Random Operators and Anderson model in dimension 1.

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There won't be notes available for this course but a bibliography is provided.

Presentation

In this course, we present techniques and results of the spectral theory of random operators which are specific to the dimension 1. We study the Anderson model which describes electronic transport in random media and the associated phenomenon of Anderson localization. Mathematically, Anderson localization is characterized by pure point spectrum and generalized eigenfunctions which decrease exponentially fast to 0 at infinity.

By focusing on the dimension 1 case, one can use tools from the dynamical systems theory to characterize the behaviour of the eigenfunctions. In particular, we use the formalism of transfer matrices and Lyapunov exponents. These exponents also provides a description of the absolutely continuous spectrum through Kotani theory.

This course combines well with the course of Frédéric Klopp, « Théorie Spectrale des opérateurs aléatoires » from the Master 2 « Mathématiques Fondamentales » of Pierre et Marie Curie University.

Contents

1. Basic theory of self-adjoints operators, spectral theorem and spectral types.
2. R.A.G.E theorem, spectral types and dynamics.
3. Discrete and continuous Anderson models, Anderson localization.
4. Random operators, ergodicity, almost-sure spectrum.
5. Tools from dynamical systems to study one-dimensional models: transfer matrices and Lyapunov exponents. Simplicity criteria for the Lyapunov spectrum and application to the discrete / continuous and scalar-valued / matrix-valued one dimensional Anderson models.
6. Kotani theory : absolutely continuous spectrum and Lyapunov exponents. Absence of diffusion.
7. Ideas of the proof of Anderson localization in dimension one: regularity of the Lyapunov exponents, Integrated Density of States ; Wegner estimates and Multi-Scale Analysis.

Prerequisites

Real and complex analysis, functional analysis. Measure theory and basic notions of probability theory.

Bibliography

- RENE CARMONA, JEAN LACROIX. Spectral Theory of Random Schrödinger Operators. *Probability and Its Applications. Birkhäuser, Boston, 1990.*
- WERNER KIRSCH. An invitation to random Schrödinger operators. In Random Schroedinger operators. *volume 25 de Panor. Syntheses, pages 1-119. Soc. Math. France, Paris, 2008.*
- MICHAEL REED, BARRY SIMON. Methods of Modern Mathematical Physics IV: Analysis of operators. *Academic Press, New York, 1978.*