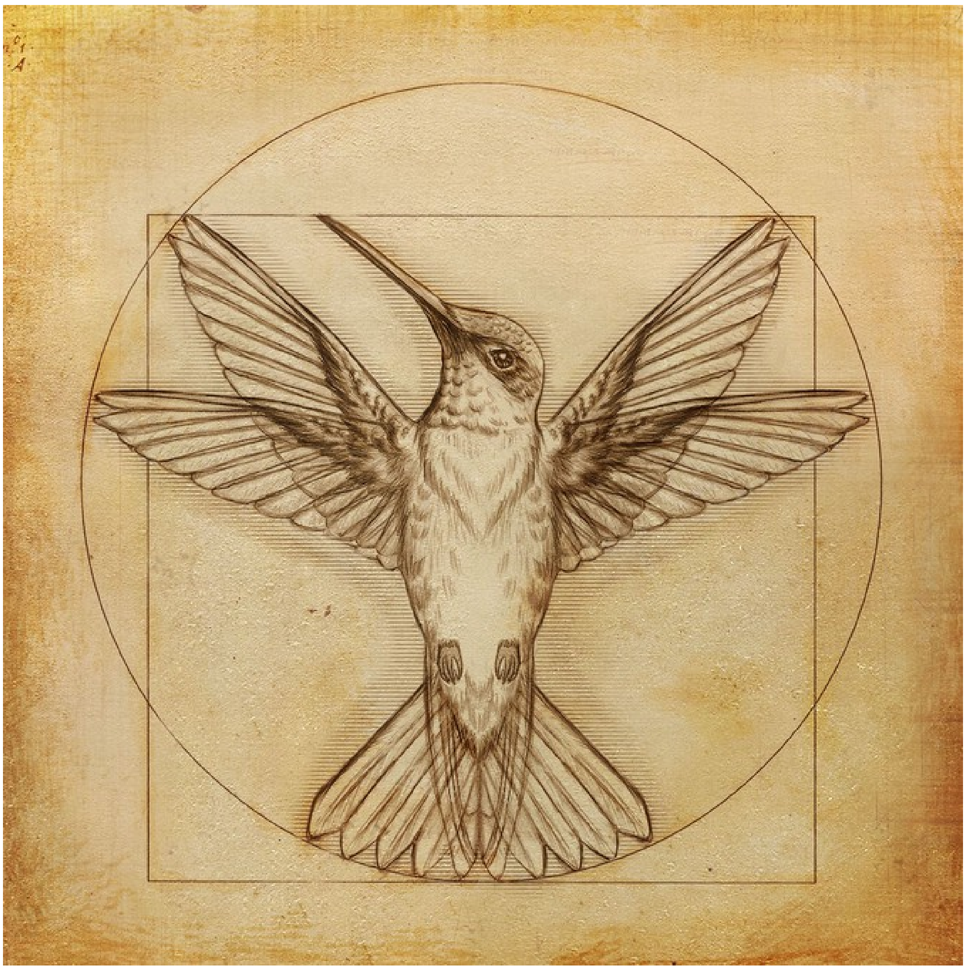
# Workshop on Non-Linear Partial Differential & Integral Equations and Applications

Université Paris Sorbonne Nord



march 19-20, 2020

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# Schedule

| time | march 19 | march 20 |
| --- | --- | --- |
| 9h30-10h00 | Breakfast and opening | Breakfast |
| 10h00-11h00 | DIEKMANN | CARILLO |
| 11h00-11h30 | Coffee break | Coffee break |
| 11h30-12h30 | BILER | CALVEZ |
| 14h00-15h00 | ESCHER | STIEHL |
| 15h00-15h30 | Coffee-Break | Coffee-Break |
| 15h30-16h00 | HAMEL | TOURNUS |
| 16h30-18h30 | free time for discussions | free time for discussions |

# Abstracts

## Large self-similar solutions of the parabolic-elliptic keller-segel model

march 19
11h30
Amphi Euler

Piotr BILER

Wroclaw University

We construct radial self-similar solutions of the, so called, minimal parabolic- elliptic Keller-Segel model in several space dimensions with radial, nonnegative initial conditions with are below the Chandrasekhar solution - the singular stationary solution of this system.

Work in collaboration with Grzegorz KARCH, Hiroshi WAKUI

march 20
11h30
B405

# Concentration dynamics in evolutionary biology

Vincent CALVEZ

Université de Lyon

We consider a non-linear model of quantitative genetics with selection and sexual mode of reproduction following the so-called infinitesimal model. We investigate the asymptotic regime of small variance in the population. We perform a perturbative analysis analogous to WKB expansion to capture the limit profile and the first order correction in the limit of small variance.

## Nonlinear Aggregation-Diffusion Equations : Stationary States, Functional inequalities & Stabilization

march 20
10h00
B405

José Antonio CARRILLO
Imperial College, London

We analyse under which conditions equilibration between two competing effects, repulsion modelled by nonlinear diffusion and attraction modelled by non-local interaction,occurs. I will discuss several regimes that appear in aggregation diffusion problems with homogeneous kernels. I will first concentrate in the fair competition case distinguishing among porous medium like cases and fast diffusion like ones. I will discuss the main qualitative properties in terms of stationary states and minimizers of the free energies.In particular, all the porous medium cases are critical while the fast diffusion are not,and they are characterized by functional inequalites related to Hardy-Littlewood-Sobolev inequalities. In the second part, I will discuss the diffusion dominated case in which this balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrisation and mass transportation techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the particular case of Newtonian interaction in two dimensions they lead to uniqueness of equilibria for any given mass up to translation and to the convergence of solutions of the associated nonlinear aggregation-diffusion equations towards this unique equilibrium profile up to translations as time tends to infinity.

This talk is based on works in collaboration with V. Calvez, M. Delgadino, J.Dolbeault, R. Frank, S. Hittmeir, F. Hoffmann, F. Patacchini, B. Volzone, and Y. Yao.

march 19
10h00
Amphi Euler

# Numerical bifurcation analysis of physiologically structured population models via pseudospectral approximation

Odo DIEKMANN

Utrecht University

As structured population models lead to infinite dimensional dynamical systems, there exist no well-tested tools for their numerical bifurcation analysis. By way of polynomial approximation of the functions that describe the population state, one can reduce to a system of ODE for which such tools are readily available. Deterministic (at the population level) physiologically structured population models can either be formulated as delay equations or as first order partial differential equations (often with the birth of new individuals described by a boundary condition). The aim of this lecture is to explain for both formulations the main ideas of pseudospectral approximation in the context of relevant examples and to highlight the potential of combining it with ODE numerical bifurcation tools.

Based on joint work with Dimitri Breda, Mats Gyllenberg, Francesca Scarabel, Rossana Vermiglio and Babette de Wolff.

## On mathematical models for dissipative microelectromechanical systems

march 19
14h00
Amphi Euler

Joachim ESCHER

Hannover University

A review of some recent results on mathematical models for microelectromechanical systems with general permittivity profile will be presented. These models consist of a quasilinear parabolic evolution problem for the displacement of an elastic membrane coupled to an elliptic moving boundary problem that determines the electrostatic potential in the region between the elastic membrane and a rigid ground plate. Results on local well-posedness, global existence, the occurrence of finite-time singularities, and convergence of solutions to those of the so-called small-aspect ratio model, respectively, are presented. Furthermore, a topic is addressed that can only be observed for non-constant permittivity profiles : different directions of the membrane's deflection or, in mathematical parlance, the sign of the solution to the nonlinear evolution problem.

march 20
14h00
B405

## A structured population model of clonal selection in acute leukemias

Thomas STIEHL

Heidelberg University

Acute leukemias are aggressive cancers of the blood-forming system. The malignant cell bulk in each individual patient is composed of multiple clones carrying different subsets of mutations. Due to competition and selection the abundance of the different clones changes over time. To better understand the mechanisms underlying this observation, we propose a multi-compartmental continuously structured population model of acute leukemias. The model consists of a system of coupled integro-differential equations. Its structure is motivated as follows :

1. The integral terms describe non-local regulatory mechanisms of the blood-forming system.
2. The multi-compartmental structure reflects the biological observation that the malignant cell bulk can be subdivided into so-called leukemic stem cells and non-stem leukemic cells. Since only stem cells can divide infinitely often, they are required for maintenance of the disease. The different compartments represent the different cell types, namely stem cells and multiple kinds of non-stem cells.
3. The continuous structure takes into account the existence of 'infinitely' many clones with different biological properties (such as proliferation rates and self-renewal probabilities).

Analysis of the model reveals that clonal selection is driven by the self-renewal probability of leukemic stem cells, i.e., the probability that progeny of a stem cell are again stem cells. The clones that possess the highest value of this parameter are ultimately selected. Furthermore, we demonstrate that the self-renewal probability and the proliferation rates of non-stem cells do not impact on the eventual outcome of clonal selection. These findings have interesting implications for the clinical management of acute leukemias.

The talk is based on a joined work with T. Lorenzi and A. Marciniak-Czochra.

# Growth-fragmentation equations : inverse problems and asymptotic behaviour.

march 20
15h30
B405

Magali TOURNUS

Université Aix-Marseille

I will consider the fragmentation equation

$$\frac{\partial f}{\partial t}(t,x) = -B(x)f(t,x) + \int_{y=x}^{y=\infty} k(y,x)B(y)f(t,y)dy,$$

and address the question of estimating the fragmentation kernel $k(y,x)$ - from measurements of the size distribution $f(t,\cdot)$ at various times. This is a natural question for any application where the sizes of the particles are measured experimentally whereas the fragmentation rates are unknown. Under the assumption of a polynomial division rate $B(x) = \alpha x^\gamma$, where $\gamma$ and $\alpha$ are known and a self-similar fragmentation kernel $k(y,x) = \frac{1}{y}\kappa(\frac{x}{y})$, we use the asymptotic behaviour to obtain a representation formula for $\kappa$. To do so, one of the delicate points is to prove that the Mellin transform of the asymptotic profile never vanishes, what we do through the Wiener-Hopf representation.

In a second part, I will present some open problems concerning asymptotic behaviours for systems.