# Implicit time-advancing applied to shallow-water problems with mobile bed

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The simulation of sediment transport is considered. The problem is modeled by means of the shallow-water equations coupled with a sediment transport equation for the morphodynamic; for this specific application we use the Exner equation and the Grass model (see e.g. [1]).
In Shallow-Water simulations, the governing equations are usually advanced in time by explicit schemes. However, if the interaction with the mobile bed is weak, the characteristic time scales of the flow and of the sediment transport can be very different introducing time stiffness in the global problem. For these case, it is of great interest to use implicit schemes. Implicit schemes might also be useful if morphodynamic models more complex than the Exner one are used, which lead to a more stiff evolution of the bed (see e.g. [1]). The aim of the present paper is to investigate the behavior of implicit linearized schemes in this context.
To discretize the equations in space we considered two different numerical schemes, both of them based on a finite-volume approach. We refer to [2-3] for details on the numerical schemes.
Starting from an explicit time-advancing algorithm, a corresponding implicit linearized scheme is derived by computing the Jacobian of the finite volume fluxes using an automatic differentiation tool [4]. Second order accuracy in time is obtained through the defect-correction technique [5].
1D problems are first considered. The results obtained by means of the explicit and implicit time-advancing schemes have been compared, both in terms of accuracy and efficiency for different types of flow/bed interaction. It is shown that the implicit scheme is cost-effective for intermediate and weak flow/bed interaction and, as a consequence, it turns out to be a good candidate to simulate flows with sediment transport. Based on these results, we extended the application and the analysis to a 2D standard test-case [2-3], which is also the benchmark #3 proposed for this workshop. The results are again evaluated in terms of accuracy, robustness and computational cost.

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