Special functions, transcendence and periods

London–Paris Number Theory seminar, 4-5th November 2024

Monday 4th November (Jussieu, room 104, towers 15-25)

14.00 Peter Jossen (King's College)

Periods, G-functions and E-functions

A gentle introduction and overview on the general theory of G-functions and E-functions, their associated differential equations and their relation to algebraic geometry. I'll give some elementary examples of extraordinary relations among values of G-functions.

15.00 Coffee break

15.30 <u>Martin Orr</u> (Manchester):

Effective height bounds for very unlikely intersections in abelian varieties

André and Bombieri's Hasse principle for relations between values of G-functions can be used to obtain theoretically effective height bounds for certain points in curves. In this talk, I will discuss how this can be applied to very unlikely intersections in an abelian variety (i.e. intersections between the curve and algebraic subgroups of large codimension).

16.45 <u>Chris Daw</u> (Reading):

Large Galois orbits under multiplicative degeneration

The Pila-Zannier strategy is a powerful technique for proving results in unlikely intersections. In this talk, I will recall the Zilber-Pink conjecture for Shimura varieties and describe how Pila-Zannier works in this setting. I will highlight the most difficult outstanding obstacle to implementing the strategy — the so-called Large Galois Orbits conjecture — and I will explain recent progress towards this conjecture, building on the works of André and Bombieri. This is joint with Martin Orr (Manchester).

Tuesday 5th November (Jussieu, room 305, towers 14-24)

 $9.30 \ Coffee$

10.00 Lucia Di Vizio (CNRS, Versailles)

Combinatorics and functional equations

11.15 Tanguy Rivoal (CNRS, Grenoble)

Diophantine approximations to values of *E*-functions

I will present various results recently obtained with Stéphane Fischler (Université Paris Saclay) concerning the Diophantine properties of values taken by E-functions at algebraic arguments. For instance, these values are not Liouville numbers, and when the E-functions have rational Taylor coefficients, their irrational values taken at rational arguments have irrationality exponent equal to 2. I will also explain why the values taken at algebraic arguments by the logarithm of a given E-function are transcendental numbers, except for possibly finitely many arguments, and I will present an irrationality measure in the rational case. Hints for the proofs will be given in each situation.<u>David Urbanik</u> (IHES)

14.30 David Urbanik (IHES)

Geometric Realizations of G-functions and Applications

We explain how to construct G-functions from degenerations of projective algebraic varieties. Our constructions give geometric interpretations of evaluations of G-functions both complex analytically and p-adically. We then explain how this can be used to understand and control special moduli, and in particular verify that CM moduli are integral for certain families of motives of non-Shimura type.