

Fundamental course

Square-tiled surfaces

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No course notes are planned.

Présentation

The dynamics in moduli spaces of translation surfaces is a research topic with some remarkable recent applications: for instance, using this theory, Delecroix-Hubert-Lelièvre confirmed in 2014 a conjecture from 1980 by two physicists – Hardy and Weber – on the abnormal rate of diffusion of typical trajectories in Ehrenfest’s wind-tree model of Lorentz gases (from Statistical Mechanics/Thermodynamics).

An interesting particular case of the results by Delecroix-Hubert-Lelièvre concerns Ehrenfest’s wind-tree model with rectangular obstacles whose dimensions are rational: indeed, the underlying translation surfaces are square-tiled surfaces in this context, so that the proof of Delecroix-Hubert-Lelièvre theorem (solving Hardy-Weber conjecture) is simpler in this setting.

This course is oriented towards the discussion of basic aspects of square-tiled surfaces: how are they defined? how to understand their homology groups? what are their applications? etc.

Content

- Square-tiled surfaces:
 - Geometric and combinatorial definitions, Examples: flat torus, L-shaped surfaces, ...
 - Automorphisms and Veech group of a square-tiled surface;
 - $SL(2, \mathbb{R})$ -orbite of a square-tiled surface;
 - Kontsevich-Zorich cocycle;
 - Volumes of moduli spaces and counting square-tiled surfaces.
- Concrete examples of square-tiled surfaces:
 - Hubert–Lelièvre and McMullen classification of square-tiled surfaces in $\mathcal{H}(2)$;
 - Some remarkable square-tiled surfaces (Eierlegende Wollmilchsau, Ornithorynque, regular origamis, etc.);
- Representations of finite groups and homology of square-tiled surfaces.

Pre-requisites

Measure theory, Real Analysis

Bibliography

- G. FORNI and C. MATHEUS, *Introduction to Teichmüller theory and its applications to dynamics of interval exchange transformations, flows on surfaces and billiards*, Journal of Modern Dynamics, vol. 8, no. 3/4, p. 271–436 (2014).