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Title of the work

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LES Turbulent shallow water equations

$$\partial_t \mathbf{W} + \partial_x(\mathbf{F}(\mathbf{W}) - \tilde{\mathbf{F}}(\mathbf{W})) + \partial_y(\mathbf{G}(\mathbf{W}) - \tilde{\mathbf{G}}(\mathbf{W})) = \mathbf{S}(\mathbf{W}) + \mathbf{Q}(\mathbf{W}) \qquad (1)$$

$\mathbf{W}$ represents the conserved variables, $\mathbf{F}$ and $\mathbf{G}$ are the convection tensor fluxes, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{G}}$ are the diffusion tensor fluxes, $\mathbf{S}$ the source terms that model the topography variations and $\mathbf{Q}$ describe the frictions and wind effects.

$$\mathbf{W} = \begin{pmatrix} \bar{h} \\ \bar{h}\tilde{u} \\ \bar{h}\tilde{v} \end{pmatrix}, \mathbf{F}(\mathbf{W}) = \begin{pmatrix} \bar{h}\tilde{u} \\ \bar{h}\tilde{u}^2 + \frac{1}{2}g\bar{h}^2 \\ \bar{h}\tilde{u}\tilde{v} \end{pmatrix}, \mathbf{G}(\mathbf{W}) = \begin{pmatrix} \bar{h}\tilde{v} \\ \bar{h}\tilde{u}\tilde{v} \\ \bar{h}\tilde{v}^2 + \frac{1}{2}g\bar{h}^2 \end{pmatrix}, \mathbf{S}(\mathbf{W}) = \begin{pmatrix} 0 \\ -g\bar{h}\partial_x \mathbf{Z} \\ -g\bar{h}\partial_y \mathbf{Z} \end{pmatrix}$$

$$\tilde{\mathbf{F}}(\mathbf{W}) = \begin{pmatrix} 0 \\ (\nu+\nu_t)\partial_x(\bar{h}\tilde{u}) \\ (\nu+\nu_t)\partial_x(\bar{h}\tilde{v}) \end{pmatrix}, \tilde{\mathbf{G}}(\mathbf{W}) = \begin{pmatrix} 0 \\ (\nu+\nu_t)\partial_y(\bar{h}\tilde{u}) \\ (\nu+\nu_t)\partial_y(\bar{h}\tilde{v}) \end{pmatrix}, \mathbf{Q}(\mathbf{W}) = \begin{pmatrix} 0 \\ -\frac{\tau_{fx}}{\rho} + \frac{\tau_{wx}}{\rho} \\ -\frac{\tau_{fy}}{\rho} + \frac{\tau_{wy}}{\rho} \end{pmatrix}.$$

Method of solution

A finite volume discretization of (**??**) or (**??**) yields

$$\mathbf{W}_i^{n+1} = \mathbf{W}_i^n - \frac{\Delta t}{|\mathcal{T}_i|} \sum_{j\in N(i)} \int_{\Gamma_{ij}} \mathcal{F}(\mathbf{W}^n;\mathbf{n})\, d\sigma + \frac{\Delta t}{|\mathcal{T}_i|} \sum_{j\in N(i)} \int_{\Gamma_{ij}} \tilde{\mathcal{F}}(\mathbf{W}^n;\mathbf{n})\, d\sigma$$
$$+ \frac{\Delta t}{|\mathcal{T}_i|} \int_{\mathcal{T}_i} \mathbf{S}(\mathbf{W}^n)\, dV + \frac{\Delta t}{|\mathcal{T}_i|} \int_{\mathcal{T}_i} \mathbf{Q}(\mathbf{W}^n)\, dV \qquad (3)$$

- **Unstructured Non-homogeneous Riemann Solver**

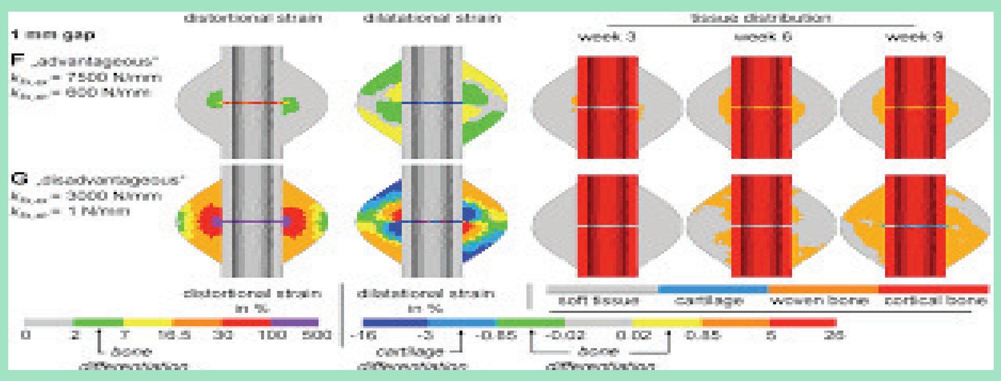
First, the advective system accounting for bed variations is projected into the normal η and the tangential τ on each edge Γ_{ij}, which gives

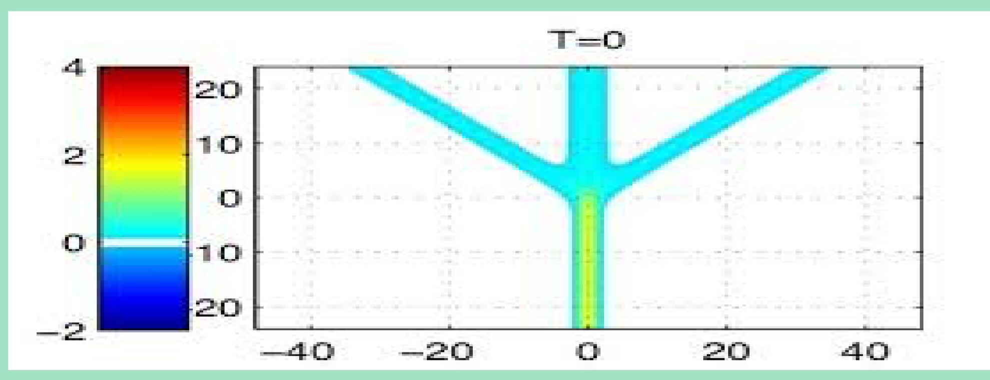
$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}_\eta(\mathbf{U}) \frac{\partial \mathbf{U}}{\partial \eta} = \mathbf{0}. \qquad (4)$$

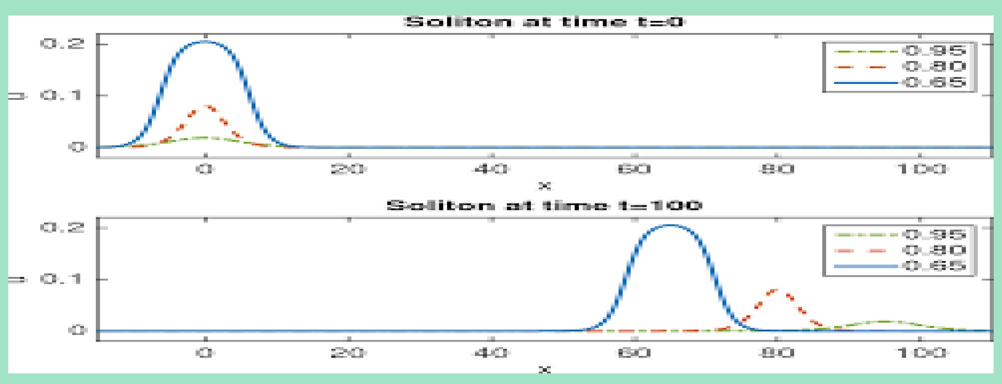
The proposed finite volume scheme consists of a predictor stage and corrector stage as

$$\mathbf{U}_{ij}^n = \frac{1}{2}\left(\mathbf{U}_i^n + \mathbf{U}_j^n\right) - \frac{1}{2}\mathrm{sgn}\left[\mathbf{A}_\eta\left(\bar{\mathbf{U}}\right)\right]\left(\mathbf{U}_j^n - \mathbf{U}_i^n\right),$$
$$\mathbf{W}_i^{n+1} = \mathbf{W}_i^n - \frac{\Delta t}{|\mathcal{T}_i|} \sum_{j\in N(i)} \mathcal{F}\left(\mathbf{W}_{ij}^n;\eta_{ij}\right)|\Gamma_{ij}| + \Delta t \mathbf{S}_i^n, \qquad (5)$$

- **Shallow water flow over porous bed**







Equations for shallow water flows in porous media

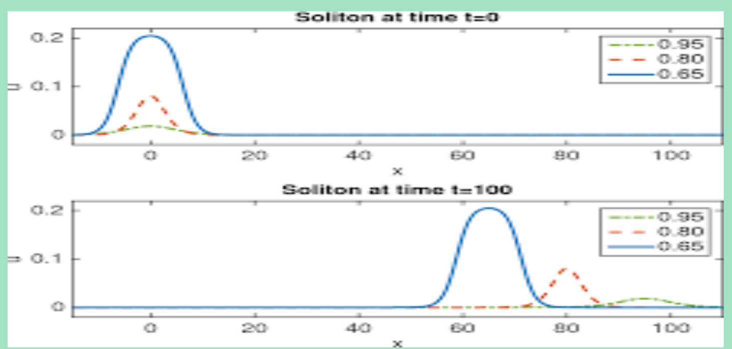
$$\partial_t \mathbf{W} + \partial_x \mathbf{F}(\mathbf{W}) + \partial_y \mathbf{G}(\mathbf{W}) = \mathbf{S}(\mathbf{W}) + \mathbf{Q}(\mathbf{W}) \qquad (2)$$

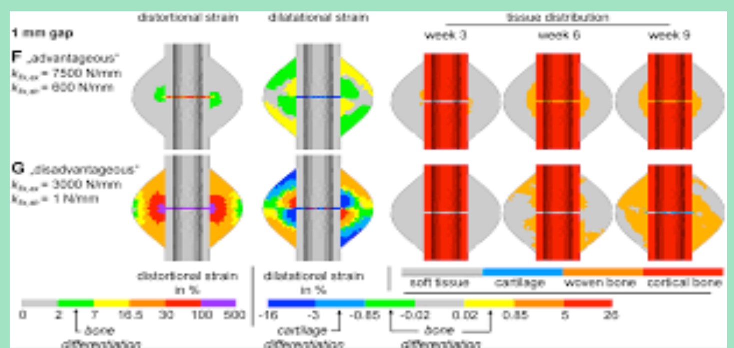
In these modified equations the porosity $\phi = \phi(x,y)$ accounts for the presence of buildings, structures, etc. that restrict the area available to water flow.

$$\mathbf{W} = \begin{pmatrix} \phi h \\ \phi hu \\ \phi hv \end{pmatrix}, \quad \mathbf{F}(\mathbf{W}) = \begin{pmatrix} \phi hu \\ \phi hu^2 + \frac{1}{2}g\phi h^2 \\ \phi huv \end{pmatrix}, \quad \mathbf{G}(\mathbf{W}) = \begin{pmatrix} \phi hv \\ \phi huv \\ \phi hv^2 + \frac{1}{2}g\phi h^2 \end{pmatrix},$$

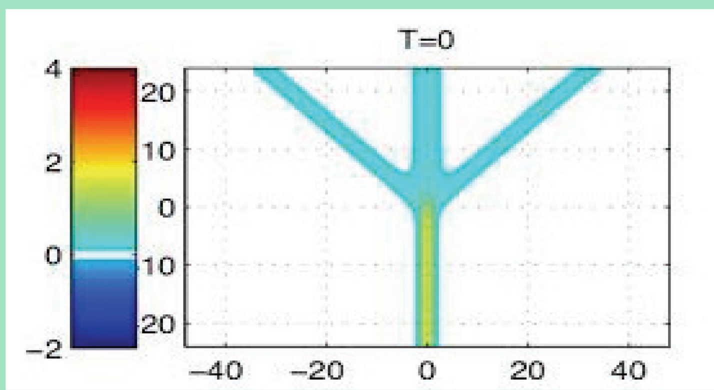
$$\mathbf{S}(\mathbf{W}) = \begin{pmatrix} 0 \\ -g\phi h\partial_x Z + \frac{1}{2}gh^2\partial_x\phi \\ -g\phi h\partial_y Z + \frac{1}{2}gh^2\partial_y\phi \end{pmatrix}, \quad \mathbf{Q}(\mathbf{W}) = \begin{pmatrix} 0 \\ -\phi\frac{\tau_{fx}}{\rho} - \frac{\tau_{dx}}{\rho} \\ -\phi\frac{\tau_{fy}}{\rho} - \frac{\tau_{dy}}{\rho} \end{pmatrix}.$$

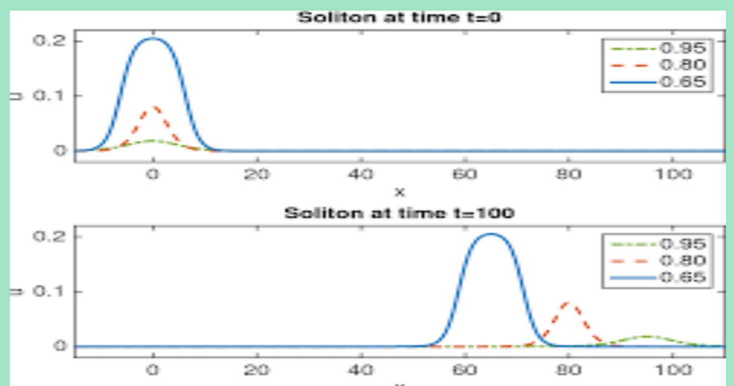
- **Turbulent shallow water flow around a cylinder**





- **Turbulent shallow water flow in a channel**





Conclusions

- Extension of the solver to coupled models of shallow water flows in porous media and bed load transport in viscous shallow water flows.
- Development of mesh adaptation techniques for the solution of shallow water equations with porosity.
- Treatment of flows over dry areas $\Longrightarrow$ Modification on the SRNH.

Perspectives

- Extension of the solver to coupled models of shallow water flows in porous media and bed load transport in viscous shallow water flows.
- Development of mesh adaptation techniques for the solution of shallow water equations with porosity.
- Treatment of flows over dry areas $\Longrightarrow$ Modification on the SRNH.

References

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3. L. Cea, M.E. Vazquez-Cendon, Unstructured finite volume discretization of two-dimensional depth-averaged shallow water equations with porosity, Int. J. Numer. Meth. Fluids, 63 903 – 930 2010.