Summer School & Workshop on

September 20 − 24, 2010

Numerical Methods for
Interactions between Sediments & Water

LAGA University Paris 13, Villetaneuse, France

http://www.ijfv.fr/ALL/MeNHydro-eng/

Benchmark 1:
Shallow water system with exact solution

The equations:

The shallow water system

$$\partial_t \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_x \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x Z \end{pmatrix}, \tag{1}$$

has to be solved in the interval $[1, 2]$ with the bed topograhy

$$Z(x) = \frac{1}{x}, \qquad x \in [1, 2], \tag{2}$$

subject to the following initial conditions

$$h(0, x) = 4 - \frac{1}{x}, \qquad u(0, x) = x, \qquad x \in [1, 2], \tag{3}$$

and boundary conditions

$$h(t, 1) = \frac{4}{1+t} - 1, \quad u(t, 1) = \frac{1}{1+t}, \quad h(t, 2) = \frac{4}{1+t} - \frac{1}{2}, \quad u(t, 2) = \frac{2}{1+t}. \tag{4}$$

The exact solution:

It is easy to verify that the above sytem of equations (1)-(4) have an exact solution given by

$$\bar{h}(t, x) = \frac{4}{1+t} - \frac{1}{x}, \qquad \bar{u}(t, x) = \frac{x}{1+t}. \tag{5}$$

The results to be presented:

The relative errors for the water hieght and velocity are defined by

$$err_h = \frac{\left| h_i^n - \bar{h}(1, x_i) \right|}{\left| \bar{h}(1, x_i) \right|}, \qquad err_u = \frac{\left| u_i^n - \bar{u}(1, x_i) \right|}{\left| \bar{u}(1, x_i) \right|}.$$

One should tabulate the L^∞, L^1 and L^2 norms of the relative errors at final time $t = 1$ using different meshes with 25, 50, 100, 200, 400 and 800 gridpoints.