# Generalized Shallow Water Model Framework

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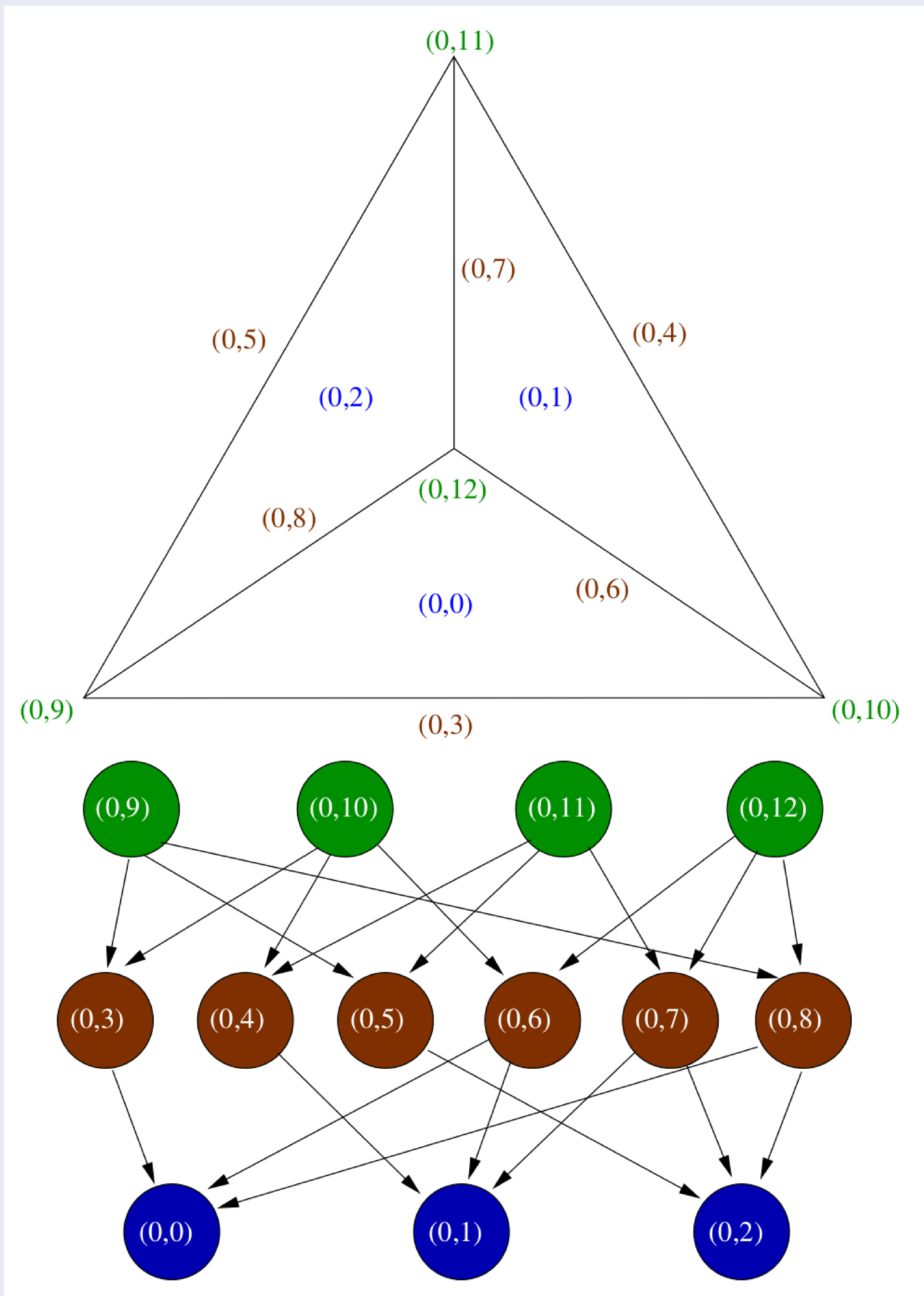
## Abstract


- The generalized shallow water model framework (GSWMF) is a generalized framework for the development and testing of numerical schemes intended for use in the dynamical cores of atmospheric models.
- Results are presented from two different schemes (Ringler, Thuburn, Klemp & Skamarock 2010 and Randall & Heikes 1995).


## GSWMF: Design

- Horizontal meshes represented as directed acyclic graphs via Sieve (see Knepley & Karpeev 2009)
- Variables (scalar, vector, array, vector component) placed arbitrarily on mesh elements
- User-provided functions define operators on these variables
- Compile and run-time polymorphism provide flexibility

Figure: Sieve representation of a mesh (from Knepley & Karpeev 2009)




## GSWMF: Implementation

- Models are written in Fortran 90 using Cheetah for compile time polymorphism; code is heavily shared between linear and non-linear models
- Uses PETSc and SLEPc to provide grid management, linear/eigenvalue solvers and I/O (see references)
- Analysis packages are written in Python using the Numpy, Scipy and Matplotlib libraries
- Currently perfect planar square, triangular and hexagonal meshes are implemented (except for hexagonal/triangular Fourier transforms)
- Adams-Bashford and Runge-Kutta explicit time stepping
- TRiSK and Randall & Heikes horizontal discretizations

## Results: Numerical Dispersion Relationships

- Dispersion relationship calculated as $\frac{d\vec{x}}{dt} = \mathbb{L}\vec{x} \rightarrow i\omega\vec{x} = \mathbf{A}\vec{x}$ (eigenvalue problem)
- Well resolved Rossby radii ($\frac{\lambda}{d} = 2.0$) and poorly resolved Rossby radii ($\frac{\lambda}{d} = 0.1$) tested
- Both C-grid (50x50) and Z-grid (18x18 and 45x45) perfect planar square meshes investigated
- Results are identical to theoretical dispersion relations (within numerical bounds)
- Spatial Fourier Transform used to determine which spatial wavenumbers each eigenvector/eigenvalue pair is associated with
- Framework can handle ANY spatial discretization that can be expressed as the above eigenvalue problem

Figure: C-Grid dispersion relations for $\frac{\lambda}{d} = 2.0$ (top) and $\frac{\lambda}{d} = 0.1$ (bottom) where $\omega = \frac{\sigma}{f}$

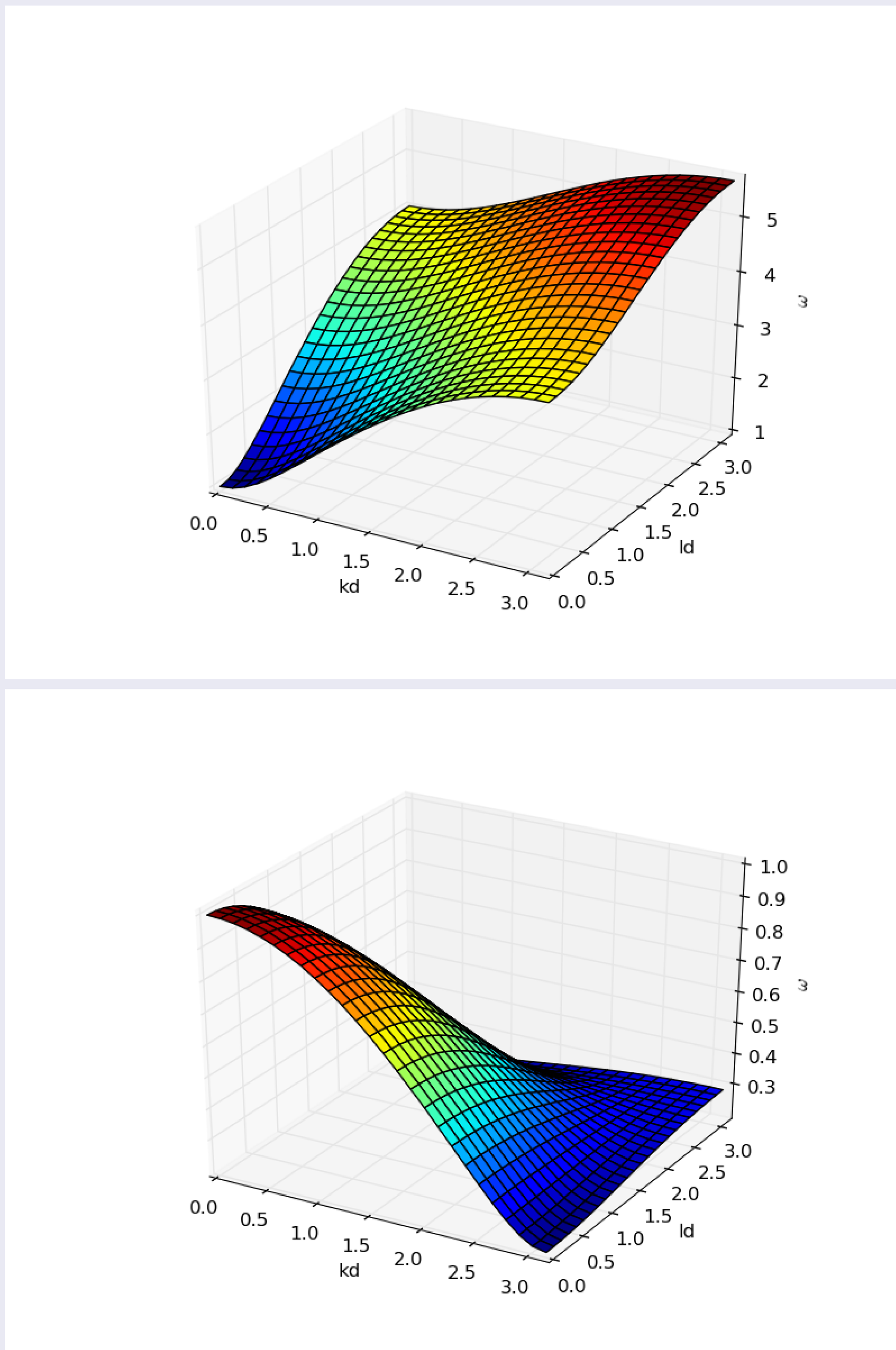
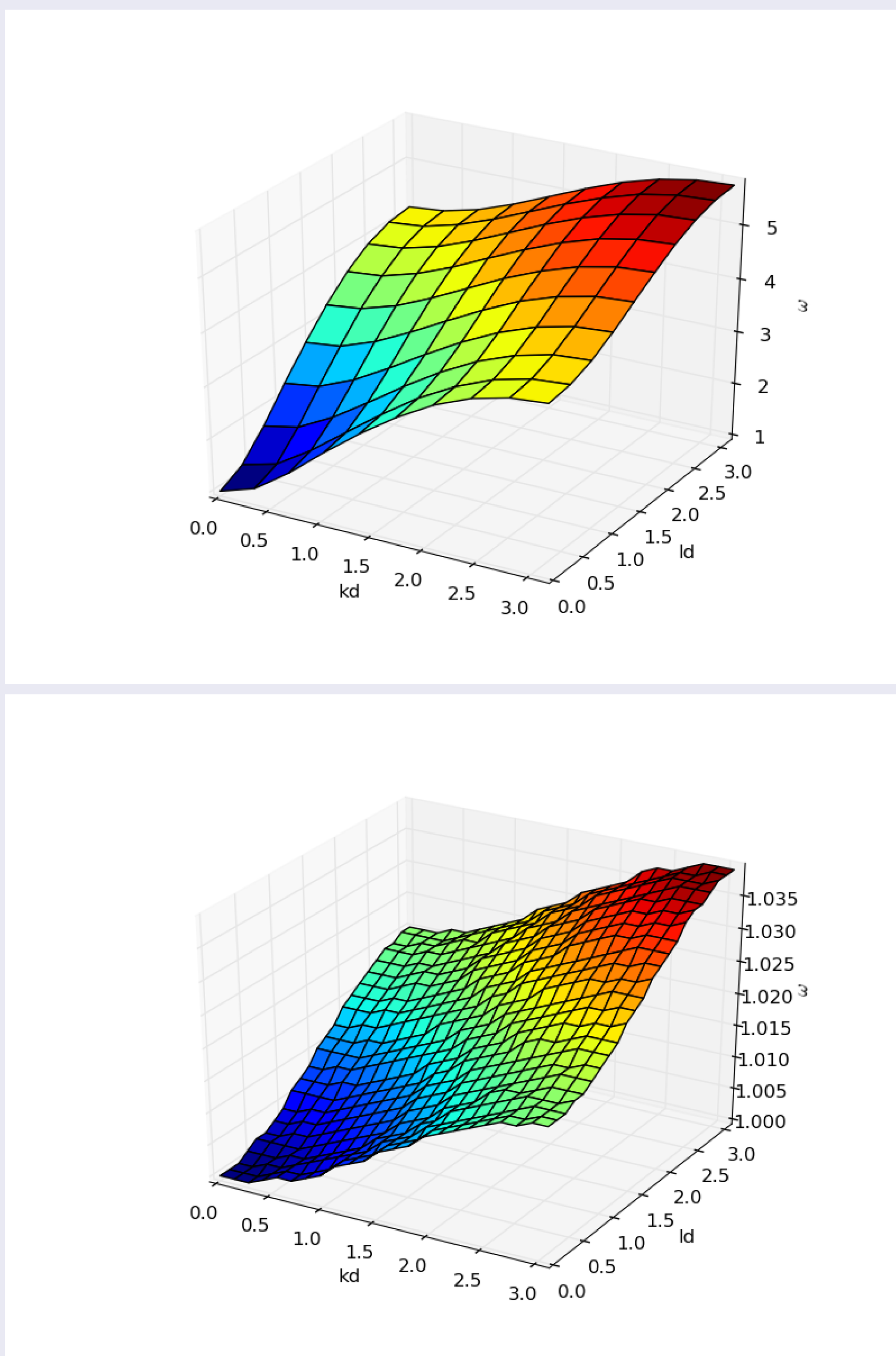


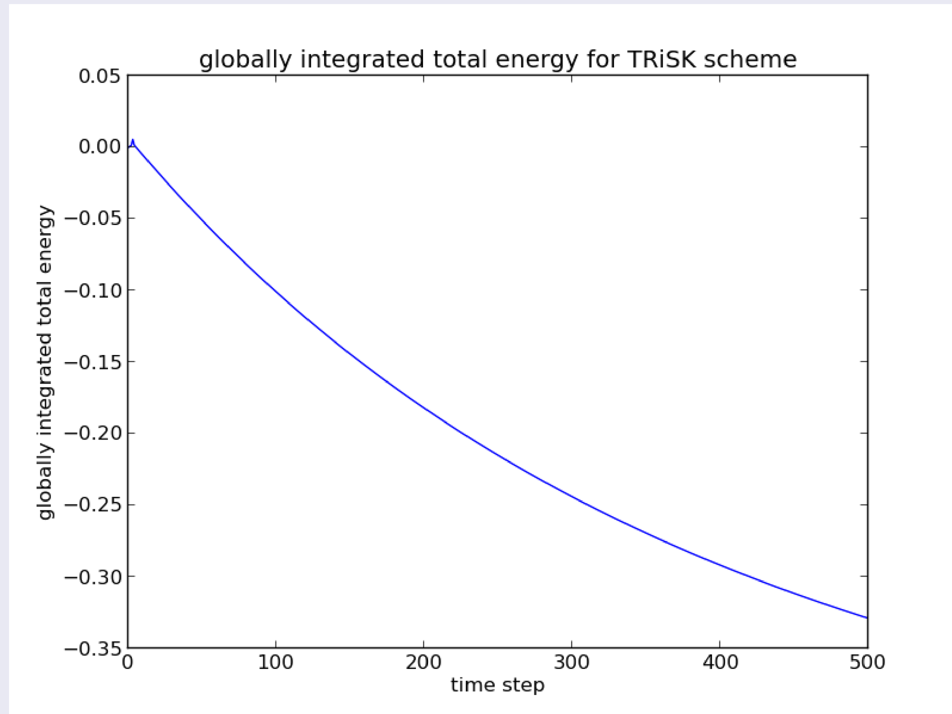

Figure: Z-Grid dispersion relations for $\frac{\lambda}{d} = 2.0$ (top) and $\frac{\lambda}{d} = 0.1$ (bottom) where $\omega = \frac{\sigma}{f}$




## Results: Nonlinear Shallow Water Equations

- Geostrophic turbulence test case
- Perfect planar square mesh (50x50), 50km grid spacing
- 3rd order Adams-Bashford time stepping, 15s time steps, 500 steps taken
- $g = 9.81ms^{-1}$ , $f = 0.0001s^{-1}$
- $H = 400.0 \pm 50m$ , $\vec{u} = \pm 0.5m/s$

Figure: Percentage change in energy for the TRiSK scheme




## GSWMF: Extensions

- Parallelization
- Additional horizontal meshes
- Additional discretization schemes
- Additional time stepping schemes
- Further support for analyzing operator matrix properties- symmetry, eigenvalues, rank, nullity, null spaces

## Future Work

- Matrix kernel analysis of horizontal discretization schemes
  - Wave dispersion properties
  - Stationary modes
  - Computational modes
- Projection of non-linear solutions onto (computational) normal modes
- Conservation properties

## Conclusions

- GSWMF provides a useful framework for inter-comparison of various numerical schemes for the nonlinear shallow water equations
- Two very different schemes (Z-grid and C-grid) can be implemented under the same code framework
- Can reproduce previous results for C-grid and Z-grid schemes (see Randall 1994)

## References


- Balay, Brown, e.t. al 2012 (PETSc)
- Hernandez, Roman & Vidal 2005 (SLEPc)
- Knepley & Karpeev 2009 (Sieve)
- Ringler, Thuburn, Klemp & Skamarock 2010 (TRiSK)
- Randall & Heikes 1995
- Randall 1994