Conference Dynamics and Geometry

Organized by H. de Thélin, T.-C. Dinh and C. Dupont

Paris, 20th - 24th June 2011

Institut Henri Poincaré, Amphithéâtre Hermite

Monday

10h00-10h50 N. Mok : Techniques of Analytic Continuation in Complex Geometry related to bounded symmetric domains (I)

Coffee Break

11h10-12h00 N. Sibony : Analysis on laminations by Riemann Surfaces (I)

14h-14h50 J.-P. Demailly : Holomorphic Morse inequalities and the Green-Griffiths conjecture (I)

15h10-16h A. Zorich : Lyapounov exponents of the Hodge bundle with respect to the Teichmuller geodesic flow (I)

Coffee Break

16h30-17h20 Y. Benoist : Stationary measures on finite volume homogeneous spaces (I)

Tuesday

9h30-10h20 Y.-T. Siu : Analytic techniques of constructing holomorphic sections of line bundles (I)

Coffee Break

10h50-11h40 N. Sibony : Analysis on laminations by Riemann Surfaces (II)

14h-14h50 N. Mok : Techniques of Analytic Continuation in Complex Geometry related to bounded symmetric domains (II)

15h10-16h Y. Pesin : Stable Ergodicity of Partially Hyperbolic Systems (I)

Coffee Break

16h30-17h20 J.-P. Demailly : Holomorphic Morse inequalities and the Green-Griffiths conjecture (II)

Wednesday

9h-9h50 A. Zorich : Lyapounov exponents of the Hodge bundle with respect to the Teichmuller geodesic flow (II)

10h10-11h Y.-T. Siu : Analytic techniques of constructing holomorphic sections of line bundles (II)

Coffee Break

11h30-12h20 J.-P. Demailly : Holomorphic Morse inequalities and the Green-Griffiths conjecture (III)

Thursday

9h30-10h20 Y. Benoist : Stationary measures on finite volume homogeneous spaces (II)

Coffee Break

10h50-11h40 N. Sibony : Analysis on laminations by Riemann Surfaces (III)

14h-14h50 Y. Pesin : Stable Ergodicity of Partially Hyperbolic Systems (II)

15h10-16h A. Zorich : Lyapounov exponents of the Hodge bundle with respect to the Teichmuller geodesic flow (III)

Coffee Break

16h20-17h10 N. Mok : Techniques of Analytic Continuation in Complex Geometry related to bounded symmetric domains (III)

17h10-18h: Open problems session: E. Bedford, D. Burns, X. Ma, J. Merker and E. Rousseau

Cocktail: 18h-19h

Friday

9h-9h50 Y.-T. Siu : Analytic techniques of constructing holomorphic sections of line bundles (III)

10h10-11h Y. Pesin : Stable Ergodicity of Partially Hyperbolic Systems (III)

Coffee Break

11h30-12h20 Y. Benoist : Stationary measures on finite volume homogeneous spaces (III)

Stationary measures on finite volume homogeneous spaces

by Yves Benoist

Abstract. Consider a Lie group G, a finite volume quotient X of G, a subgroup H of G, and a probability measure m on H whose support is compact and generates H. Assume that the Zariski closure of the adjoint group of H is semisimple and connected.

In this course I will explain a series of paper with Jean-Francois Quint in which we study the dynamics of H on X. We classify the m-stationary probability measures on X and prove Furstenberg Stiffness Conjecture: all of them are H-invariant. As a consequence, we describe the H-orbit closures in X and deduce various equidistribution results on X.

References

[1] Benoist Y. and Quint J.-F., Mesures stationnaires et fermés invariants des espaces homogènes (I) et (II).

Holomorphic Morse inequalities and the Green-Griffiths conjecture

by Jean-Pierre Demailly

Lecture 1. "Asymptotic cohomology and holomorphic Morse inequalities"

Abstract. Definition of asymptotic cohomology and its basic properties, relation with holomorphic Morse inequalities, formula for the volume of a line bundle, converse to the Andreotti-Grauert theorem in dimension 2.

Lecture 2. "Directed varieties, jet differentials and hyperbolic algebraic manifolds"

Abstract. An introduction to the geometric tools needed for the study of Kobayashi hyperbolicity and the related conjectures of Kobayashi, Lang and Green-Griffiths. Proof of the fundamental vanishing theorem for entire curves. Curvature calculations for jet bundles.

Lecture 3. "Probabilistic estimates for the cohomology of jet differentials"

Abstract. Using holomorphic Morse inequalities, one can derive very precise estimates for the growth of cohomology of jet differential sheaves on an arbitrary compact directed variety. This leads to the solution of an important step of the Green-Griffiths conjecture, namely the algebro-differential degeneracy of entire curves drawn in a complex algebraic variety of general type. ¿From this, one can derive the hyperbolicity of generic hypersurfaces of high degree in projective space.

References

[1] Jean-Pierre Demailly, (all papers can be found on my web page http://www-fourier.ujf-grenoble.fr/ demailly), Champs magnétiques et inégalités de Morse pour la d"-cohomologie, Ann. Inst. Fourier (Grenoble) 35 (1985) 189-229

[2] Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials, Proceedings of Symposia in Pure Math., Vol. 62.2 (AMS Summer Institute on Algebraic Geometry, Santa Cruz, July 1995), ed. J. Kollár, R. Lazarsfeld, (1997), 285-360

[3] Holomorphic Morse inequalities and asymptotic cohomology groups: a tribute to Bernhard Riemann, Prépublication Institut Fourier, February 23, 2010, math.CV:hal-00462954,

[4] A converse to the Andreotti-Grauert theorem, Prépublication Institut Fourier, October 11, 2010, math.AG:hal-00536317

[5] Holomorphic Morse inequalities and the Green-Griffiths-Lang conjecture, Prépublication Institut Fourier, November 15, 2010, math.AG:hal-00536319

[6] Simone Diverio, Joël Merker, Erwan Rousseau, Effective algebraic degeneracy, arXiv:0811.2346 (math.AG), Invent. Math. 180 (2010) 161-223.

[7] Joël Merker, Low pole order frames on vertical jets of the universal hypersurface, arXiv:0805.3987 (math.AG), Ann. Inst. Fourier Grenoble, 59 (2009) 1077-1104

Techniques of Analytic Continuation in Complex Geometry related to bounded symmetric domains

by Ngaiming Mok

Abstract. Bounded symmetric domains are the Harish-Chandra realizations of Hermitian symmetric manifolds of the semisimple and noncompact type as bounded domains. We will consider the standard realization of such domains as circular domains using the Harish-Chandra realization, e.g., the complex unit ball is the Harish-Chandra realization in the rank-1 case.

In three lectures we will study questions on the analytic continuation of germs of holomorphic maps satisfying certain geometric constraints. These contraints may arise from Algebraic Geometry or from Complex Differential Geometry. The case of $f : (D; 0) \to (\Omega; 0)$ between bounded symmetric domains $D \Subset \mathbb{C}^n$ and $\Omega \Subset \mathbb{C}^N$ in their Harish-Chandra realization is of particular interest because of applications to the study of finite-volume quotients of bounded symmetric domains, which are quasi-projective manifolds very often corresponding to moduli spaces of algebro-geometric objects. In many cases the results generalize to a much broader setting.

Lecture 1 Extension principle for germs of holomorphic maps respecting geometric structures

Lecture 2 Alexander-type extension theorems for irreducible bounded symmetric domains

Lecture 3 Extension of germs of holomorphic isometries with respect to the Bergman metric

Brief description of each of the 3 lectures

Lecture 1: *Extension principle for germs of holomorphic maps respecting geometric structures*
Let S be a Hermitian symmetric manifold of the compact type of rank ≥ 2, and $D \Subset \mathbb{C}^n \subset S$ be the inclusions consisting of the Harish-Chandra embedding $D \Subset \mathbb{C}^n$ and the Borel Embedding $D \subset S$. S carries a holomorphic G-structure for some reductive Lie Group $G \subset GL(n; \mathbb{C})$ in the sense that there is a holomorphic reduction of the frame bundle from $GL(n; \mathbb{C})$ to G. This G-structure is also referred to as the S-structure. Using cohomological methods Ochiai [Oc70] proved that, given a biholomorphism $f : U \cong V$ between two non-empty connected open subsets of S preserving the S-structure, f extends to an automorphism $F : S \cong S$. Ochiai's Theorem can be incorporated as a very special case of an extension result, called Cartan-Fubini Extension Principle, when one interprets the preservation of S-structures as the preservation of privileged sets of tangent vectors. This

was accomplished by Hwang-Mok [HM04] for Fano manifolds of Picard number 1 under very mild conditions via the study of the variety of minimal rational tangents (VMRT), viz., the set $\mathcal{C}_x(S) \subset \mathbb{P}T_x(S)$ consisting of vectors tangent to minimal rational curves (projective lines). To explain the argument for S-structures, the work of Hwang-Mok [HM01] in the special case where the VMRT satisfies some Gauss-map condition is sufficient; in fact the use of Harish-Chandra coordinates further simplifies the proof. Recently, [HM01] was generalized by Hong-Mok [HoM10] to the non-equidimensional case, where a relative version of the Gauss-map condition was imposed on a pair consisting of a VMRT and a linear section (cf. also Mok [Mk08, §7]). We will highlight the key arguments involving holomorphic foliations and deformation of minimal rational curves, and give some geometric applications.

<u>Lecture 2</u>: *Alexander-type extension theorems for irreducible bounded symmetric domains*
Let $n \geq 2$, $b_i \in \partial B^n$; $i = 1, 2$; and U_i be an open neighborhood of b_i in $\mathbb{C}^n$. Let $f : U_1 \cong U_2$ be a biholomorphism such that $f(U_1 \cap \partial B^n) \subset U_2 \cap \partial B^n$. Then, by Alexander's Theorem, f extends to an automorphism $F : B^n \cong B^n$. For the case of a bounded symmetric domain $D \Subset \mathbb{C}^n$ of rank ≥ 2 in its Harish-Chandra realization, Henkin-Tumanov [TK82,83] proved an analogue by considering $b_i \subset \partial D$ belonging to the Shilov boundary $Sh(D)$, subject to the condition that $f(U_1 \cap \partial D) \subset U_2 \cap \partial D$ and $f(U_1 \cap Sh(D)) \subset U_2 \cap Sh(D))$. Recently, Mok-Ng [MN10] proved a strengthened Alexander-type theorem for $\mathrm{rank}(D) \geq 2$ by considering smooth points $b_i \subset \mathrm{Reg}(\partial D)$ in place of points on $Sh(D)$. The proof makes use of the fine structure of ∂D (cf. Wolf [Wo72]), the method of Mok-Tsai [MT91] concerning boundary values of proper holomorphic maps on D, and Ochiai's Theorem as in Lecture 1.

The original motivation for proving the strengthened Alexander-type extension theorem was a problem raised by Clozel-Ullmo [CU03] in Arithmetic Dynamics, where the authors wanted to characterize commutants of certain Hecke correspondences on D/Γ. They reduced the problem to characterizing germs of measure-preserving holomorphic maps $f : (D, \lambda d\mu_D; 0) \to (D, d\mu_D; 0) \times \cdots \times (D, d\mu_D; 0)$, where $d\mu_D$ denotes the Bergman volume form on D. Thus, f induces a holomorphic isometry between some Hermitian holomorphic vector bundles. Lifting the map to unit sphere bundles, we apply the extension result of Webster-Huang (Huang [Hu94]) on strongly pseudoconvex algebraic hypersurfaces to prove that Graph(f) extends as an affine-algebraic variety. Using functional identities arising from the measure-preserving property, we reduce the problem to the strengthened Alexander-type extension theorem.

<u>Lecture 3</u>: *Extension of germs of holomorphic isometries with respect to the Bergman metric*
Another outgrowth of Clozel-Ullmo [CU03] is the question of characterizing germs of holomorphic isometries $f : (D, \lambda ds_D^2; 0) \to (\Omega, ds_\Omega; 0)$, where ds_D^2 denotes the

Bergman metric on ds^2_D, etc., and $\Omega = D \times \cdots \times D$. We consider the much more general problem, where $D \Subset \mathbb{C}^n$, $\Omega \Subset \mathbb{C}^N$ are bounded domains, subject to the condition that the Bergman kernel function $K_D(z,w)$ resp. $K_\Omega(\zeta,\xi)$ extends to a rational function in $(z,\overline{w})$ resp. $(\zeta,\overline{\xi})$. Whenever (D, ds^2_D) and (Ω, ds^2_Ω) are complete, we prove in Mok [Mk] that f extends to a proper holomorphic isometry $F : (D, \lambda ds^2_D) \to (\Omega, ds^2_\Omega)$ in such a way that $\mathrm{Graph}(F) \subset D \times \Omega \Subset \mathbb{C}^n \times \mathbb{C}^N$ extends as an affine-algebraic variety (cf. Mok [Mk02] for a simple special case), generalizing results in [CU03] where D is the unit disk and Ω is a polydisk.

Interior extension results on $\mathrm{Graph}(f) \subset D \times \Omega$ already follow from the seminal work of Calabi [Ca53] on holomorphic isometries in which the *diastasis* was introduced, when a bounded domain G is holomorphically and canonically embedded into the infinite-dimensional projective space $\mathbb{P}(H^2(D)^\star)$. By polarization we make use of holomorphic functional identities arising from equating diastases, and interpret $\mathrm{Graph}(f)$ as a subset of the set of common zeros of the polarized functional identities. It may happen nonetheless that the functional identities are not sufficient to cut the zero locus down to the dimension of $\mathrm{Graph}(f)$, but we prove that, by adjoining additional constraints coming from certain extremal functions in $H^2(G)$ we can cut down the zero locus to the dimension of $\mathrm{Graph}(f)$, which yields the algebraic extension.

Main References (available at [http://www.hku.hk/math])

[HoM10] Hong, J.; Mok, N.: Analytic continuation of holomorphic maps respecting varieties of minimal rational tangents and applications to rational homogeneous manifolds, *J. Diff. Geom.* **86**(2010), 539-567. (*Lecture 1*)

[MN10] Mok, N.; Ng, S.-C.: Germs of measure-preserving holomorphic maps from bounded symmetric domains to their Cartesian products, to appear in *J. Reine Angew. Math.* (*Lecture 2*)

[Mk] Mok, N.: Extension of germs of holomorphic isometries up to normalizing constants with respect to the Bergman metric. Preprint. (*Lecture 3*)

Supplementary References to lectures

[Mk08] Mok, N.: Geometric structures on uniruled projective manifolds defined by their varieties of minimal rational tangents, Proceedings of the Conference “Géometrie differentielle, Physique mathématique, Mathématique et Société”, *Astérisque*, **322**(2008), Volume II, 151-205, published by Société Mathématique de France. (*Lecture 1*, available at [http://www.hku.hk/math])

[HM04] Hwang, J.-M.; Mok, N.: Birationality of the tangent map for minimal rational curves, Special Volume dedicated to Professor Yum-Tong Siu on his sixtieth birthday, *Asian J. Math.* **8**(2004), 51-64. (*Lecture 1*, available at [http://www.hku.hk/math])

[Mk02] Mok, N.: Local holomorphic isometric embeddings arising from correspondences in the rank-1 case, in Contemporary Trends in Algebraic Geometry and Algebraic Topology, ed. S.-S. Chern, L. Fu and R. Hain, Nankai Tracts in Mathematics, Vol.5, World Scientific, New Jersey 2002, pp.155-166. (*Lecture 3*)

References on Backgrounds

[Al74] Alexander, E.: Holomorphic mappings from the ball and the polydisc, *Math. Ann.* **209**(1974), 249-256.

[Ca53] Calabi, E.: Isometric imbedding of complex manifolds, *Ann. Math.* **58**(1953), 1-23.

[CU03] Clozel, L.; Ullmo, E.: Correspondances modulaires et mesures invariantes, *J. Reine Angew. Math.* **558**(2003), 47-83.

[HM01] Hwang, J.-M.; Mok, N.: Cartan-Fubini type extension of holomorphic maps for Fano manifolds of Picard number 1, *J. Math. Pure Appl.* **80**(2001), 563-575.

[Hu94] Huang, X.: On the mapping problem for algebraic real hypersurfaces in the complex spaces of different dimensions, *Ann. de l'Inst. Fourier*, **44**(1994), 433-463.

[MT91] Mok, N.; Tsai, I.-H.: Rigidity of convex realizations of irreducible bounded symmetric domains of rank ≥ 2, *J. Reine Angew. Math.* **431**(1992), 91-122.

[Oc70] Ochiai, T.: Geometry associated with semisimple flat homogeneous spaces, *Trans. Amer. Math. Soc.* **152**(1970), 159-193.

[TK82] Tumanov, A. E.; Khenkin, G.M.: Local characterization of analytic automorphisms of classical domains (Russian), *Dokl. Akad. Nauk SSSR* **267**(1982), 796-799; English translation: *Math. Notes* **32**(1982), 849-852.

[TK83] Tumanov, A. E.; Khenkin, G.M.: Local characterization of holomorphic automorphisms of Siegel domains (Russian), *Funktsional. Anal. i Prilozhen* **17** (1983), 49-61; English translation: *Functional Anal. Appl.* **17**(1983), 285-294.

[Wo72] Wolf, J. A.: Fine structure of Hermitian symmetric spaces, in Geometry of Symmetric Spaces, ed. Boothby-Weiss, Marcel-Dekker, New York, 1972, pp.271-357.

Stable Ergodicity of Partially Hyperbolic Systems

by Yakov Pesin

Abstract. I will introduce the concept of stable ergodicity in smooth dynamics and I will discuss the sytable ergodicity problem for partially hyperbolic systems both conservative and dissipative. I will state the Pugh-Shub Stable Ergodicity conjecture and will outline two different approaches to it. The first one is due to the original work of Pugh and Shub and is based on a new idea in studying absolute continuity of invariant foliations in the case of partial hyperbolicity. The second approach originated in work of Alves, Bonatti and Viana as well as Burns, Dolgopyat and myself and is based on the study of Lyapunov exponents in the central direction of the system. This approach can also be used to study stable ergodicity of partially hyperbolic attractors. As a by-product of this approach we obtain some criteria for existence of Sinai-Ruelle-Bowen measures for these attractors.

References.

1. J. Alves, C. Bonatti and M. Viana, "SRB measures for partially hyperbolic systems whose central direction is mostly expanding", Inv. Math., no.140 (2000) 351–398.

2. C. Bonatti, M. Viana, "SRB measures for partially hyperbolic systems whose central direction is mostly contracting", Israel J. Math., no.115 (2000) 157–193.

3. M. Brin, Ya. Pesin, "Partially hyperbolic dynamical systems", Math. USSR Izv. v. 38 (1974) 170–212.

4. K. Burns, D. Dolgopyat, Ya. Pesin, "Partial hyperbolicity, Lyapunov exponents and stable ergodicity", J. Stat. Phys., no.109 (2002) 927–942.

5. K. Burns, D. Dolgopyat, Ya. Pesin, M. Pollicott, "Stable ergodicity for partially hyperbolic attractors with negative central exponents", J. Modern Dynamics, v. 2, no.1 (2008) 1–19.

6. K. Burns, A. Wilkinson, "On the ergodicity of partially hyperbolic systems", Ann. Math., no.171 (2010) 451–489.

7. K. Burns, C. Pugh, A. Wilkinson, "Stable ergodicity and Anosov flows", Topology no.39 (2000) 149-159.

8. Ph. Didier, "Stability of accessibility", Ergod. Th. Dyn. Syst., v. 23, no.6 (2003) 1717–1731.

9. D. Dolgopyat, A. Wilkinson, "Stable accessibility is $C1$ dense", Asterisque, no.287 (2003) 33–60.

10. M. Grayson, C. Pugh, M. Shub, "Stably ergodic diffeomorphisms", Ann. Math., no.140 (1994), 295–329.

11. M. Hirsch, C. Pugh, M. Shub, "Invariant manifolds", Lect. Notes in Math., no.583, Springer-Verlag, 1977.

12. C. Pugh, M. Shub, "Stably ergodic dynamical systems and partial hyperbolicity", J. of Complexity no.13 (1997) 125–179.

13. C. Pugh, M. Shub, "Stable ergodicity and julienne quasi-conformality", JEMS, no.2 (2000) 1–52.

14. F. Rodriguez Hertz, "Stable ergodicity of certain linear automorphisms of the torus", Ann. Math., v. 162, no.1 (2005) 65–107.

15. F. Rodriguez Hertz, M. Rodriguez Hertz, R. Ures, Accessibility and stable ergodicity for partially hyperbolic diffeomorphisms with 1d-center bundle", Invent. Math., no.172 (2008) 353–381

16. M. Shub, A. Wilkinson, "Pathological foliations and removable zero exponents", Invent. Math. (2001) 495-508.

17. A. Wilkinson, "Stable ergodicity of the time-one map of a geodesic flow", Ergod. Th. Dyn. Syst. v. 18 no.6 (1998) 1545-1588

Analysis on laminations by Riemann Surfaces

by Nessim Sibony

Abstract. Consider the polynomial differential equation in $\mathbb{C}^2$

$$\frac{dz}{dt} = P(z,w), \qquad \frac{dw}{dt} = Q(z,w).$$

The polynomials P and Q are holomorphic, the time is complex. We want to study the global behavior of the solutions. It is convenient to consider the extension as a foliation in the projective plane $\mathbb{P}^2$. There are however singular points. When the line at infinity is invariant, Il'yashenko has shown that generically leaves are dense and that the foliation is ergodic. This follows from the study of the holonomy on the invariant line. But generically on the vector field, there is no invariant line and even no invariant algebraic curve as shown by Jouanolou. This example is a special case of a lamination (with singularities) by Riemann surfaces. In particular, one can consider similar questions in any number of dimensions.

In order to understand their dynamics we will need some analysis on such objects. We will discuss the following topics.

1. Singularities of holomorphic foliations, linearization problems. Algebraic solutions. Examples.

2. Holonomy, closed directed currents, harmonic currents ($\partial\overline{\partial}$-closed currents) directed by the lamination. Intersection Theory in dimension 2.

3. Heat equation on a lamination. The directed $\partial\overline{\partial}$-closed current replaces the manifold and we solve the heat equation with respect to the current. This is useful because in presence of singularities, the leaves are not of bounded geometry.

4. Geometric ergodic theorems. For compact Riemann surface laminations with singularities we get an ergodic theorem with more geometric flavor than the ones associated to a diffusion. Let $(X, \mathcal{L}, E)$ be a lamination by Riemann surfaces. Assume for simplicity that the singularity set E of $\mathcal{L}$ is a finite set of points, then directed $\partial\overline{\partial}$-closed currents exist.

Every hyperbolic leaf L is covered by the unit disc $\mathbb{D}$. Let $\phi_a : \mathbb{D} \to L_a$ denote a universal covering map of the leaf L_a passing through a with $\phi_a(0) = a$. We consider the associated measure

$$m_{a,R} := \frac{1}{M_R}(\phi_a)_*\big(\log^+ \frac{r}{|\zeta|}\omega_P\big) \quad \text{with} \quad R := \log\frac{1+r}{1-r}$$

which is obtained by averaging until "hyperbolic time" R along the leaves. Here, M_R is a constant to normalize the mass. Recall that ω_P denotes the Poincaré metric on $\mathbb{D}$ and also on the leaves of X.

Theorem. (Dinh-Nguyên-Sibony [2]) *Let* $(X, \mathcal{L}, E)$ *be a compact lamination with linearizable isolated singularities in a complex manifold* M *and* ω_P *the Poincaré metric on the leaves. Let* T *be an extremal positive harmonic current of Poincaré mass* 1 *on* $(X, \mathcal{L}, E)$ *without mass on the union of parabolic leaves. Then for almost every point* $a \in X$ *with respect to the measure* $m_P := T \wedge \omega_P$*, the measure* $m_{a,R}$ *defined above converges to* m_P *when* $R \to \infty$.

For holomorphic foliations in the projective plane $\mathbb{P}^2$, with only hyperbolic singularities and without algebraic leaves (this is the generic case) it was shown by J.-E. Fornaess and the author in [6] that one has unique ergodicity, i.e., there is a unique $\partial\overline{\partial}$-closed current, of mass one, directed by the foliation. So the weighted averages considered above converge and have a unique limit.

5. Entropy of Hyperbolic foliations. A notion of entropy for foliations (without singularities) with leaves of dimension l, was introduced by Ghys-Langevin-Walczak in [9]. I will discuss a notion of entropy for hyperbolic foliations by Riemann surfaces $(X, \mathcal{L}, E)$ in a complex manifold M (using hyperbolic time). A compact hyperbolic lamination transversally of class $\mathcal{C}^{2+\delta}$ has finite entropy. When $\dim_{\mathbb{C}} M = 2$, a compact hyperbolic lamination transversally of class $\mathcal{C}^{2+\delta}$ with hyperbolic singularities has finite entropy. In particular, a generic foliation by Riemann surfaces in $\mathbb{P}^2$ has positive finite entropy. The presence of singular points makes the proof quite delicate. The question is related to the transverse regularity of the Poincaré metric on leaves. This is work in progress with T.-C. Dinh and V.-A. Nguyên [3].

Most of the results are based on joint works with J.-E. Fornaess, T.-C. Dinh and V.-A. Nguyên. We give below some references.

References

[1] Candel A. and Conlon L., *Foliations. II.* Graduate Studies in Mathematics, **60**. American Mathematical Society, Providence, RI, 2003.

[2] Dinh T.-C., Nguyên V.-A. and Sibony N., Heat equation and ergodic theorems for Riemann surface laminations, *preprint*, 2010. `arXiv:1004.3931.`

[3] —–, Entropy for hyperbolic Riemann surface laminations, preprint, 2011.

[4] Fornæss J.-E. and Sibony N., Harmonic currents of finite energy and laminations, *Geom. Funct. Anal.*, **15** (2005), no. 5, 962-1003.

[5] —–, Riemann surface laminations with singularities, *J. Geom. Anal.*, **18** (2008), no. 2, 400-442.

[6] —–, Unique ergodicity of harmonic currents on singular foliations of $\mathbb{P}^2$, *Geom. Funct. Anal.*, **19** (2010), 1334-1377.

[7] Fornæss J.-E., Sibony N. and Wold E.-F., Examples of minimal laminations and associated currents, *preprint*, 2010. `arXiv:1002.2779.`

[8] Ghys É., Laminations par surfaces de Riemann, *Dynamique et géométrie complexes (Lyon, 1997)*, *Panor. Synthèses*, **8**, Soc. Math. France, Paris, 1999.

[9] Ghys É., Langevin R. and Walczak P., Entropie géométrique des feuilletages, *Acta Math.* **160** (1988), no. 1-2, 105-142.

[10] Ilyashenko Y. and Yakovenko S., *Lectures on analytic differential equations*, Graduate Studies in Math., **86**, A.M.S., 2008.

[11] Sullivan D., Cycles for the dynamical study of foliated manifolds and complex manifolds, *Invent. Math.*, **36** (1976), 225-255.

Analytic techniques of constructing holomorphic sections of line bundles

by Yum-Tong Siu

Abstract. Problems of algebraic and complex geometry such as the effective Matsusaka big theorem, the Fujita conjecture, the deformational invariance of plurigenera, the finite generation of the canonical ring, and the abundance conjecture all hinge on the construction of holomorphic sections of line bundles satisfying certain prescribed conditions. This series of three lectures deals with the following analytic techniques of constructing holomorphic sections of line bundles and their applications to the above problems.

(i) The vanishing theorem with L^2 estimates of $\bar\partial$ (from the completion of squares in a quadratic polynomial with symbols of differential operators as variables, Morrey's technique of partial integration-by-parts to handle the boundary term, and the smoothing of nonnegatively curved metric by holomorphic vector fields on Stein subdomains).

(ii) The theorem of Riemann-Roch (in particular, in the asymptotic formulation) to compare the number of variables and the number of linearly independent equations to obtain nontrivial solutions of a system of linear equations.

(iii) Extension and interpolation results for holomorphic sections with given values on hypersurfaces by means of the hyperbolic geometry of the punctured disk.

(iv) The relation between nonnegatively curved metrics and multivalued holomorphic sections. Construction of holomorphic sections for deformational invariance of plurigenera from the addition of one more copy of the canonical line bundle in the direction of going from metrics to sections.

(v) The variation of Hodge structure and the second fundamental form of a holomorphic subbundle of a flat bundle with indefinite flat metric.

(vi) Direct construction of sections, up to a flat bundle, from curvature current of a nonnegatively curved metric of minimal singularity and the arithmetic techniques of Gelfond-Schneider, Lang, Bombieri, Briskorn, and Simpson to handle the flat bundle.

(vii) Integral values for the numerical Kodaira dimension from fibrations defined by asymptotic behavior of Lelong numbers and from the the exterior products of copies of the curvature current of a nonnegatively curved metric of minimal singularity.

Lyapounov exponents of the Hodge bundle with respect to the Teichmuller geodesic flow

by Anton Zorich

Abstract. I plan to speak about dynamics of the Teichmuller geodesic flow in the moduli space. The main goal is to present the recent result obtained jointly with A. Eskin and with M. Kontsevich on the explicit formula for the sum of the Lyapunov exponents.

The plan is to use the first two lectures for a short survey of the Teichmuller dynamics, formulate the principal results and discuss the applications. The last lecture would be used to tell some words on the key technical ingredients of the computation of the Lyapunov exponents: on comparison of the determinants of the flat and hyperbolic Laplacians on a Riemann surface, and on the geometric compactification of the moduli space of quadratic differentials.

Participants

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Youssef Barkatou
Eric Bedford
Yves Benoist
Pierre Berger
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Cinzia Bisi
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Corentin Boissy
Araceli Bonifant
Dan Burns
Jérôme Buzzi
Serge Cantat
Marc Chaperon
Dan Coman
Sylvain Crovisier
Xiaojun Cui
Bernardo da Costa
Lionel Darondeau
Sandrine Daurat
Jean-Pierre Demailly
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