Calculus of Variations and Minimization

The main focus of this course is that part of the Calculus of Variations that deals with the minimization of integral type functionals. Thus equally important topics like critical points or optimization will not be discussed.

Rather than emphasize the most general concepts, we will essentially limit our investigations to the rather ubiquitous case of energies with integrands that only depend on the gradient of scalar or vector-valued test functions.

We will first study lower semi-continuity as the basic tool for the direct method of the Calculus of Variations. We will then address convexity and duality which are at the root of the existence theory for minimizers in the scalar-valued case, or still quasi-convexity, the vectorial counterpart.

The next topic will be relaxation which palliates the lack of lower semi-continuity, resulting in scalar convexification and vectorial quasi-convexification.

Finally the students will choose between the following two topics: regularity for minimizers, or Gamma-convergence for the study of parameter-dependent functionals (with periodic homogenization as main application).

This course will complement (or be complemented by) that entitled “Partial Differential Equations”. The only prerequisite is some familiarity with classical Sobolev spaces.

References:

Andrea Braides, Anneliese Defranceschi: Homogenization of Multiple Integrals, Oxford Lecture Series in Mathematics and its Applications, vol.12, Oxford Press, 1998.

Gianni Dal Maso: An Introduction to Gamma-convergence, Progress in NonlinearDifferential Equations and Applications, vol. 8, Birkhaüser, 1993.

Mariano Giaquinta: Multiple Integrals in the Calculus of Variations and Nonlinear Elliptic Systems, Annals of Mathematics Studies, vol. 105,
Princeton University Press, 1983.