Matrix sign function and Roe scheme. Application to the bilayer shallow water system with sediment transport

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Abstract:

In this work a model defined by the bilayer shallow water system coupled with a sediment transport equation (see [1], [2], [3], [5]) is presented. This model is a particular case of a general class of hyperbolic systems of conservations laws with coupled and source terms and could be re-written under the form 1D hyperbolic systems in non-conservative form:

$$\frac{\partial W}{\partial t} + \mathcal{A}(W)\frac{\partial W}{\partial x} = 0, \quad x \in \mathbf{R},\ t > 0. \tag{1}$$

The non-conservative product $\mathcal{A}(W)W_x$ makes difficult the definition of weak solutions for this kind of systems. After the theory developed by Dal Maso, LeFloch and Murat, a definition of non-conservative products as Borel measures is introduced, which is based on the selection of a family of paths in the phases space.

As it is well-known, the presence of source or coupling terms can affect the quality of the results obtained when steady or nearly steady state solutions are approximated. To handle such problems, the concept of *well-balanced* schemes, that is, schemes that preserves equilibria to some extent, has been considered by several authors. Parés and Castro (see [4]) have extended the concept of well-balancing to non-conservative systems of the form (1), for which generalized Roe schemes have been developed.

Roe method is defined in terms of the absolute value of Roe matrix. Then, it is neccesary to computes the eigenvalues and eigenvectors of Roe matrix in order to define the absolute value matrix.

In this work we consider a form to rewrite Roe method for non-conservative systems in terms of the sign Roe matrix. And we propose an iterative algorithm that allows to obtain the sign Roe matrix as the limit of the process (see [6]).

References

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