0.408
3000	9 006 001	18 000 000	0.756
4000	16 008 001	32 000 000	1.24
5000	25 010 001	50 000 000	1.912

Table 7: Tessellation of $[-1,1]^2$ by simplices

```
from fc_hypermesh.benchs import bench_gen
bench_gen(3, 'simplicial', [[-1,1],[-1,1],[-1,1]],
range(40,190,20))
```

Listing 13: Tessellation of $[-1,1]^3$ by simplices

N	n_q	n_{me}	Python
40	68 921	384 000	0.222
60	226 981	1 296 000	0.296
80	531 441	3 072 000	0.393
100	1 030 301	6 000 000	0.52
120	1 771 561	10 368 000	0.71
140	2 803 221	16 464 000	0.977
160	4 173 281	24 576 000	1.351
180	5 929 741	34 992 000	1.836

Table 8: Tessellation of $[-1,1]^3$ by simplices

```
from fc_hypermesh.benchs import bench_gen
bench_gen(4, 'simplicial', [[-1,1],[-1,1],[-1,1],[-1,1]],
[10,20,25,30,35,40])
```

Listing 14: Tessellation of $[-1,1]^4$ by simplices

N	n_q	n_{me}	Python
10	14 641	240 000	0.323
20	194 481	3 840 000	0.591
25	456 976	9 375 000	1.012
30	923 521	19 440 000	1.764
35	1 679 616	36 015 000	2.952

Table 9: Tessellation of $[-1,1]^4$ by simplices

```

from fc_hypermesh.benchs import bench_gen
bench_gen(5, 'simplicial', [[-1,1], [-1,1], [-1,1], [-1,1], [-1,1]],
         range(2,14,2))

```

Listing 15: Tessellation of $[-1,1]^5$ by simplices

N	n_q	n_{me}	Python
2	243	3 840	0.538
4	3 125	122 880	0.528
6	16 807	933 120	0.657
8	59 049	3 932 160	0.815
10	161 051	12 000 000	1.471
12	371 293	29 859 840	3.076

Table 10: Tessellation of $[-1,1]^5$ by simplices

References

- [1] F. Cuvelier and G. Scarella. Vectorized algorithms for regular tessellations of d-orthotopes and their faces. http://www.math.univ-paris13.fr/~cuvelier/-docs/reports/HyperMesh/HyperMesh_0.0.4.pdf, 2016.