Propagation of Coherent States in Quantum Mechanics

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There exists a deep connection between classical mechanics and quantum mechanics, that can be informally stated as follows : "when the Planck constant $\hbar$ is negligible compared to the characteristic parameters of a given quantum system, the latter tends to evolve according to the classical laws". This principle, named the *Bohr correspondence principle*, can be given a rigorous meaning thanks to a branch of mathematics called *semiclassical analysis*, which I will introduce at the beginning of the talk.

The classical-quantum connection becomes even more noticeable when the wave function of the quantum system is a *gaussian coherent state*, that is, a minimiser of the uncertainty principle

$$\Delta_x \Delta_\xi \geqslant \frac{\hbar}{2}. \tag{1}$$

The aim of the talk is to study the Schrödinger equation

$$\begin{cases} i\varepsilon \partial_t \psi(t) = \hat{H}_t \psi(t) \\ \psi(0) = \varphi_z \end{cases} \tag{2}$$

in the semiclassical regime $\varepsilon \to 0$, where the initial state φ_z is a gaussian coherent state. More precisely, assuming that the solution $\psi_z(t)$ of (2) is well-defined, I will explain how to obtain an asymptotic expansion of $\psi_z(t)$ in the ε variable, at any order.

Finally, depending on the remaining time, I will briefly explain how the mathematical objects presented in the talk can be transposed to the framework of Quantum Field Theory, as the first aim of my PhD is to establish a similar result in this context.