Abstracts of the March 27, 2025 session of the Paris-London Analysis Seminar

Louis Ioos (CY Cergy University)

Looking for canonical Kähler metrics through balanced projective embeddings

Abstract. The search for canonical Kähler metrics on projective manifolds is an attempt to extend the uniformization theorem for Riemann surfaces to general dimensions. This search has made significant progresses in the last decades, culminating in what is now called the Yau-Tian-Donaldson program. In this talk, I will explain the role played in this program by the notion of balanced projective embeddings, first introduced by Bourguignon, Li and Yau, and show how they apply to the study of Kähler-Ricci solitons.

Laura Monk (University of Bristol)

Typical hyperbolic surfaces have an optimal spectral gap

Abstract. The first non-zero Laplace eigenvalue of a hyperbolic surface, or its spectral gap, measures how well-connected the surface is: surfaces with a large spectral gap are hard to cut in pieces, have a small diameter and fast mixing times. For large hyperbolic surfaces (of large area or large genus g, equivalently), we know that the spectral gap is asymptotically bounded above by $1/4$. The aim of these talks is to present joint work with Nalini Anantharaman, where we prove that most hyperbolic surfaces have a near-optimal spectral gap. That is to say, we prove that, for any $\varepsilon > 0$, the Weil-Petersson probability for a hyperbolic surface of genus g to have a spectral gap greater than $1/4 - \varepsilon$ goes to one as g goes to infinity. This statement is analogous to Alon's 1986 conjecture for regular graphs, proven by Friedman in 2003. I will present our approach, which shares many similarities with Friedman's work, and introduce new tools and ideas that we have developed in order to tackle this problem.

Stéphane Nonnemacher (Université Paris-Saclay, Orsay)

Random eigenstates of the Quantum Cat Map

Abstract. Long standing conjectures in Quantum Chaos concerns the equidistribution and statistical properties of eigenstates of quantized chaotic systems, in the semiclassical/small wavelength regime. On the macroscopic scale, one expects Quantum Unique Ergodicity: all eigenmodes should asymptotically equidistribute across the classically allowed phase space. At the microscopic (or wavelength) scale, the eigenmodes are expected to enjoy the same statistical properties as monochromatic random waves (Berry's random wave conjecture).

So far, results on Berry's conjecture have been obtained for random quasimodes of chaotic systems (e.g. the Laplacian on a closed manifold of negative curvature), localized in certain spectral windows. The reason to consider quasimodes is due to the fact that for these systems, the eigenvalues are expected to be nondegenerate, preventing any freedom in the definition of eigenmodes.
The quantized hyperbolic automorphisms of the 2-torus, also known as "Quantum Cat Maps", form a toy model of quantized chaotic systems, which enjoys atypical properties. In particular, in the semiclassical limit, the model can enjoy "maximally large" spectral multiplicities (in the context of hyperbolic surfaces, such multiplicities would saturate Bérard's bound for the remainder in Weyl's law).
These large multiplicities allow us to consider random eigenbases of the Quantum Cat Map. We prove that, with high probability, those random eigenbases are equidistributed at macroscopic and down to mesoscopic (algebraically small) scales. We also show that the local statistical properties of these random eigenstates converge to those of standard Gaussian random states, which, for this toy model, is the analogue of Berry's random wave model.
This is joint work with Nir Schwartz.

Sebastian van Strien (Imperial College London)
The Thurston algorithm for real entire transcendental post-singularly finite maps

Abstract. Thurston's characterisation theorem gives a necessary and sufficient condition for when a branched covering map of the sphere (for which the orbits of the branch points have finite cardinality) can be realised by a rational map. In spite of progress, an analogous result for entire maps of the complex plane is not yet known. In this talk, I will discuss a somewhat different approach in the setting of real entire maps whose post-singular set is real and has finite cardinality.