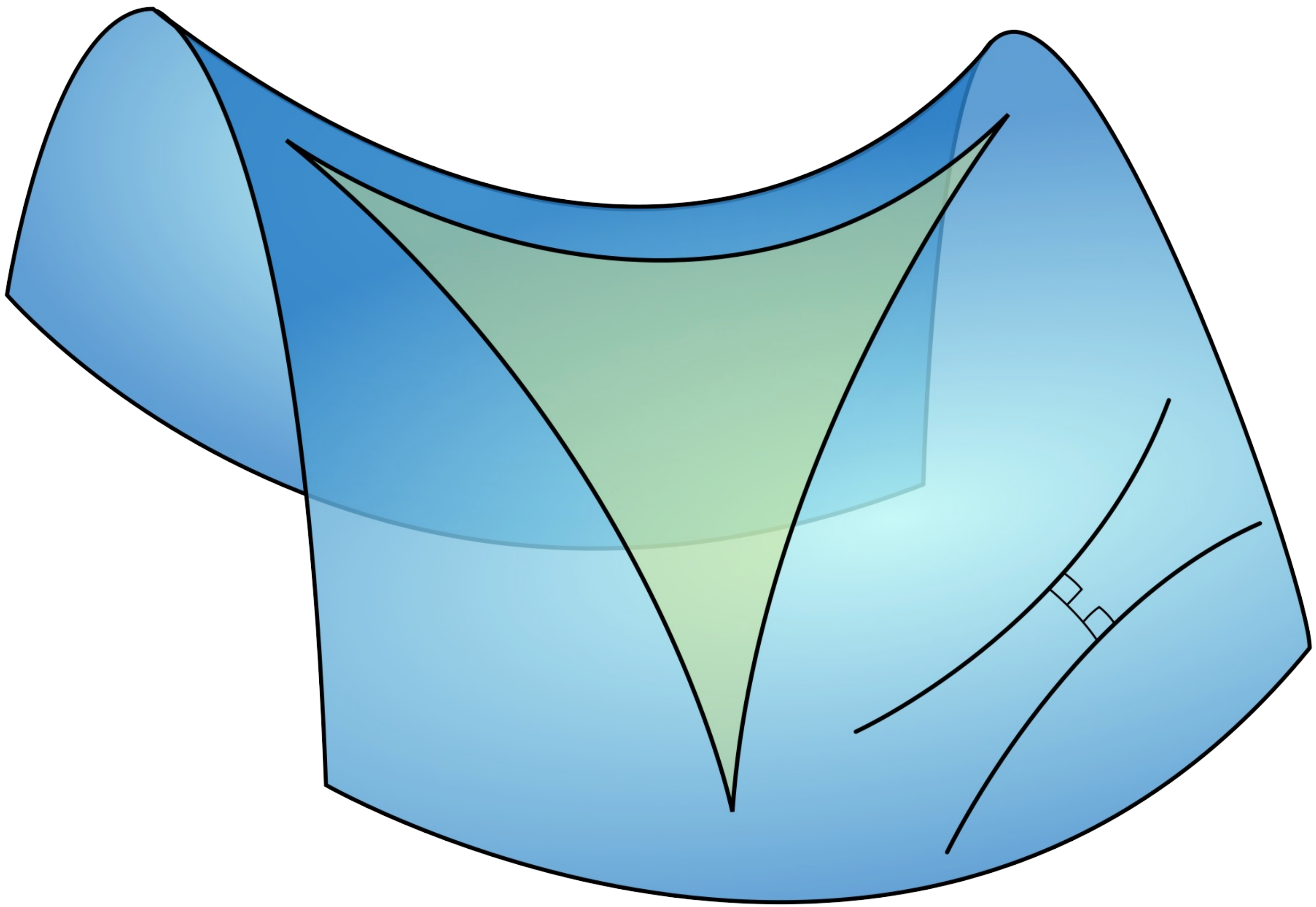
Géométrie & Spectres des Surfaces Hyperboliques Aléatoires

Aussois 12,2025

Definition a hyperbolic surface is a surface which locally looks like the hyperbolic plane



G. Meyer

[math.GT] 2 Nov 2010

PANTS DECOMPOSITIONS OF RANDOM SURFACES

LARRY GUTH, HUGO PARLIER[†], AND ROBERT YOUNG

Abstract. Our goal is to show, in two different contexts, that "random" surfaces have large pants decompositions. First we show that there are hyperbolic surfaces of genus g for which any pants decomposition requires curves of total length at least $g^{7/6-\varepsilon}$. Moreover, we prove that this bound holds for most metrics in the moduli space of hyperbolic metrics equipped with the Weil-Petersson volume form. We then consider surfaces obtained by randomly gluing euclidean triangles (with unit side length) together and show that these surfaces have the same property.

Any surface of genus g, $g \geq 2$, can be decomposed into three-holed spheres (colloquially, pairs of pants). We say that a surface has pants length $\leq l$ if it can be divided into pairs of pants by curves each of length $\leq l$. We say that a surface has total pants length $\leq L$ if it can be divided into pairs of pants by curves with the sum of the lengths $\leq L$. The pants length and total pants length measure the size an[d]
divide the surface into simp[le]
how big the pants length c[...]
understand how big the tot[al]
use a random construction [...]

To put the paper in con[text]
length. In [Ber74, Ber85], [...]

[math.GN] 10 Dec 2010

Growth of Weil-Petersson volumes and random hyperbolic surfaces of large genus

Maryam Mirzakhani*

December 13, 2010

1 Introduction

In this paper, we investigate the geometric properties of hyperbolic surfaces by studying the lengths of simple closed geodesics. The moduli space $\mathcal{M}_{g,n}$ of complete hyperbolic surfaces of genus $g \geq 2$ with n punctures, is equipped with a natural notion of measure, which is induced by the *Weil-Petersson* symplectic form $\omega_{g,n}$ (§2). By a theorem of Wolpert, this form is the symplectic form of a Kähler noncomplete metric on the moduli space $\mathcal{M}_{g,n}$. We describe the

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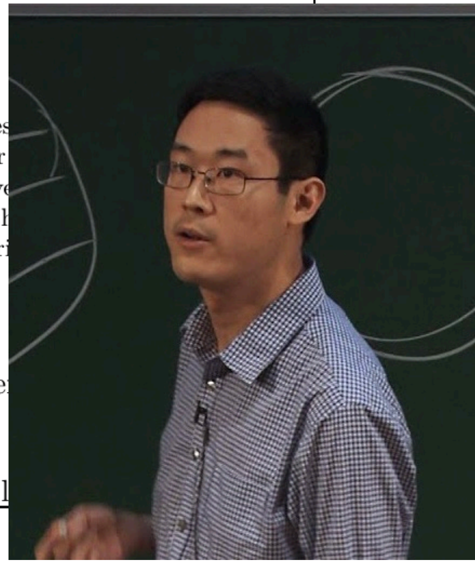

goal is to … ndom" surfaces
First we … of genus g for
equires cu … eover, we prove
etrics in … quipped with th
e then con … ng euclidean tri
ther and … property.

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at a sur … be divided
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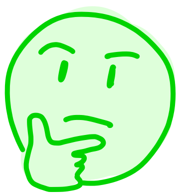
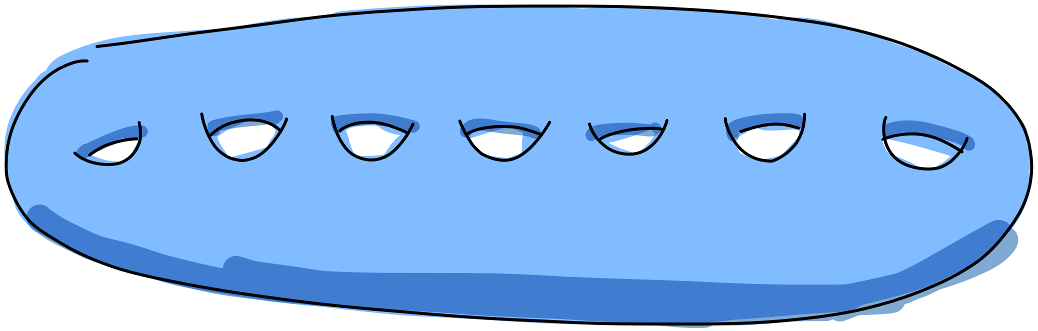


What does a random hyperbolic surface of LARGE genus look like?

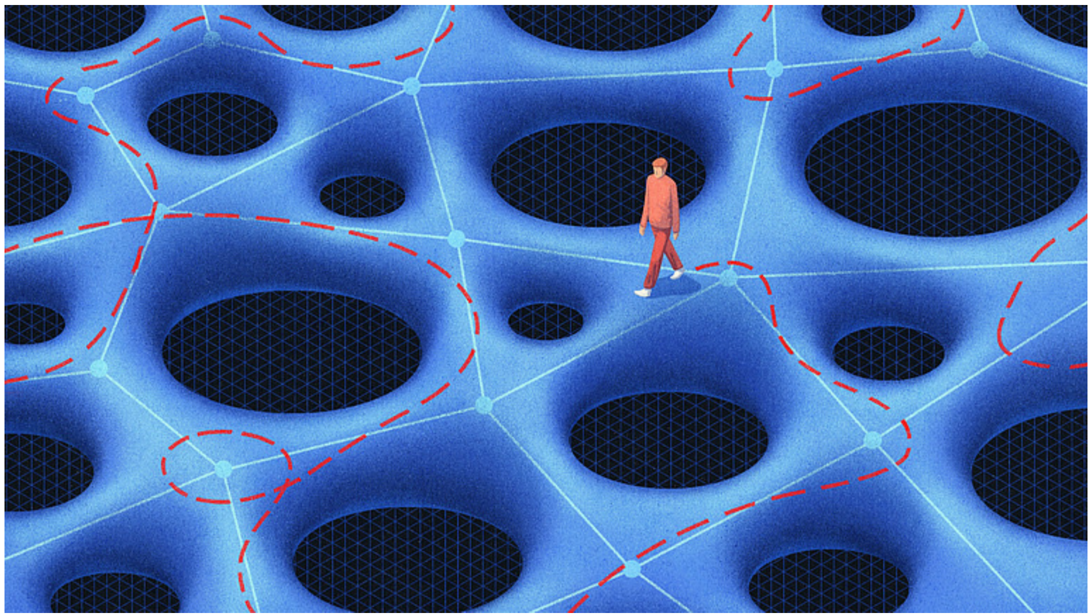
Theorem (Mirzakhani, 2010)

With proba $\to 1$ when the genus $g \to \infty$ a random WP hyperbolic surface of genus g has

1) diameter $< 40 \log g$



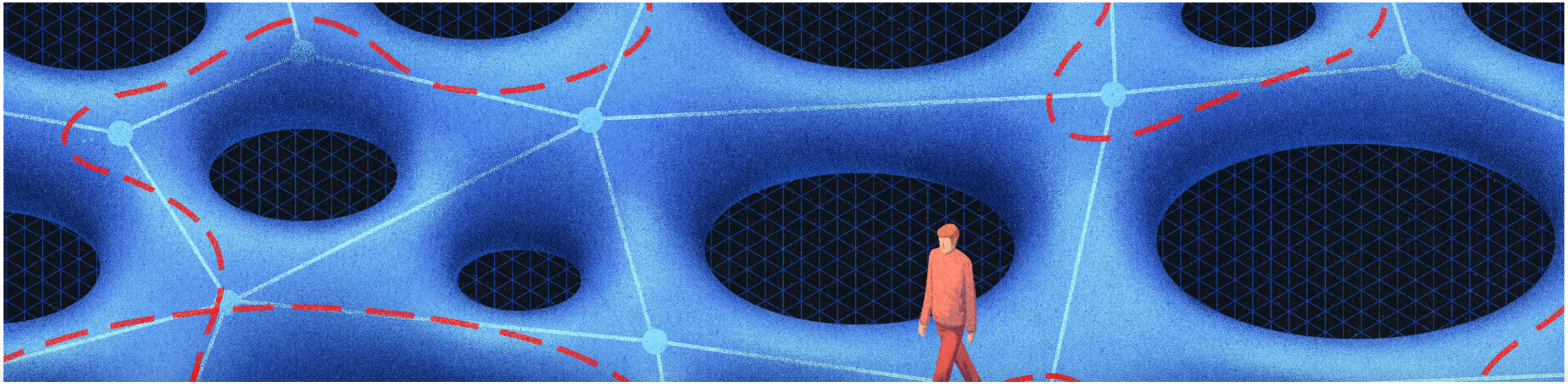
diam $\propto g$



TOPOLOGY

Surfaces Beyond Imagination Are Discovered After Decades-Long Search

Using ideas borrowed from graph theory, two mathematicians have shown that extremely complex surfaces are easy to traverse.



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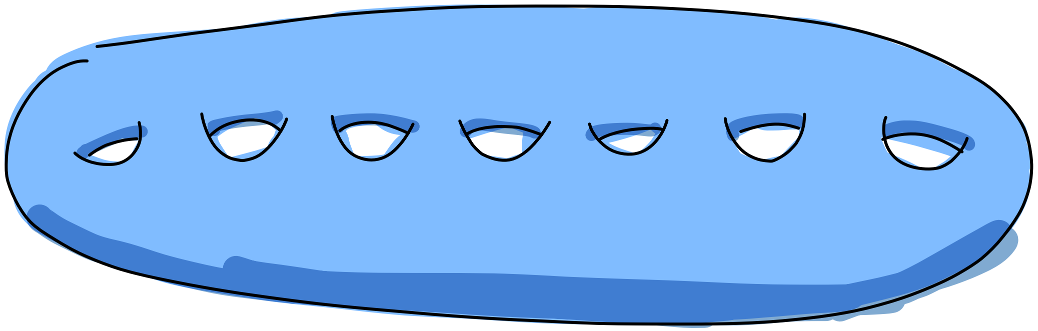

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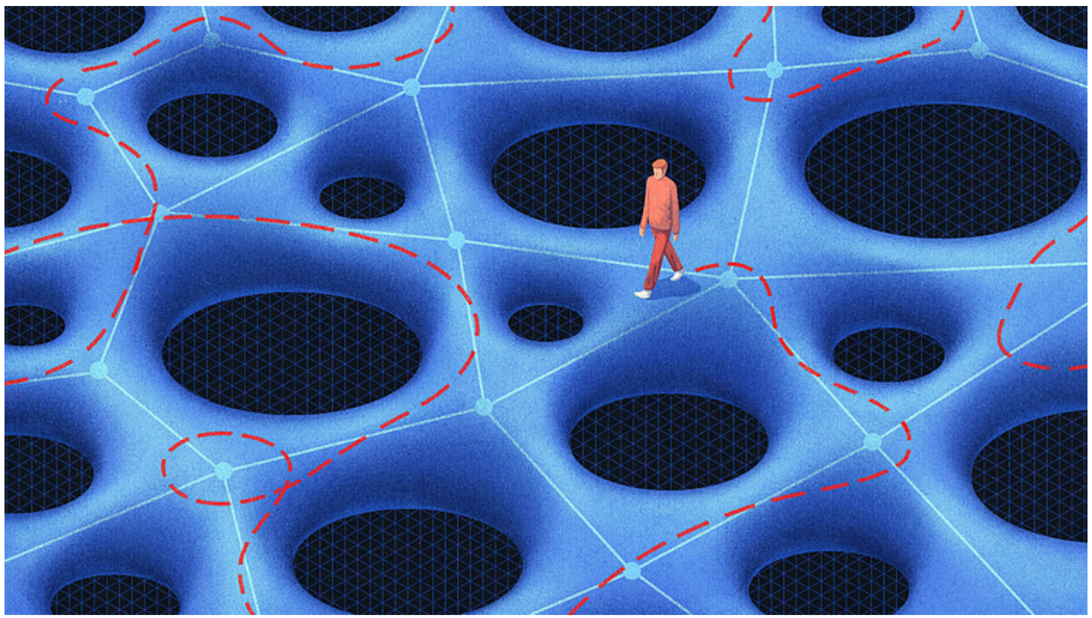

diam $\propto \sqrt{g}$

Theorem (Mirzakhani, 2010)

With proba $\to 1$ when the genus $g \to \infty$

a random WP hyperbolic surface of genus g has

1) diameter $< 40 \log g$

2) spectral gap λ_1

smallest > 0 eigenvalue of Δ

- counting of closed geodesics
- Cheeger constant
- Brownian motion

Theorem (Mirzakhani, 2010)

With proba $\to 1$ when the genus $g \to \infty$
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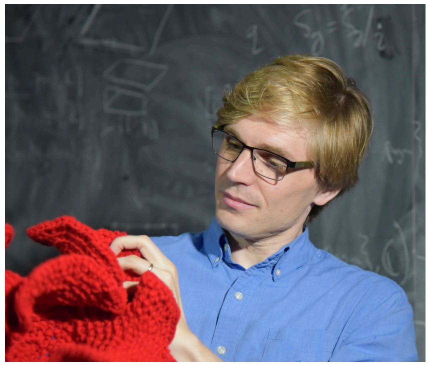
smallest > 0 eigenvalue of Δ


2021: Wu – Xue

Lipnowski – Wright

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Lipnowski-Wright '23 Anantharaman-Monk

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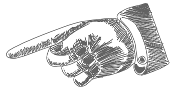
2) spectral gap $\lambda_1 > \cancel{0.002}$ $\quad \cancel{\frac{3}{16} - \varepsilon}$ $\quad \cancel{\frac{2}{9}}$ $\quad \frac{1}{4} - \varepsilon$  optimal

smallest > 0 eigenvalue of Δ

2021: Wu – Xue '25

Lipnowski – Wright '23 Anantharaman – Monk

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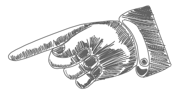
1) diameter $< 40 \log g$ $\quad \cancel{\frac{3}{16} - \varepsilon} \quad \cancel{\frac{2}{9}} \quad \frac{1}{4} - \varepsilon$  optimal

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a few months ago...

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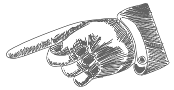

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2021: Wu–Xue

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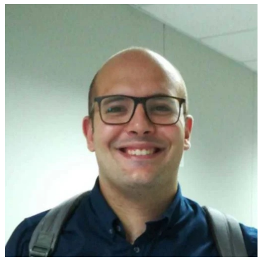
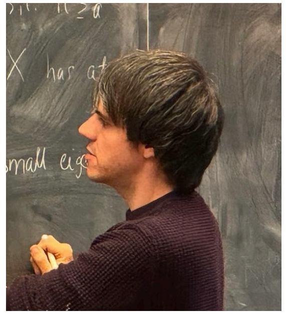
'25 Anantharaman–Monk

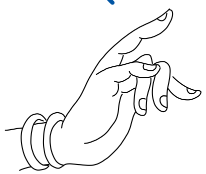
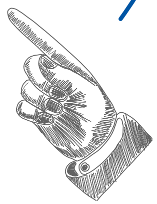
a few months ago Hide–Macera–Thomas





Weil-Petersson random hyperbolic surfaces

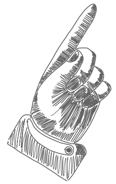
Model $(\Omega, \mathbb{P})$

sample space   proba measure on Ω

$$\Omega = \{\text{hyperbolic surface of genus } g\} / \text{isometry} = \mathcal{M}_g$$

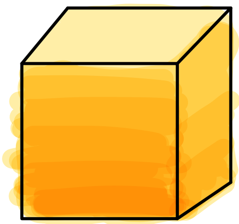
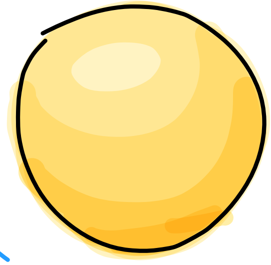
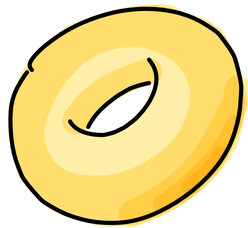
$\mathcal{M}_g$ moduli space

Moduli space $\mathcal{M}_g$ orbifold



 $\mathcal{M}_g$ is "almost" a manifold (of $\dim_{\mathbb{R}} = 6g-6$

 It's a bit complicated ... χ Euler char

$\chi\Big($  $\Big) = \chi\Big($  $\Big) = 2$, $\chi\Big($  $\Big) = 0$

$\dim 2 : \chi = \#\text{vertices} - \#\text{edges} + \#\text{faces} \overset{\text{Euler}}{=} 2-2g$

$$\chi(\mathcal{M}_2) = -\frac{1}{240} \approx -4.17 \times 10^{-3}$$

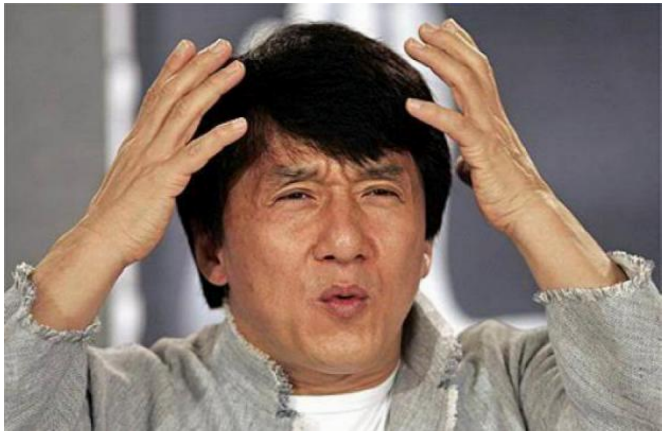
$$\chi(\mathcal{M}_4) \approx -6.94 \times 10^{-4}$$

$$\chi(\mathcal{M}_8) \approx -3.17 \times 10^{-2}$$

$$\chi(\mathcal{M}_{16}) \approx -1.57 \times 10^{7}$$

$$\chi(\mathcal{M}_{32}) \approx -5.28 \times 10^{34}$$

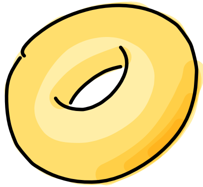
$$\chi(\mathcal{M}_{64}) \approx -3.26 \times 10^{109}$$

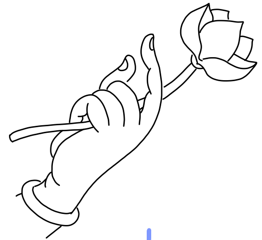


universal cover $\widetilde{M}_g = T_g$ Teichmüller space

$M_g = T_g / \pi_1(M_g)$

mapping class group





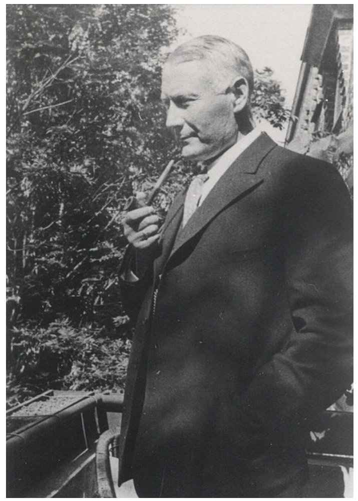
$T_g \underset{\text{homeo}}{\sim} \left(\mathbb{R}_{>0} \times \mathbb{R}\right)^{3g-3}$

Fenchel-Nielsen coordinates

$M_g \approx \mathbb{T}^2 \simeq \mathbb{R}^2 / \mathbb{Z}^2$

$T_g \approx \widetilde{\mathbb{T}}^2 \simeq \mathbb{R}^2$

$\pi_1(M_g) \approx \pi_1(\mathbb{T}^2) \simeq \mathbb{Z}^2$



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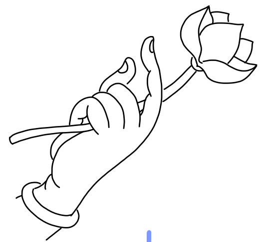
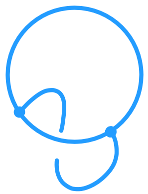


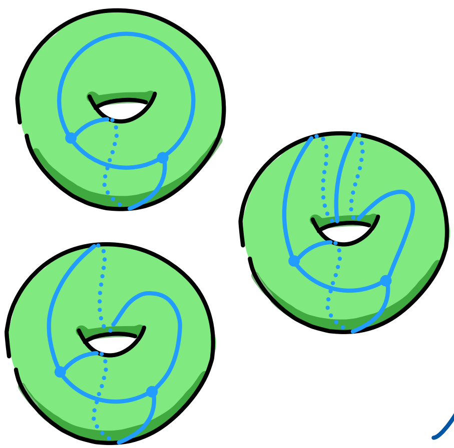
$M_g \approx \{$  $\}$

map

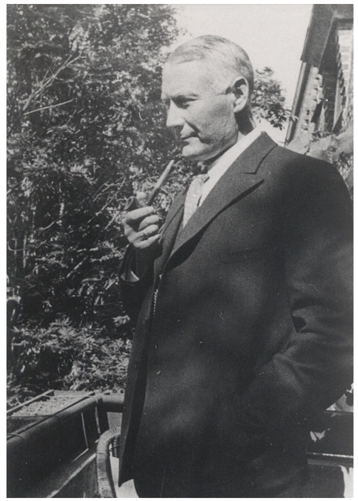
$\sim \Big|$ homeo

$$\left(\mathbb{R}_{>0} \times \mathbb{R}\right)^{3g-3}$$

Fenchel-Nielsen coordinates

$T_g \approx \{$  $\}$

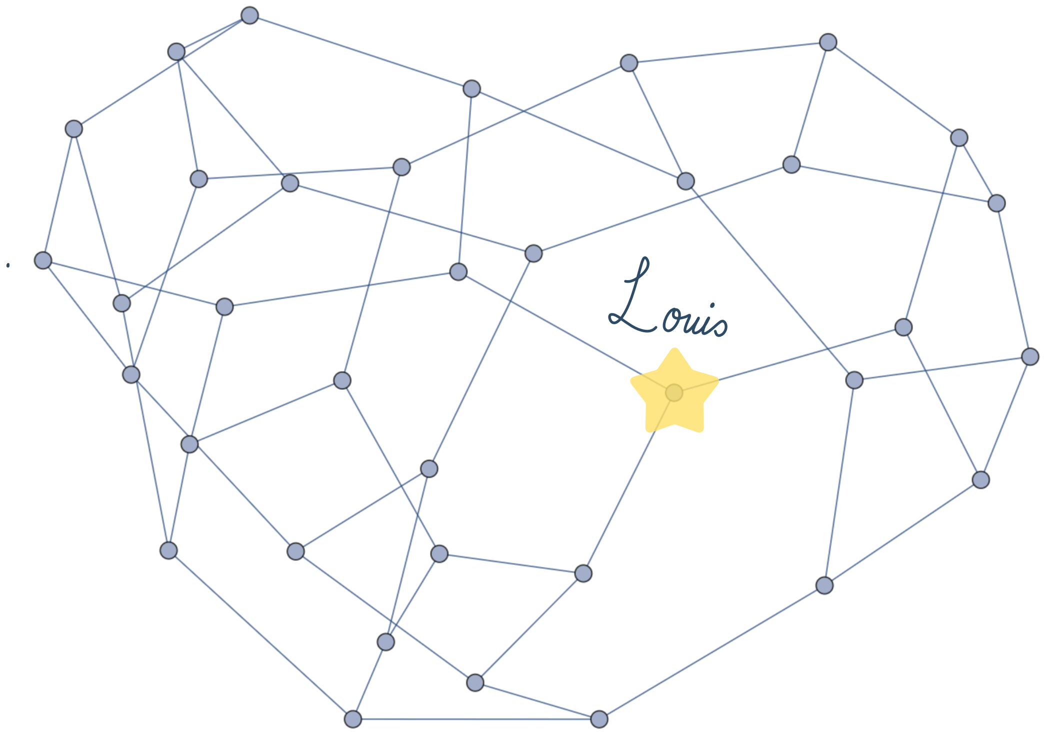
map + embedding



Teichmüller space : space of marked hyperbolic surfaces of genus g

Fix a topological surface Σ_g

$$\mathcal{T}_g := \left\{ (X,\varphi) \;\middle|\; \varphi : \Sigma_g \xrightarrow{\text{homeo}} X \right\} \Big/ \sim$$



$(X,\varphi) \sim (X',\varphi')$ if $\varphi' \circ \varphi^{-1}$ isotopic to an isometry

Teichmüller space : space of **marked** hyperbolic surfaces of genus g

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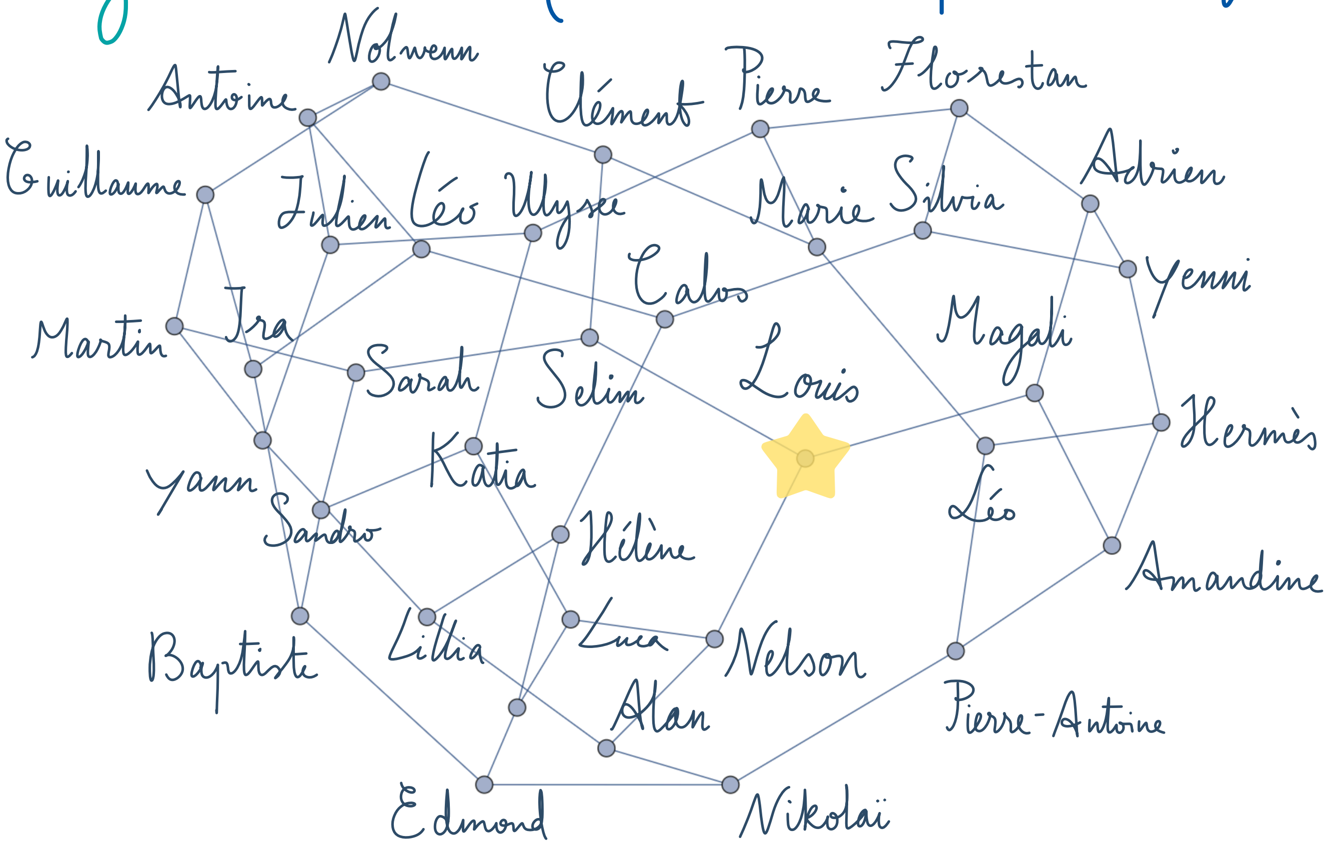
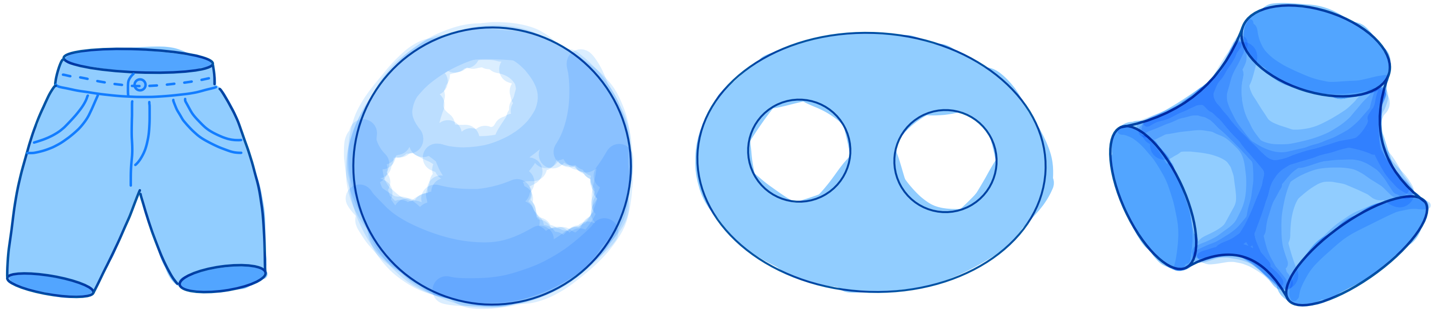


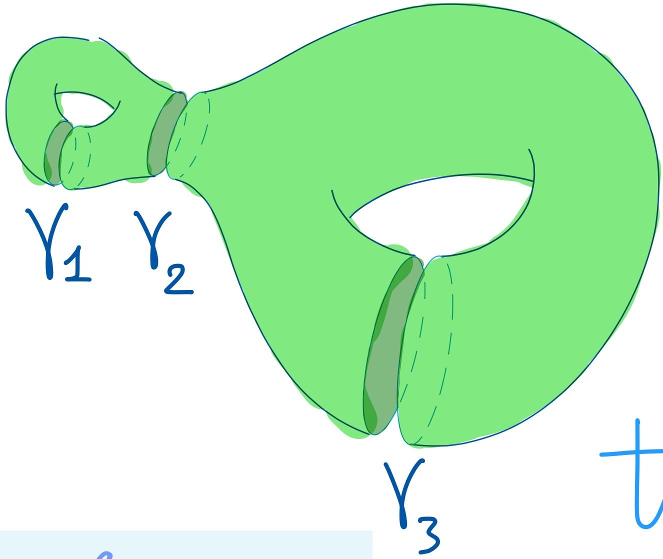
$(X,\varphi) \sim (X',\varphi')$ if $\varphi' \circ \varphi^{-1}$ isotopic to an isometry

pair of pants
$\parallel$
sphere with 3 holes



pants decomposition

$\gamma_1, \gamma_2, \ldots, \gamma_{3g-3}$



γ_i simple

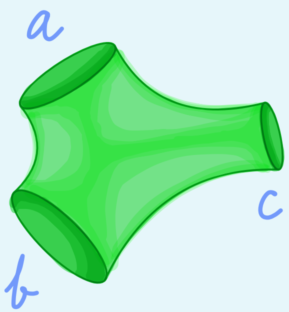
no self-intersection

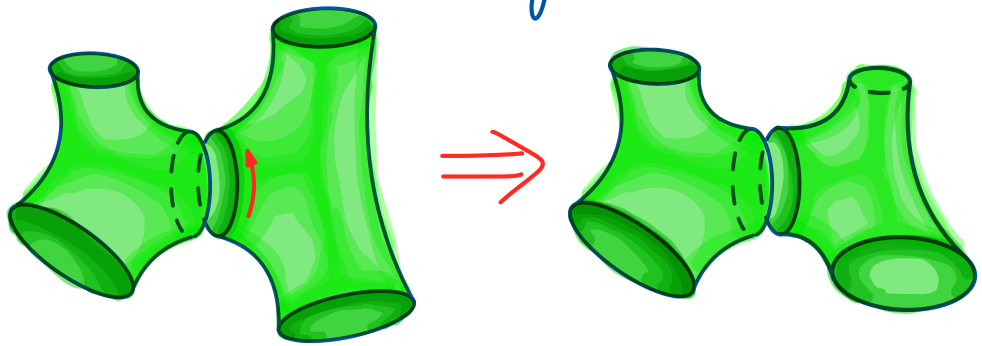
$\gamma_i \cap \gamma_j = \emptyset$

length of $\gamma_i = l_i$

twist along $\gamma_i = \tau_i$

Fact $\forall\, a, b, c \in \mathbb{R}_{>0}$ $\exists!$ hyperbolic



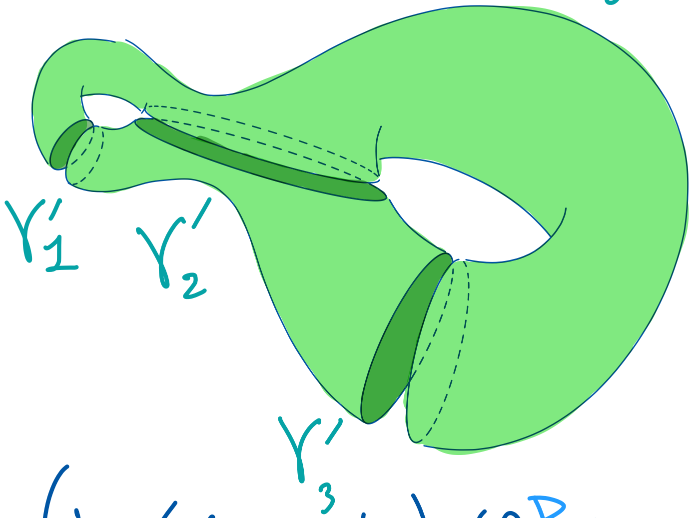


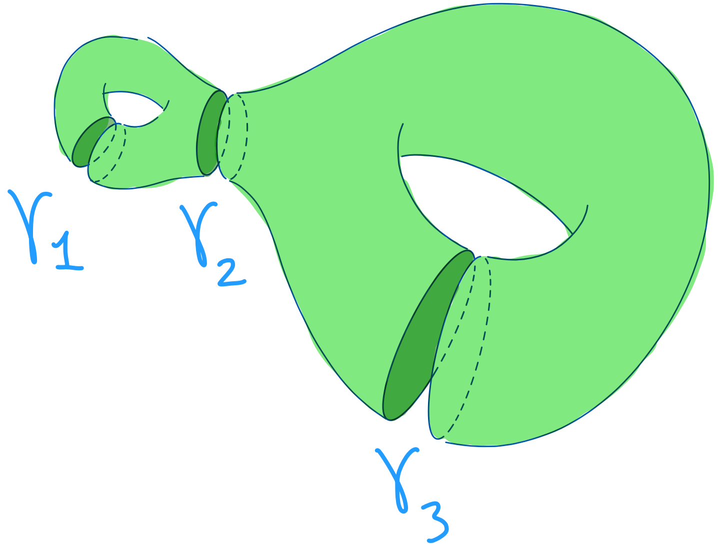
$$\left\{ (l_1, \tau_1, \ldots, l_{3g-3}, \tau_{3g-3}) \;\middle|\; \begin{array}{l} l_i \in \mathbb{R}_{>0} \\ \tau_i \in \mathbb{R} \end{array} \right\} \xrightarrow[\text{Fenchel-Nielsen}]{} T_g$$

a pants decomposition $\mathcal{P}=\{\gamma_1,\dots,\gamma_{3g-3}\} \Rightarrow$

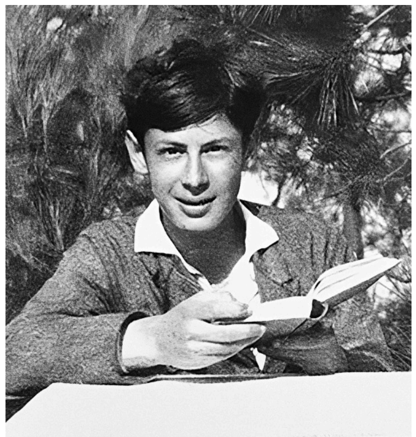
$$\begin{tikzcd} (\mathbb{R}_{>0}\times\mathbb{R})^{3g-3} \arrow[d, shift right=2, "\varphi^{\mathcal{P}}"'] \arrow[d, shift left=2, "\varphi^{\mathcal{P}'}"] \\ \mathcal{T}_g \end{tikzcd}$$

$\mathcal{P}'=\{\gamma_1',\dots,\gamma_{3g-3}'\}$





- (Wolpert) $\varphi^{\mathcal{P}}_*(\mu_{Leb})=\varphi^{\mathcal{P}'}_*(\mu_{Leb})$ = Weil–Petersson measure



a pants decomposition $\mathcal{P} = \{\gamma_1, \dots, \gamma_{3g-3}\} \Rightarrow$

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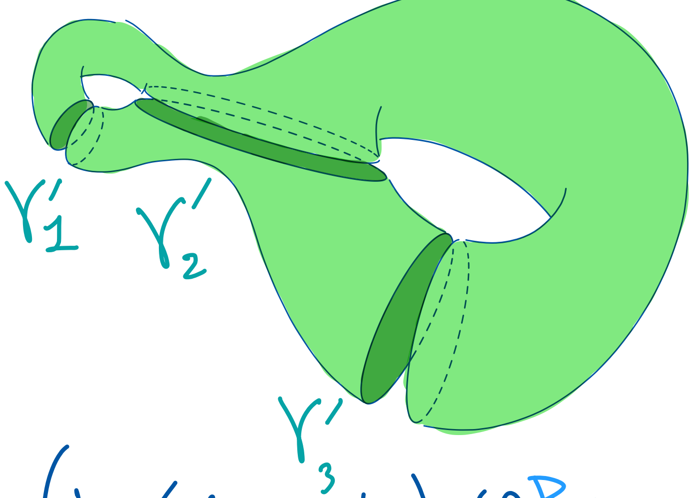

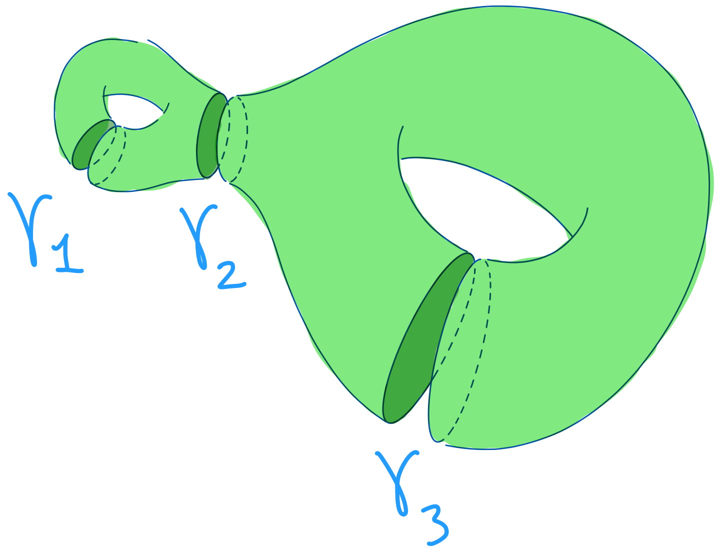

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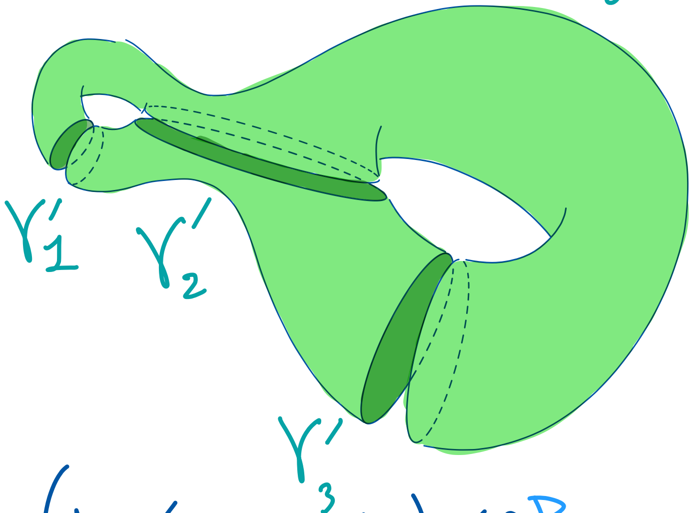
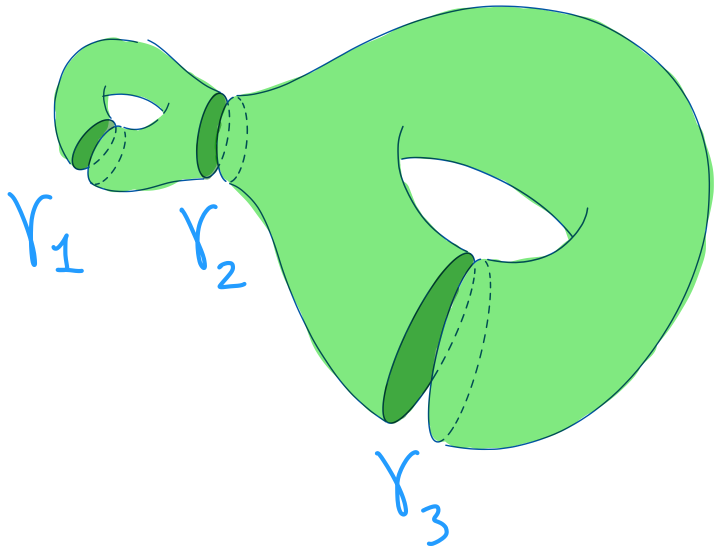
- μ_{WP} is invariant under $\pi_1(\mathcal{M}_g) \Rightarrow \mu_{WP}$ descends to $\mathcal{M}_g$

$$\mathcal{M}_g = \mathcal{T}_g / \pi_1(\mathcal{M}_g)$$

a pants decomposition $\mathcal{P} = \{\gamma_1, \dots, \gamma_{3g-3}\} \Rightarrow (\mathbb{R}_{>0} \times \mathbb{R})^{3g-3}$

$\mathcal{P}' = \{\gamma'_1, \dots, \gamma'_{3g-3}\}$





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- μ_{WP} is invariant under $\pi_1(\mathcal{M}_g) \Rightarrow \mu_{WP}$ descends to $\mathcal{M}_g$
- $\mu_{WP}(\mathcal{M}_g) < \infty$ $\Longrightarrow$ WP model for random hyp surf

not compact

Theorem (Mirzakhani 2010) Weil–Petersson measure

$$\frac{1}{\mu_{WP}(\mathcal{M}_g)} \int_{\mathcal{M}_g} \mathbb{1}_{\{S \in \mathcal{M}_g \mid \mathrm{diam}(S) < 40 \log g\}}(X)\, dX \xrightarrow[g \to \infty]{} 1$$

Notation X_g a WP random hyperbolic surface of genus g

$$\mathbb{P}\left(\mathrm{diam}(X_g) < 40 \log g\right) \xrightarrow[g \to \infty]{} 1$$

LENGTHS OF CLOSED GEODESICS ON RANDOM SURFACES OF LARGE GENUS

MARYAM MIRZAKHANI AND BRAM PETRI

ABSTRACT. We prove Poisson approximation results for the bottom part of the length spectrum of a random closed hyperbolic surface of large genus. Here, a random hyperbolic surface is a surface picked at random using the Weil-Petersson volume form on the corresponding moduli space. As an application of our result, we compute the large genus limit of the expected systole.

1. INTRODUCTION

In this paper, we study the distribution of short closed geodesics on random hyperbolic surfaces. Our definition of a random surface is as follows. First of all, we consider for every $g \geq 2$ the moduli space $\mathcal{M}_g$ of closed hyperbolic surfaces of genus g. Its universal cover, the Teichmüller space $\mathcal{T}_g$ comes with a symplectic form ω_g, called the Weil-Petersson symplectic from. The associated volume form descends to $\mathcal{M}_g$ and is of finite total volume. This means that we obtain a probability measure $\mathbb{P}_g$ on $\mathcal{M}_g$ by defining

$$\mathbb{P}_g[A] = \frac{\mathrm{vol}_{\mathrm{WP}}(A)}{\mathrm{vol}_{\mathrm{WP}}(\mathcal{M}_g)}$$

for every measurable set $A \subseteq \mathcal{M}_g$, where $\mathrm{vol}_{\mathrm{WP}}(A)$ denotes the Weil-Petersson volume of A. Our main goal is now to combine methods from probability theory and Weil-Petersson geometry to estimate probabilities of the form

$$\mathbb{P}_g[X \in \mathcal{M}_g \text{ has } k \text{ closed geodesics of length } \leq L].$$

Fix $0 \leq a < b$

$$N_{[a,b)}(X) := \#\left\{\gamma \begin{array}{l}\text{primitive closed}\\ \text{geodesic on } X\end{array} \;\middle|\; a \leq \ell(\gamma) < b\right\}$$

Theorem (Mirzakhani - Petri, 2017)

Fix $0 \leq a < b$

$Y \sim \mathrm{Poi}(\lambda)$ if $\mathbb{P}(Y=k) = e^{-\lambda}\dfrac{\lambda^k}{k!}, \forall k \in \mathbb{Z}_{\geq 0}$

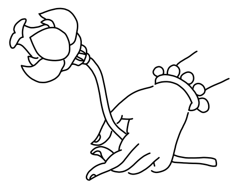
$$N_{[a,b)}(X) := \#\left\{ \gamma \ \begin{matrix}\text{primitive closed} \\ \text{geodesic on } X\end{matrix} \ \middle|\ a \leq \ell(\gamma) < b \right\}$$

<u>Theorem</u> (<u>Mirzakhani–Petri, 2017</u>)

X_g a WP random surface of genus g

$$N_{[a,b)}(X_g) \xrightarrow[g \to \infty]{(d)} \mathrm{Poi}\left(\int_a^b \overbrace{\frac{\cosh(x)-1}{x}}^{\lambda(x)\ =} dx \right)$$

Length Spectrum



a multiset

$$\Lambda(X) := \left\{ \ell(\gamma) \in \mathbb{R}_{>0} \;\middle|\; \gamma \text{ primitive closed geodesic on } X \right\}$$

Theorem (Mirzakhani–Petri, 2017)

X_g a WP random surface of genus g.

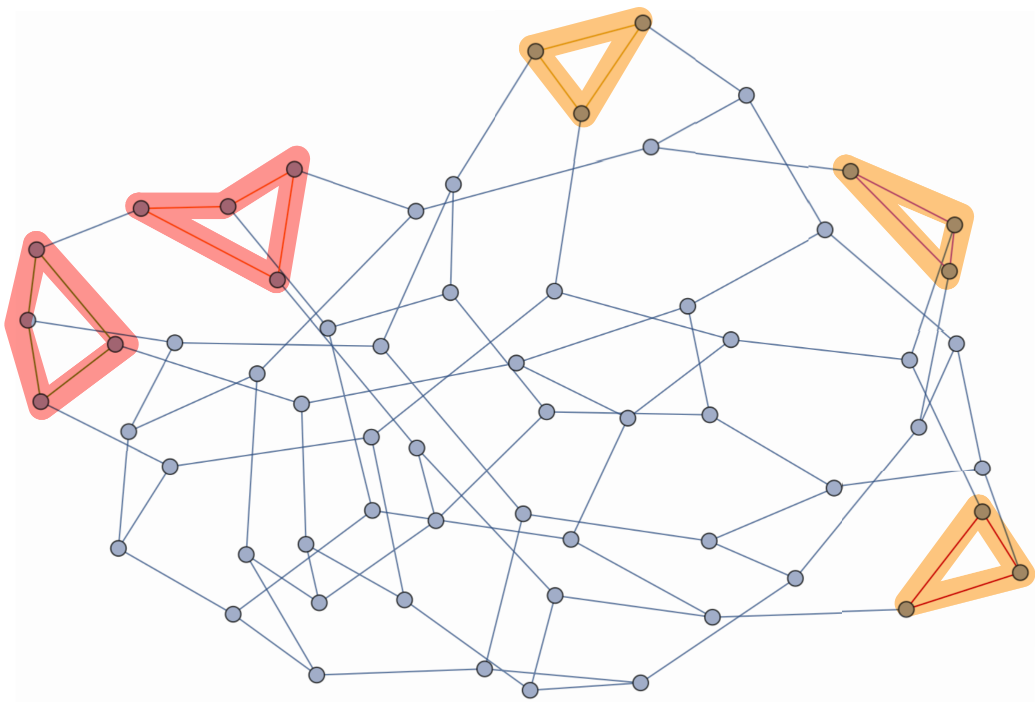
Regarded as a point process on $\mathbb{R}_{>0}$,

$$\Lambda(X_g) \xrightarrow[g\to\infty]{(d)} \text{Poisson point process with intensity } \lambda.$$

$$\frac{\cosh(x)-1}{x} = \lambda$$

Remark G a graph, $k \geq 1$ an integer

$$\mathcal{N}_k(G) := \{\gamma \text{ cycle in } G \mid \ell(\gamma) = k\}$$



$k = 3, 4$

Remark G a graph, $k \geq 1$ an integer

$$N_k(G) := \{\gamma \text{ cycle in } G \mid \ell(\gamma) = k\}$$

Theorem (Bollobás, Wormald, ≈ 1980)

G_v a uni ... aph with v vertices

Remark G a graph, $k \geq 1$ an integer

$$N_k(G) := \{ \gamma \text{ cycle in } G \mid \ell(\gamma) = k \}$$

Theorem (Bollobás, Wormald, ≈ 1980)

G_v a uniform random 3-regular graph with v vertices

For any integer $k \geq 3$,

$$N_k(G_v) \xrightarrow[v \to \infty]{(d)} \mathrm{Poi}\left(\frac{2^k}{2k}\right)$$

Remark

$$\mathbb{E}\left(N_{[0,L]}(X_g)\right) \xrightarrow[g\to\infty]{} \int_0^L \lambda(x)\,dx \underset{L\to\infty}{\sim} \frac{e^L}{2L}$$

Theorem (≈ 1960 Huber, Selberg, Margulis, Chaubet…)



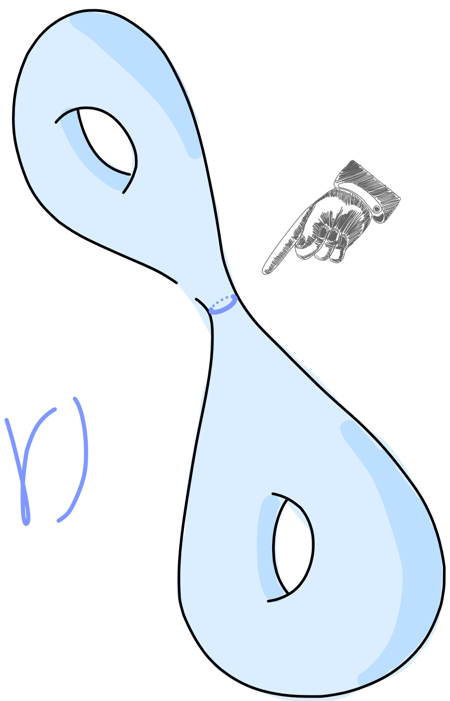

Remark

$$\mathbb{E}(N_{[0,L]}(X_g)) \xrightarrow[g\to\infty]{} \int_0^L \lambda(x)\,dx \underset{L\to\infty}{\sim} \frac{e^L}{2L}$$

Theorem (≈ 1960 Huber, Selberg, Margulis, Chaubet...)

For *any* hyperbolic surface X

$$N_{[0,L]}(X) \underset{L\to\infty}{\sim} \frac{e^L}{2L}$$



Corollary

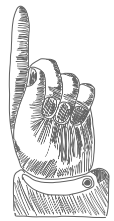
$$\mathrm{sys}(X) := \min_{\gamma \text{ geod on } X} \ell(\gamma)$$

$$\mathbb{E}(\mathrm{sys}(X_g)) \xrightarrow[g\to\infty]{} 1.615...$$

What is a map?

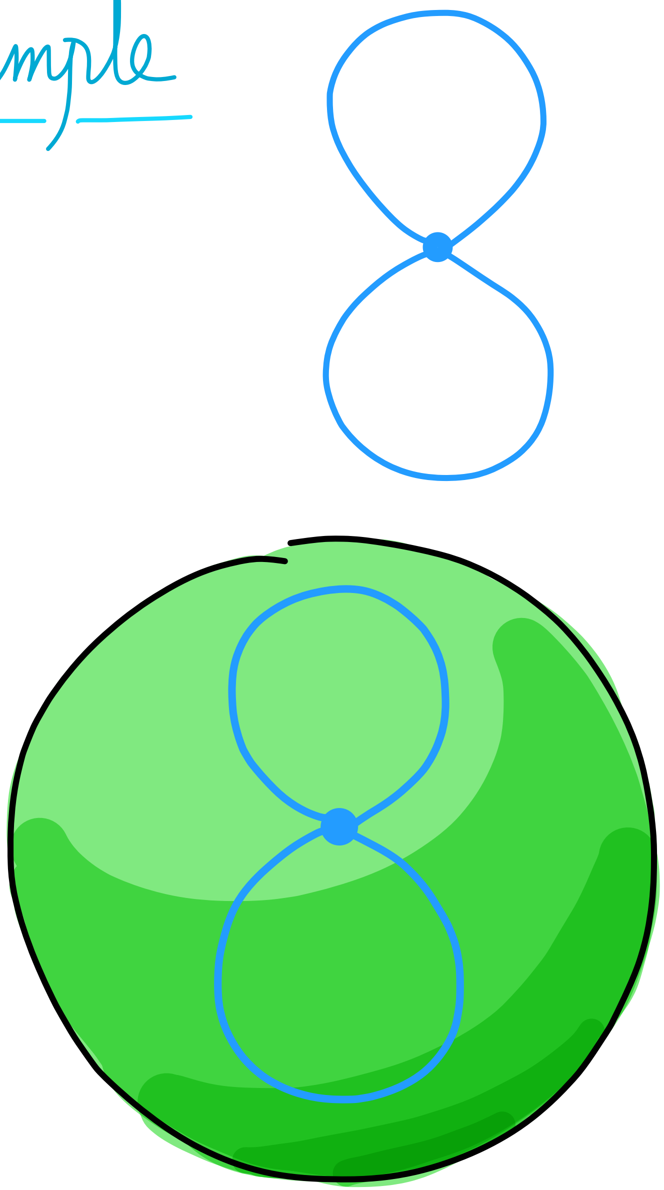
What is a map?

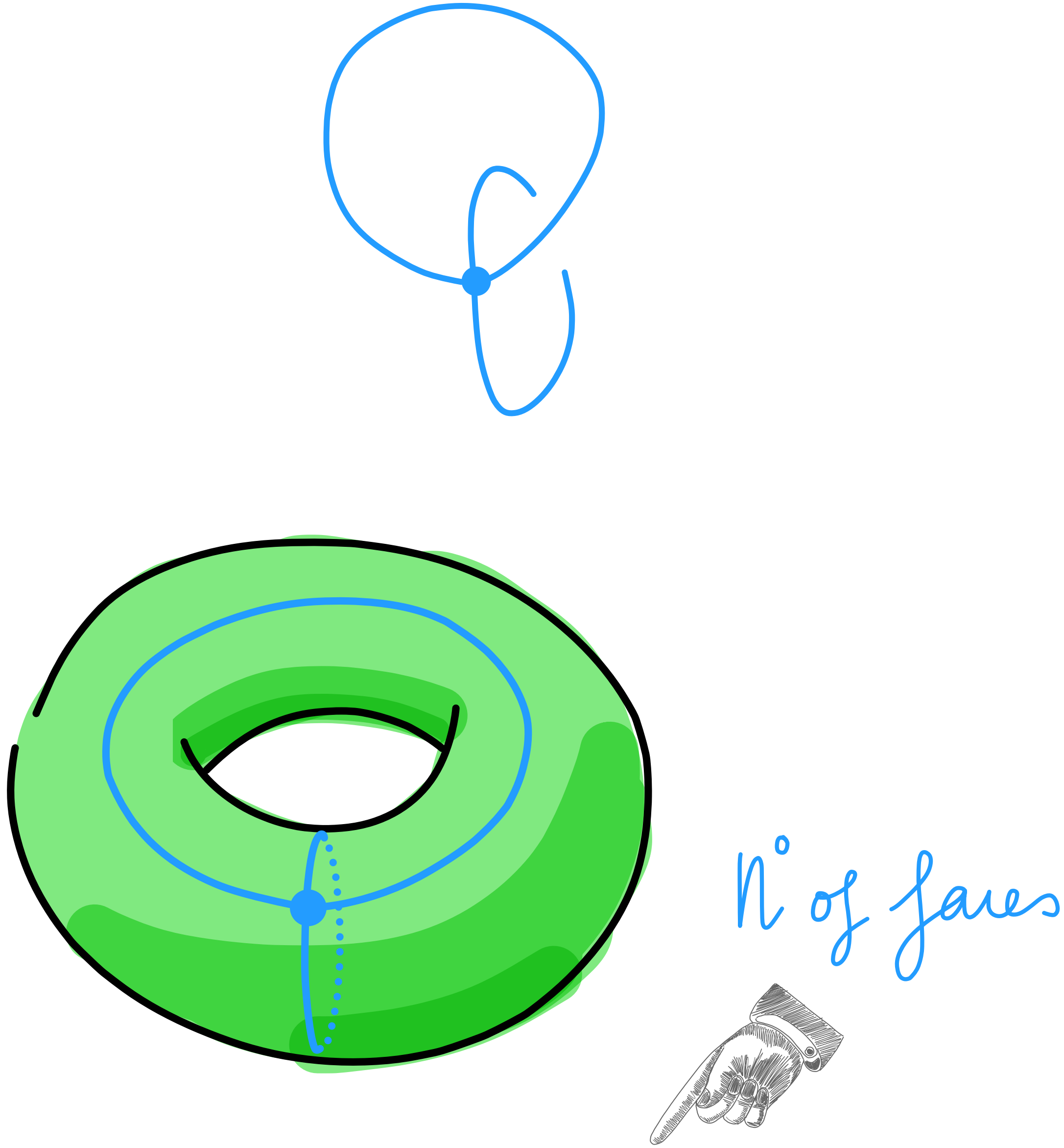
A map is a graph G drawn on a surface S such that $S \setminus G$ is a disjoint union of polygons



faces

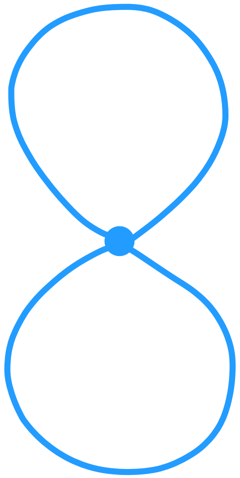
Example





$g = 0, \; n = 3$ $\qquad$ $g = 1, \; n = 1$

Example

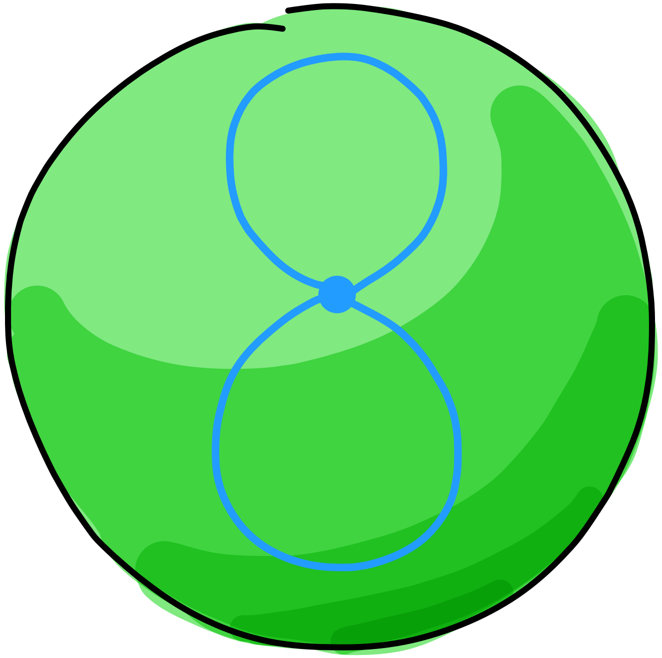


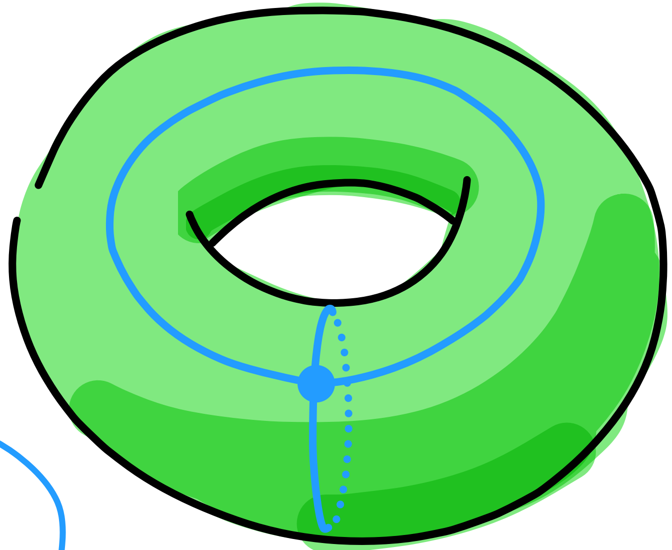
Def Unicellular if $n = 1$

$g = 0,\ n = 1$

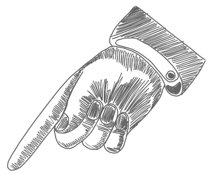
$\Downarrow$

plane trees

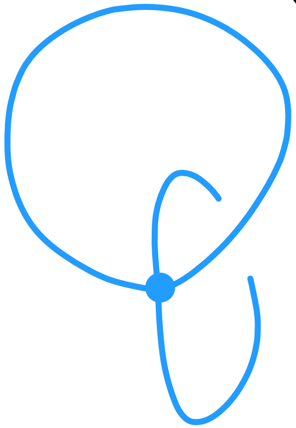




n° of faces



$g = 0,\ n = 3$



$g = 1,\ n = 1$

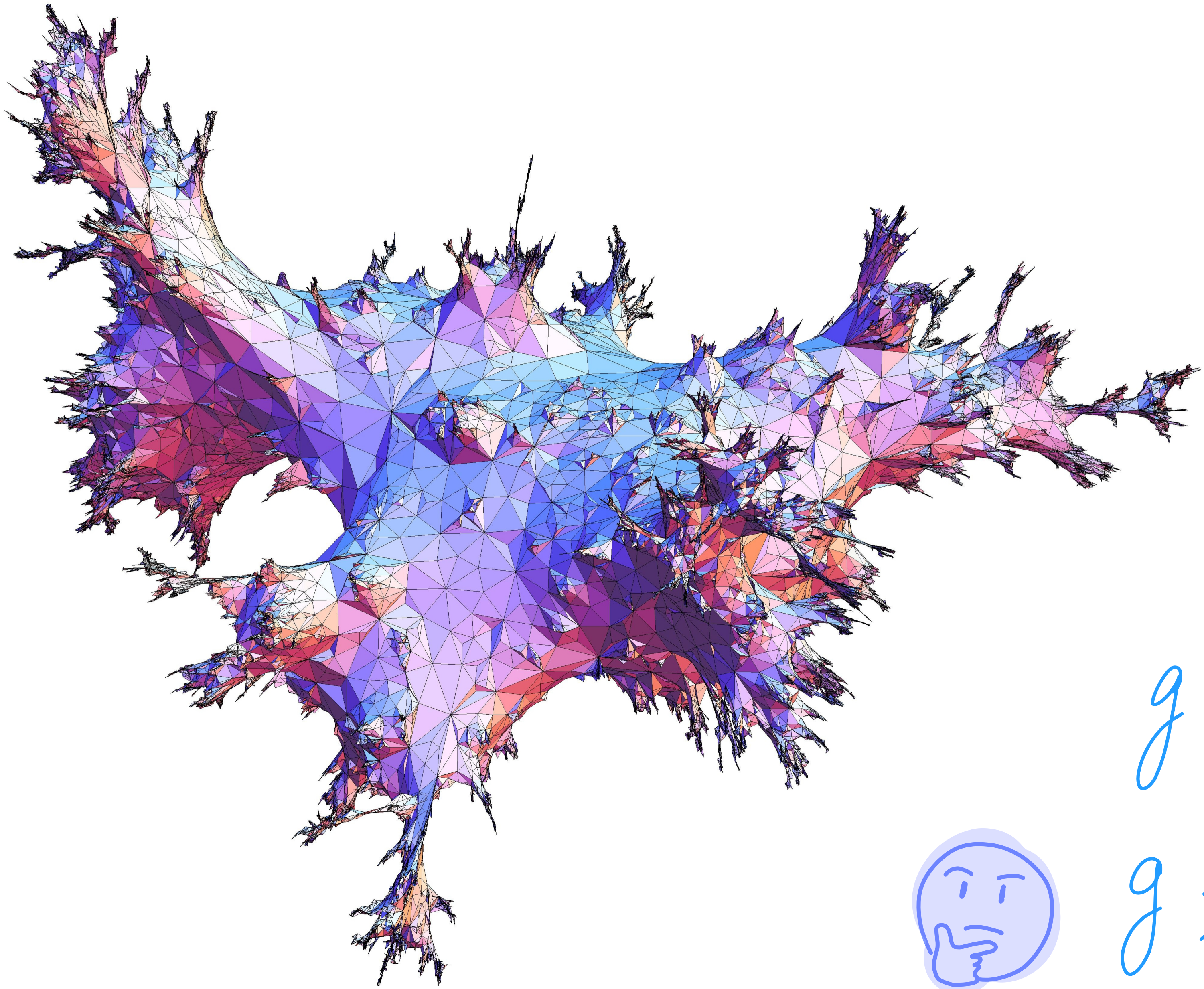
Tutte

Le Gall, Miermont

(1960s)

(2011)

Brownian sphere : scaling limit of uniform random plane ~~triangulations~~ quadrangulations



$g = 0$

$g \gg 0$

UNICELLULAR MAPS VS HYPERBOLIC SURFACES IN LARGE GENUS: SIMPLE CLOSED CURVES

SVANTE JANSON AND BAPTISTE LOUF

ABSTRACT. We study uniformly random maps with a single face, genus g, and size n, as $n, g \to \infty$ with $g = o(n)$, in continuation of several previous works on the geometric properties of "high genus maps". We calculate the number of short simple cycles, and we show convergence of their lengths (after a well-chosen rescaling of the graph distance) to a Poisson process, which happens to be exactly the same as the limit law obtained by Mirzakhani and Petri (2019) when they studied simple closed geodesics on random hyperbolic surfaces under the Weil–Petersson measure as $g \to \infty$.

This leads us to conjecture that these two models are somehow "the same" in the limit, which would allow to translate problems on hyperbolic surfaces in terms of random trees, thanks to a powerful bijection of Chapuy, Féray and Fusy (2013).

1. INTRODUCTION

1.1. **Combinatorial maps.** Maps are defined as gluings of polygons forming a (compact, connected, oriented) surface. They have been studied extensively in the past 60 years, especially in the case of planar maps, i.e., maps of the sphere. They were first approached from the combinatorial point of view, both enumeratively, starting with [32], and bijectively, starting with [30].

More recently, relying on previous combinatorial results, geometric properties of large random maps have been studied. More precisely, one can study the geometry of random maps picked uniformly in certain classes, as their size tends to infinity. In the case of planar maps, this culminated in the identification of two types of "limits" (for two well defined topologies on the set of planar maps): the local limit (the $UIPT$[1] [2]) and the scaling

$n=1$

$\mathcal{U}$ a unicellular map of genus g with v vertices

$$\mathcal{N}_{[a,b)}(\rho\cdot\mathcal{U}) := \#\{\gamma_{\text{cycle in }\mathcal{U}} \mid a \leq \rho\cdot\ell(\gamma) < b\}$$

$$\rho = \sqrt{12g/v}$$

Theorem (Janson–Louf, 2021)

$n=1$

$\mathcal{U}$ a unicellular map of genus g with v vertices

$$N_{[a,b)}(\rho\cdot\mathcal{U}) := \#\{\gamma \text{ cycle in } \mathcal{U} \mid a \le \rho\cdot\ell(\gamma) < b\}$$

$$\rho = \sqrt{12g/v}$$

Theorem (Janson–Louf, 2021)

$v, g \to \infty$ with $g = o(v)$

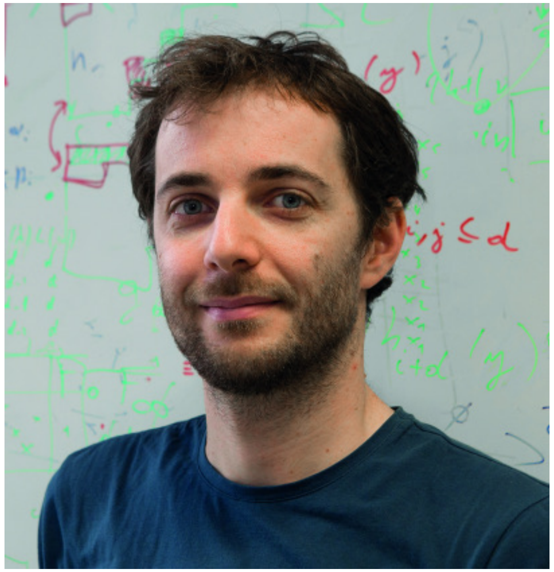
$\mathcal{U}_{v,g}$ a uniform unicellular map of genus g with v vertices

$$N_{[a,b)}(\rho\cdot\mathcal{U}_{v,g}) \xrightarrow[g\to\infty]{(d)} \mathrm{Poi}\left(\int_a^b \lambda(x)\,dx\right)$$

One word about the proof

a magic bijection due to

Chapuy — Féray — Fusy

One word about the proof

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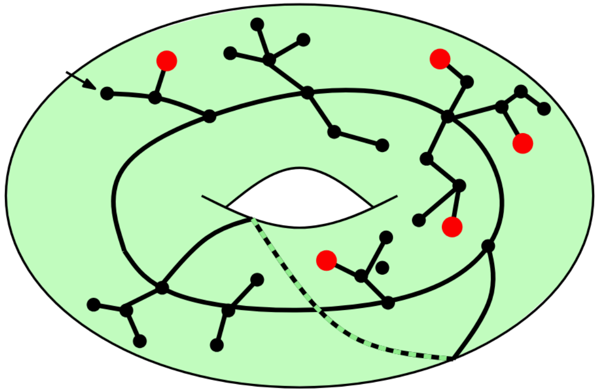
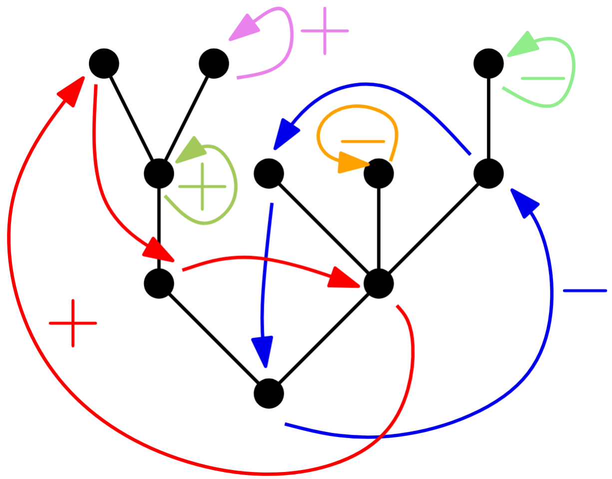
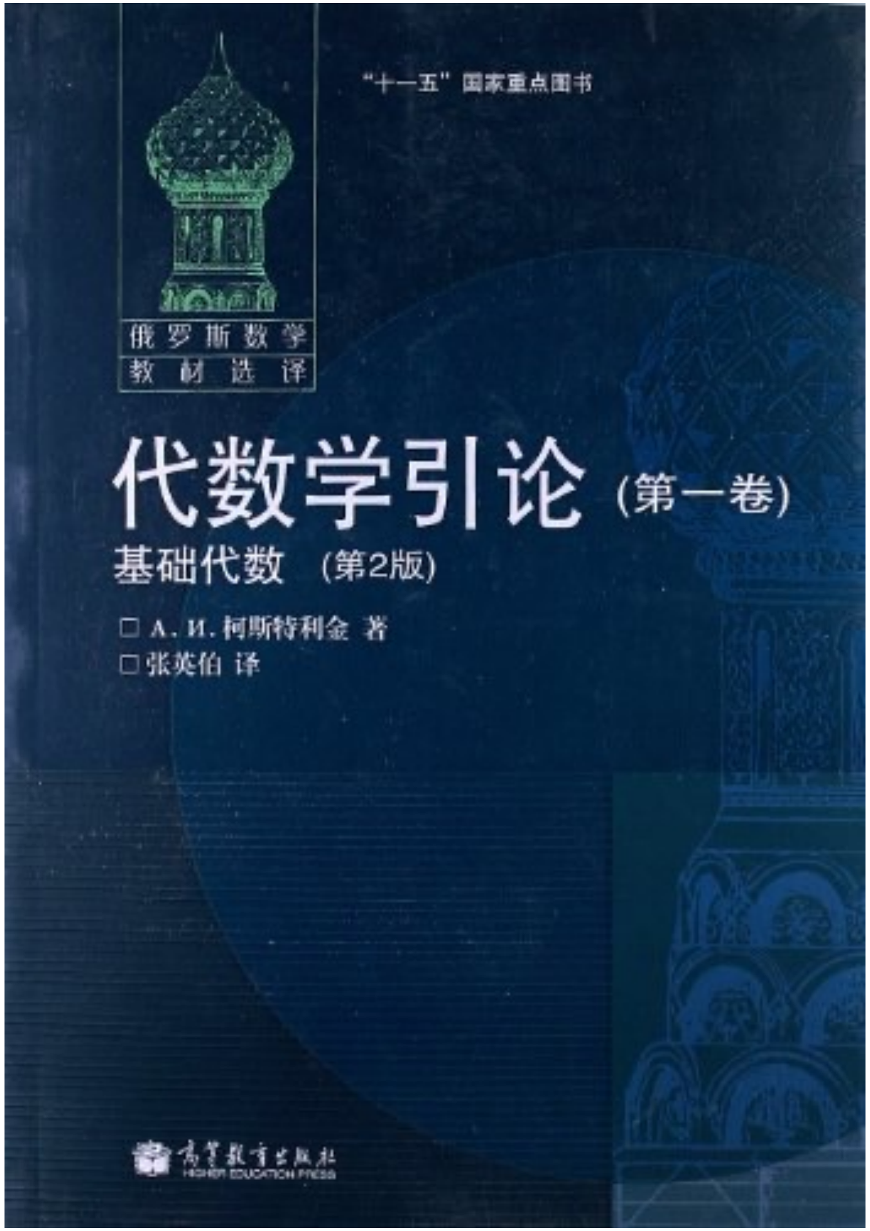


bij $\longleftrightarrow$



unicellular map

plane tree + permutation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ную подошли к формулировке главной задачи.

Положим
$$A_n = \min_{f \in \mathcal{P}_n(1)} A_f.$$

Разумеется, $0 \leqslant A_n \leqslant 1$. Согласно замечаниям, сделанным выше, $A_2 = 1$, в то время как $A_n < 1$ при $n \geqslant 3$.

Требуется доказать, что *при любом n имеет место неравенство $A_n > 0$*. Хорошо бы также *оценить A_n как функцию от n*.

Пока не доказано даже, что $A_3 > 0$.

* * *

“Ещё многое имею сказать вам,
но вы теперь не можете вместить”.

Еванг. от Иоанна, 16:12

12 “I have much more to tell you, but ~~you cannot bear it now~~.

I'm running out of time ...

$$A_n = \min_{f \in \mathcal{P}_n(1)} A_f.$$

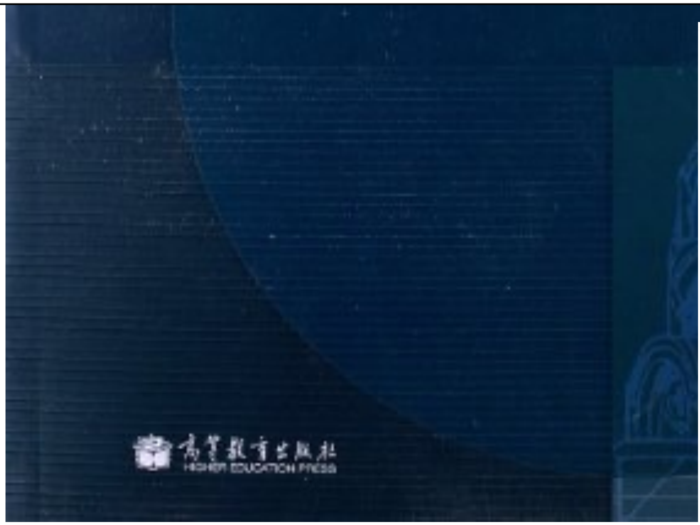
当然 $0 \leqslant A_n \leqslant 1$. 根据上述评注, $A_2 = 1$, 而当 $n \geqslant 3$ 时, $A_n < 1$.

需要证明, 对于任意 n, 不等式 $A_n > 0$ 成立. 将 A_n 看作 n 的函数 也是很好估计的.

暂时甚至连 $A_3 > 0$ 也没有证明.

“还有许多问题我愿意告诉你们,
但是你们现在尚不能接受.”

摘自福音书 Иоанн 16:12



Thank you!

Merci!