The Paris-London Number Theory Seminar

Séance XXX: Courbes elliptiques, variétés abeliennes, points de torsion, hauteurs

Salle B407, LAGA, Université Sorbonne Paris Nord

Lundi 29 Novembre

11:30 : **Rendez-vous at the Gare du Nord** (inside, next to the Starbucks):

departure all together to the conference venue

12:30–14:00 : Déjéneur de bienvenu, salle F002 de l'Institut Galilée

14:00–15:00 : Richard Griffon (Université Clermont–Auvergne)
Isogenies of elliptic curves over function fields
I will report on a recent work, joint with Fabien Pazuki, in which we study elliptic curves over function fields and the isogenies between them. More specifically, we prove analogues in the function field setting of two famous theorems about isogenous elliptic curves over number fields. The first of these describes the variation of the Weil height of the j-invariant of elliptic curves within an isogeny class. And our second main result is an "isogeny estimate" in the spirit of theorems by Masser–Wüstholz and by Gaudron–Rémond. The function field versions of these theorems, though they have a similar flavour to their number field counterparts, display some striking differences with them. After stating our results and giving quick sketches of their proof I will, time permitting, mention a few Diophantine applications.

15:00–15:30 : Caffè !

15:30–16:30 : Rachel Newton (King's College London)
Explicit uniform bounds for Brauer groups of singular $K3$ surfaces
By analogy with Merel's theorem on torsion groups of elliptic curves, Várilly-Alvarado has conjectured that Brauer groups (modulo constants) of $K3$ surfaces over number fields are bounded by a number that only depends on degree of the field and the isomorphism class of the Néron-Severi lattice. Orr and Skorobogatov proved this conjecture for $K3$ surfaces of CM type, showing the existence of a bound that only depends on the degree of the number field. I will present joint work with Francesca Balestrieri and Alexis Johnson in which we re-prove Várilly-Alvarado's conjecture for singular $K3$ surfaces, this time with an explicit bound. This bound is very large in general but can be improved dramatically in certain cases, e.g. if the geometric Picard group is generated by divisors defined over the base field. When combined with results of Kresch–Tschinkel and Poonen–Testa–van Luijk, this shows that the Brauer–Manin sets for these varieties are effectively computable.

16:45–17:45 : Diego Izquierdo (École Polytechnique)
On the Cassels–Tate pairing: from the classical theory to more recent developments

The Tate–Shafarevich conjecture states that the first Tate–Shafarevich group of an abelian variety over a number field should always be finite. Assuming it holds, Cassels and Tate then proved a duality theorem relating the Tate–Shafarevich group of a given abelian variety to the Tate–Shafarevich group of the dual abelian variety. Such a duality encodes interesting information about rational points for curves and for torsors under abelian varieties. In the talk, after recalling this very classical situation, I will present more recent developments about the Cassels–Tate duality over other arithmetical fields that behave quite differently from number fields.

Mardi 30 Novembre

8:40 : **RENDEZ-VOUS AT THE GARE DU NORD** (INSIDE, NEXT TO THE STARBUCKS):

DEPARTURE ALL TOGETHER TO THE CONFERENCE VENUE

10.00–11.00 : Adam Morgan (University of Glasgow)

Integral Galois module structure of Mordell–Weil groups

Let $E/\mathbb{Q}$ be an elliptic curve, G a finite group and V a fixed finite dimensional rational representation of G. As we run over G-extensions $F/\mathbb{Q}$ with $E(F)\otimes\mathbb{Q}$ isomorphic to V, how does the $\mathbb{Z}[G]$-module structure of $E(F)$ vary from a statistical point of view? I will report on joint work with Alex Bartel in which we propose a heuristic giving a conjectural answer to an instance of this question, and make progress towards its proof. In the process I will relate the question to quantifying the failure of the Hasse principle in certain families of genus 1 curves, and explain a close analogy between these heuristics and Stevenhagen's conjecture on the solubility of the negative Pell equation.

11:00–11:30 : CAFFÈ !

11:30–12:30 : Celine Maistret (University of Bristol)

Parity of ranks of abelian surfaces

Let K be a number field and A/K an abelian surface. By the Mordell–Weil theorem, the group of K-rational points on A is finitely generated and as for elliptic curves, its rank is predicted by the Birch and Swinnerton–Dyer conjecture. A basic consequence of this conjecture is the parity conjecture: the sign of the functional equation of the L-series determines the parity of the rank of A/K.

Assuming finiteness of the Shafarevich–Tate group, we prove the parity conjecture for principally polarized abelian surfaces under suitable local constraints. Using a similar approach we show that for two elliptic curves E_1 and E_2 over K with isomorphic 2-torsion, the parity conjecture is true for E_1 if and only if it is true for E_2. In both cases, we prove analogous unconditional results for Selmer groups.

12:30–14:00 : DÉJÉNEUR ET CAFÉ, SALLE F002 DE L'INSTITUT GALILÉE

14:00–15.00 : Marco D'Addezio (Institut de Mathématiques de Jussieu)

Parabolicity conjecture of F-isocrystals

I will present recent developments in the theory of overconvergent F-isocrystals, the p-adic analogue of ℓ-adic lisse sheaves. For the most part of the talk I will focus on the parabolicity conjecture, a conjecture proposed by Crew in '92 on the algebraic monodromy groups of F-isocrystals. In the second part I will explain some applications of the conjecture to abelian varieties and Igusa varieties.

Le séminaire de Théorie de Nombres Paris–Londres est soutenu par l'Institut de Mathématiques de Jussieu–Paris Rive Gauche, le département de Mathématiques d'Orsay, le Laboratoire Analyse Géométrie Applications, l'ANR COLOSS, l'Heilbronn Institute for Mathematical Research